234

96-234

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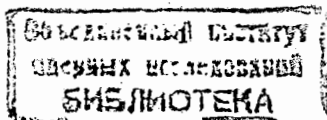
SECOND-CLASS CONSTRAINTS
AND LOCAL SYMMETRIES

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1 Introduction

In his basic works [1] on the generalized Hamiltonian formalism Dirac has shown that from the presence of first-class constraints in a theory the existence of the local symmetry group follows, a rank of which is determined by the number of first-class primary constraints. In the same place it is pointed out that,"possibly, all the first-class secondary constraints are to be attributed to a class of generators of transformations which are not related with a change of the physical state" (Dirac's hypothesis). In connection with the importance of constructing gauge transformations this hypothesis has brought about a rather excited discussion [2]-[13]. In papers [4]-[6] it is queried. And in refs.[14]-[16] one even asserts that second-class constraints contribute also to a generator of gauge transformations which become global in the absence of first-class constraints [14]. The generalized Hamiltonian dynamics of systems with constraints of first and second class is at all studied relatively weakly up to now. For example, only recently there have appeared the real schemes of separation of constraints into the first- and second-class ones [17]-[19]. Explicit form of the local-symmetry transformations is needed in both the traditional Dirac approach and, e.g., for realization of the presently popular BRST-BFV methods of covariant quantization [20]-[22].

In our previous papers [10]-[13], we have suggested a method of constructing the generator of gauge transformations for singular Lagrangians only with first-class constraints. At present, we extend our scheme also to the theories with second-class constraints; and moreover, in the given work we consider theories with an algebra of constraints of special form, when first-class primary constraints are the ideal of quasi-algebra of all the first-class constraints. A majority of the physically interesting theories satisfy this condition. The general case (without restrictions on the algebra of constraints) will be investigated in a subsequent paper. The local-symmetry transformations is looked for here from the requirement of a quasi-invariance (within a surface term) of the action functional under these transformations. To elucidate a role of second-class constraints

in the local-symmetry transformations, we consider first- and second-class constraints on the same basis in the hypothetical generator of these transformations. We prove that the second-class constraints do not contribute to the local-symmetry transformation law and, thus, the Dirac hypothesis holds true in this case (i.e. the transformation generator is a linear combination only of all the first-class constraints).

The paper is organized as follows. Section 2 is devoted to constructing the local-symmetry transformation generator in the theories with constraints of first and second class and to proving of that the latter are in no way responsible for this symmetry. These derivations are based substantially on results of our previous paper [19] (below cited as paper I) on the separation of constraints into the first- and second-class ones and on properties of the canonical set of constraints. In the 3rd section our results are examplified with a number of model Lagrangians [14], the Chern–Simons theory and spinor electrodynamics.

2 Local Symmetry Transformations

Let us consider a dynamical system with the canonical set $(\Phi_\alpha^{m_\alpha}, \Psi_{a_i}^{m_{a_i}})$ of first- and second-class constraints, respectively ($\alpha = 1, \cdots, F$, $m_\alpha = 1, \cdots, M_\alpha$; $a_i = 1, \cdots, A_i$, $m_{a_i} = 1, \cdots, M_{a_i}$, $i = 1, \cdots, n$). Passing to this set from the initial one is always possible in an arbitrary case by the method developed in paper I.

A group of phase-space coordinate transformations, that maps each solution of the Hamiltonian equations of motion into the solution of the same equations, will be called the symmetry transformation. Under these transformations the action functional is quasi-invariant within a surface term.

Consider the action

$$S = \int_{t_1}^{t_2} dt\, (p\dot{q} - H_T), \qquad (1)$$

where

$$H_T = H + u_\alpha \Phi_\alpha^1, \qquad (2)$$

$$H = H_c + \sum_{i=1}^{n} (\mathbf{K}^{1\ i})^{-1}_{b_i\ a_i} \{\Psi_{a_i}^i, H_c\} \Psi_{b_i}^1$$

is a first-class function [1], H_c is the canonical Hamiltonian, u_α are the Lagrange multipliers.

We shall require a quasi-invariance of the action S with respect to transformations:

$$\begin{cases} q_i' = q_i + \delta q_i, & \delta q_i = \{q_i, G\}, \\ p_i' = p_i + \delta p_i, & \delta p_i = \{p_i, G\}. \end{cases} \qquad (3)$$

The generator G will be looked for in the form

$$G = \varepsilon_\alpha^{m_\alpha} \Phi_\alpha^{m_\alpha} + \eta_{a_i}^{m_{a_i}} \Psi_{a_i}^{m_{a_i}}. \qquad (4)$$

In contrast to our previous works, in expression (4) for G the second term with constraints $\Psi_{a_i}^{m_{a_i}}$ is added, because we will elucidate a role of second-class constraints under these transformations.

So, under transformations (3) we have

$$\delta S = \int_{t_1}^{t_2} dt\, [\delta p\, \dot{q} + p\, \delta\dot{q} - \delta H_T] = \int_{t_1}^{t_2} dt\, \Big[\frac{d}{dt}(p\, \frac{\partial G}{\partial p} - G) + \frac{\partial G}{\partial t} + \{G, H_T\} \Big]. \quad (5)$$

From (5) we see: in order that the transformations (3) were the symmetry ones, it is necessary

$$\frac{\partial G}{\partial t} + \{G, H_T\} \overset{\Sigma_1}{=} 0. \qquad (6)$$

That the last equality must be realized on the primary-constraint surface Σ_1, can be easily interpreted if one remembers that the surface Σ_1 is the whole $(q, \dot{q})$-space image in the phase space. Since under the operation of the local-symmetry transformation group the $(q, \dot{q})$-space is being mapped into itself in a one-to-one manner, therefore the one-to-one mapping of Σ_1 into itself corresponds to this in the phase space. Therefore, at looking for the generator G it is natural to require also the primary-constraint surface Σ_1 to be conserved under transformations (3) and (4), i.e. the requirement (6) must be supplemented by the demands

$$\{G, \Psi_{a_i}^1\} \overset{\Sigma_1}{=} 0, \qquad (7)$$

$$\{G, \Phi_\alpha^1\} \overset{\Sigma_1}{=} 0, \qquad (8)$$

Note in connection with the relations (7) and (8) that the symmetry group of the action functional for dynamical system is the symmetry group of

the motion equations obtained from the variational principle (the inverse is incorrect in the general case). Since the constraint equations $\Phi_\alpha^1 = 0$ and $\Psi_{a_i}^1 = 0$ are contained in a system of the motion equations, the relations (7) and (8) are an expression of this group property [1].

Furhter we shall use the following Poisson brackets among the canonical constraint set (Φ, Ψ) and H established in paper I:

$$\{\Phi_\alpha^{m_\alpha}, H\} = g^{m_\alpha m_\beta}{}_\beta \, \Phi_\beta^{m_\beta}, \quad m_\beta = 1, \cdots, m_\alpha + 1, \tag{9}$$

$$\{\Psi_{a_i}^{m_{a_i}}, H\} = \bar{g}^{m_{a_i} m_\alpha}_{a_i \ \alpha} \, \Phi_\alpha^{m_\alpha} + \sum_{k=1}^{n} h^{m_{a_i} m_{b_k}}_{a_i \ \ b_k} \, \Psi_{b_k}^{m_{b_k}}, \quad m_{b_n} = m_{a_i} + 1, \tag{10}$$

$$\{\Phi_\alpha^{m_\alpha}, \Phi_\beta^{m_\beta}\} = f^{m_\alpha m_\beta m_\gamma}_{\alpha \ \beta \ \gamma} \, \Phi_\gamma^{m_\gamma}, \tag{11}$$

$$\{\Psi_{a_i}^{m_{a_i}}, \Psi_{b_k}^{m_{b_k}}\} = \bar{f}^{m_{a_i} m_{b_k} m_\gamma}_{a_i \ \ b_k \ \ \gamma} \, \Phi_\gamma^{m_\gamma} + \sum_{l=1}^{n} k^{m_{a_i} m_{b_k} m_{c_l}}_{a_i \ \ b_k \ \ c_l} \, \Psi_{c_l}^{m_{c_l}} + D^{m_{a_i} m_{b_k}}_{a_i \ \ b_k}, \tag{12}$$

where the structure functions, generally speaking, depend on q and p and, besides, one can see that

$$g^{m_\alpha m_\beta}_{\alpha \ \beta} = 0, \quad \text{if } m_\alpha + 2 \leq m_\beta, \tag{13}$$

$$\begin{cases} \bar{g}^{m_{a_i} m_\alpha}_{a_i \ \alpha} = 0, & \text{if } m_\alpha \geq m_{a_i}, \\[2mm] h^{m_{a_i} m_{b_k}}_{a_i \ \ b_k} = 0, & \text{if } m_{a_i} + 2 \leq m_{b_k} \text{ or if } a_i = b_k, \ m_{a_i} = M_{a_i}, \\[1mm] & m_{b_k} \geq M_{a_i}, \end{cases} \tag{14}$$

$$f^{1 \ m_\beta m_\gamma}_{\alpha \ \beta \ \gamma} = 0 \quad \text{for } m_\gamma \geq 2, \tag{15}$$

$$\begin{cases} F^{M_{a_i} - l \ l+1}_{a_i \ \cdot \ b_i} = (-1)^l \, F^{1 \ M_{b_i}}_{a_i \ b_i}, & l = 0, 1, \cdots, M_{a_i} - 1, \\[2mm] F^{j \ k}_{a_i \ b_i} = 0, & \text{if } j + k \neq M_{a_i} + 1, \\[2mm] F^{m_{a_i} m_{b_k}}_{a_i \ \ b_k} = 0, & \text{if } a_i, b_k \text{ refer to different chains (or doubled} \\[1mm] & \text{chains) of second-class constraints } \left(D^{m_{a_i} m_{b_k}}_{a_i \ \ b_k} \stackrel{\Sigma}{=} F^{m_{a_i} m_{b_k}}_{a_i \ \ b_k} \right). \end{cases} \tag{16}$$

The equality (15) reflects the first-class primary constraints to make a subalgebra of quasi-algebra of all the first-class constraints. The equalities

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[1] The relations of type (7) and (8) on the secondary constraints are not imposed in accordance with reasoning after formula (6).

(16) express partly the structure of the canonical second-class constraints established in paper I.

So, from eqs.(6) and (4) with taking account of (9)-(12) we write down

$$\left(\dot{\varepsilon}_\alpha^{m_\alpha} + \varepsilon_\beta^{m_\beta} g^{m_\beta m_\alpha}_{\beta \ \ \alpha} + \sum_{i=1}^{n} \eta_{a_i}^{m_{a_i}} \bar{g}^{m_{a_i} m_\alpha}_{a_i \ \ \alpha} \right) \Phi_\alpha^{m_\alpha}$$

$$+ \sum_{i=1}^{n} \left(\dot{\eta}_{a_i}^{m_{a_i}} + \sum_{k=1}^{n} \eta_{b_k}^{m_{b_k}} h^{m_{b_k} m_{a_i}}_{b_k \ \ a_i} \right) \Psi_{a_i}^{m_{a_i}} \tag{17}$$

$$+ u_\alpha \{G, \Phi_\alpha^1\} \stackrel{\Sigma_1}{=} 0.$$

Taking into consideration (8), we have

$$u_\alpha \{G, \Phi_\alpha^1\} \stackrel{\Sigma_1}{=} 0. \tag{18}$$

Then, in view of the functional independence of constraints $\Phi_\alpha^{m_\alpha}$ and $\Psi_{a_i}^{m_{a_i}}$, in order to satisfy the equality (17) one must demand the coefficients of constraints $\Phi_\alpha^{m_\alpha}$ $(m_\alpha \geq 2)$ and $\Psi_{a_i}^{m_{a_i}}$ $(m_{a_i} \geq 2)$ to vanish.

Before analyzing these conditions to satisfy the equality (17), let us consider in detail the conditions of the primary-constraints surface conservation starting from (7). Its realization would mean the presence of the following equalities:

$$\varepsilon_\alpha^{m_\alpha} \{\Phi_\alpha^{m_\alpha}, \Psi_{a_i}^1\} \stackrel{\Sigma_1}{=} 0, \qquad \sum_{k=1}^{n} \eta_{b_k}^{m_{b_k}} \{\Psi_{b_k}^{m_{b_k}}, \Psi_{a_i}^1\} \stackrel{\Sigma_1}{=} 0. \tag{19}$$

The first requirement (19) may be always realized by vanishing the Poisson brackets with the help of the corresponding transformation of equivalence as it is made in the previous section.

Since we take that passing to the canonical constraints set of paper I has been performed, in the second equality (19) for each value of a_i in the double sum over k and over b_k the only non-vanishing Poisson brackets are those at $b_k = a_i$, $M_{b_k} = i$, therefore

$$\eta_{a_i}^i = 0 \quad \text{for } i = 1, \cdots, n, \tag{20}$$

i.e. we have determined that in expression (4) the coefficients of those i-ary constraints, which are the final stage of each chain of second class constraints, and of those second-class primary constraints, which do not

generate the secondary constraints, disappear. Now we consider the requirement of vanishing the coefficients of constraints $\Psi_{a_i}^{m_{a_i}}$ ($m_{a_i} \geq 2$, $i = 2, \cdots, n$, $a_i = 1, \cdots, A_i$) in eq. (17):

$$
\begin{cases}
\dot{\eta}_{a_n}^n + \eta_{b_n}^n h_{b_n}^n {}_{a_n}^n + \eta_{b_n}^{n-1} h_{b_n}^{n-1} {}_{a_n}^n = 0, \\[6pt]
\dot{\eta}_{a_n}^{n-1} + \eta_{b_n}^n h_{b_n}^n {}_{a_n}^{n-1} + \eta_{b_n}^{n-1} h_{b_n}^{n-1} {}_{a_n}^{n-1} + \eta_{b_n}^{n-2} h_{b_n}^{n-2} {}_{a_n}^{n-1} = 0, \\[6pt]
\quad \vdots \\[6pt]
\dot{\eta}_{a_2}^2 + \eta_{b_2}^2 h_{b_2}^2 {}_{a_2}^2 + \eta_{b_2}^1 h_{b_2}^1 {}_{a_2}^2 = 0.
\end{cases}
\tag{21}
$$

In this system of equations the number of unknown functions exceeds the number of equations by the number of the second-class primary constraints which make up the constraint chains. However, we have already established the result (20) for senior terms of each subsystem of equations for a_i. Inserting the values $\eta_{a_n}^n = 0$ into the first line of system (21), we obtain a system of A_n algebraic linear homogeneous equations for A_n unknowns $\eta_{b_n}^{n-1}$ that has only a trivial solution $\eta_{b_n}^{n-1} = 0$ since $\det\|h_{b_n}^{n-1} {}_{a_n}^n\| \neq 0$ (see below). Using this result in the second line of (21), we obtain a system of analogous equations for unknowns $\eta_{b_n}^{n-2}$. Its solution is $\eta_{b_n}^{n-2} = 0$ since $\det\|h_{b_n}^{n-2} {}_{a_n}^{n-1}\| \neq 0$. Continuing successively this process we shall deduce that all quantities $\eta_{a_i}^{m_{a_i}}$ vanish, i.e. the second-class constraints do not contribute to the generator of local-symmetry transformations.

Now we shall show that

$$
\det\|h_{b_i}^{i-k-1} {}_{a_i}^{i-k}\| \neq 0
\tag{22}
$$

since a set of all constraints consists of independent functions. We shall apply a method by contradiction, i.e. suppose the indicated determinant to vanish. Further from the relation (10) we have

$$
\{\Psi_{a_i}^{i-k-1}, H_c\} \overset{\Sigma_{i-k-1}}{=} h_{a_i}^{i-k-1} {}_{b_m}^{i-k} \Psi_{b_m}^{i-k}
\tag{23}
$$

where Σ_{i-k-1} is the surface of all constraints up to and including the $i-k-1$ stage. But the assumption of vanishing the determinant of the matrix $\|h_{b_i}^{i-k-1} {}_{a_i}^{i-k}\|$ means a linear dependence of its some rows or columns:

$$
h_{b_i}^{i-k-1} {}_{a_m}^{i-k} = C_{a_i s_i} h_{s_i}^{i-k-1} {}_{b_m}^{i-k}.
\tag{24}
$$

Inserting (24) into the right-hand side of (23) we obtain

$$
\{\Psi_{a_i}^{i-k-1}, H_c\} \overset{\Sigma_{i-k-1}}{=} C_{a_i s_i} h_{s_i}^{i-k-1} {}_{b_m}^{i-k} \Psi_{b_m}^{i-k} \overset{\Sigma_{i-k-1}}{=} \{C_{a_i s_i} \Psi_{s_i}^{i-k-1}, H_c\}.
$$

From here write down

$$
\{\Psi_{a_i}^{i-k-1} - C_{a_i s_i} \Psi_{s_i}^{i-k-1}, H_c\} \overset{\Sigma_{i-k-1}}{=} 0.
$$

The last equality means that

$$
\Psi_{a_i}^{i-k-1} = C_{a_i s_i} \Psi_{s_i}^{i-k-1} + c_{a_i}^{m_{a_i}} \Psi_{a_i}^{i-m_{a_i}} + d_\alpha^{m_\alpha} \Phi_\alpha^{i-m_\alpha},
$$

$$
m_{a_i} = k+1, \cdots, i-1, \qquad m_\alpha = k+1, \cdots, i-1,
$$

where $c_{a_i}^{m_{a_i}}$ and $d_\alpha^{m_\alpha}$ are arbitrary functions of q and p, non-vanishing on the constraint surface Σ_{i-k-1}. Thus, we have arrived at the contradiction with the condition of independence of constraints. This proves the validity of (22).

Returning to the second condition (8) of the primary-constraint surface conservation under local-symmetry transformations, we see that it (and from here, too, the equality (18)) will be fulfilled if

$$
\{\Phi_\alpha^1, \Phi_\beta^{m_\beta}\} = f_\alpha {}_\beta^{m_\beta} {}_\gamma^1 \Phi_\gamma^1.
\tag{25}
$$

This relation emerged already earlier [10] in the case of dynamical systems only with the constraints of first class and ensured the conservation of the primary-constraint surface Σ_1 under the local-symmetry transformations. Here it means a quasi-algebra of special form where the first-class primary constraints make an ideal of quasi-algebra formed by all first-class constraints, also in the presence of second-class constraints.

To determine the multipliers $\varepsilon_\alpha^{m_\alpha}$ in the generator (4), now we have only the requirement of vanishing the coefficients of constraints $\Phi_\alpha^{m_\alpha}$ in (17) [10]:

$$
\dot{\varepsilon}_\alpha^{m_\alpha} + \varepsilon_\beta^{m_\beta} g_\beta^{m_\beta m_\alpha} {}_\alpha = 0, \qquad m_\beta = m_\alpha - 1, \cdots, M_\alpha.
\tag{26}
$$

In the system of equations (26), the number of unknowns exceeds the number of equations by the number $F = A - \sum_{i=1}^n A_i$ of the first-class primary constraints, therefore the system (26) may be solved to within F arbitrary functions. This proves that the rank of the local-symmetry transformation quasigroup is defined by the number of first-class primary

constraints also in the presence of second-class constraints. We shall remind, for completeness, how one make use of this system of equations [11]. We write down (26) as

$$\dot{\varepsilon}_{\alpha}^{M_\alpha} + \varepsilon_{\beta}^{M_\alpha} g_{\beta}^{M_\alpha \ M_\alpha}{}_\alpha + \varepsilon_{\beta}^{M_\alpha-1} g_{\beta}^{M_\alpha-1 \ M_\alpha}{}_\alpha = 0,$$

$$\dot{\varepsilon}_{\alpha}^{M_\alpha-1} + \varepsilon_{\beta}^{M_\alpha} g_{\beta}^{M_\alpha \ M_\alpha-1}{}_\alpha + \varepsilon_{\beta}^{M_\alpha-1} g_{\beta}^{M_\alpha-1 \ M_\alpha-1}{}_\alpha + \varepsilon_{\beta}^{M_\alpha-2} g_{\beta}^{M_\alpha-2 \ M_\alpha-1}{}_\alpha = 0,$$

$$\vdots \tag{27}$$

$$\dot{\varepsilon}_{\alpha}^{2} + \varepsilon_{\beta}^{M_\alpha} g_{\beta}^{M_\alpha \ 2}{}_\alpha + \varepsilon_{\beta}^{M_\alpha-1} g_{\beta}^{M_\alpha-1 \ 2}{}_\alpha + \varepsilon_{\beta}^{1} g_{\beta}^{1 \ 2}{}_\alpha = 0,$$

$$\alpha, \beta = 1, \cdots, F.$$

Taking $\varepsilon_\alpha \equiv \varepsilon_\alpha^{M_\alpha}$ as arbitrary functions and inserting them into the first line of system (27) we obtain a system of F inhomogeneous algebraic linear equations for F unknowns $\varepsilon_{\beta}^{M_\alpha-1}(\beta = 1, \cdots, F)$. Solving this system of equations (we have $\det \|g_{\beta}^{M_\alpha-1 \ M_\alpha}{}_\alpha\| \neq 0$ [11]) and inserting this result into the second line of (27), we obtain again a system of F inhomogeneous algebraic linear equations for F unknowns $\varepsilon_{\beta}^{M_\alpha-2}$ that must be solved ($\det\|g_{\beta}^{M_\alpha-2 \ M_\alpha-1}{}_\alpha\| \neq 0$). The result must be inserted into the following line of system (27), etc., up to the last line which gives a system of F equations for F unknowns ε_{β}^{1}. Solving this last system of equations we shall express, finally, all $\varepsilon_{\alpha}^{m_\alpha}$ in terms of $\varepsilon_\alpha(t), g_{\beta}^{m_\beta m_\alpha}{}_\alpha$ and their derivatives:

$$\varepsilon_{\alpha}^{m_\alpha} = B_{\alpha}^{m_\alpha m_\beta}{}_\beta \varepsilon_{\beta}^{(M_\alpha-m_\beta)}, \qquad m_\beta = m_\alpha, \cdots, M_\alpha \tag{28}$$

(in formula (28) the summation runs also over m_β), where

$$\varepsilon_{\beta}^{(M_\alpha-m_\beta)} \equiv \frac{d^{M_\alpha-m_\beta}}{dt^{M_\alpha-m_\beta}} \varepsilon_\beta(t), \qquad \varepsilon_\beta(t) \equiv \varepsilon_{\beta}^{M_\beta}$$

and $B_{\alpha}^{m_\alpha m_\beta}{}_\beta$ are, generally speaking, functions of q and p and their derivatives up to the order $M_\alpha - m_\alpha - 1$. Note that the condition

$$\det\|g_{\beta}^{M_\alpha-k-1 \ M_\alpha-k}{}_\alpha\| \neq 0 \quad (k = 0, 1, \cdots, M_\alpha - 2),$$

which is needed for the system of equations (26) to have a solution, is proved as a consequence of the functional independence of all constraints – in the same way as in the case of dynamical systems with the constraints of first class only [10], and in the same way as the similar condition (22)

for the system of equations (21). So, the generator of the local-symmetry transformations takes the form

$$G = B_{\alpha}^{m_\alpha m_\beta}{}_\beta \phi_{\alpha}^{m_\alpha} \varepsilon_{\beta}^{(M_\alpha-m_\beta)}, \qquad m_\beta = m_\alpha, \cdots, M_\alpha. \tag{29}$$

The obtained generator (29) satisfies the group property

$$\{G_1, G_2\} = G_3, \tag{30}$$

where the transformation G_3 (29) is realized by carrying out two successive transformations G_1 and G_2 (29). The amount of group parameters $\varepsilon_\alpha(t)$ which determine the rank of the quasigroup of these transformations equal the number of primary constraints of first class. As can be seen from formula (29), the transformation law may include both arbitrary functions $\varepsilon_\alpha(t)$ and their derivatives up to and including the order $M_\alpha - 1$; the highest derivatives $\varepsilon_\alpha^{(M_\alpha-1)}$ should be always present.

Thus, we have derived the generator of the local-symmetry transformations and proved that there is no influence of second-class constraints on these transformations from requirements of the action quasi-invariance and of conservation of the primary-constraint surface under local-symmetry transformations and on the basis of properties of the completely-separated (into first- and second-class) constraint set.

Notice that the corresponding transformations of local symmetry in the Lagrangian formalism are determined by following way:

$$\delta q_i(t) = \{q_i(t), G\}\Big|_{p=\frac{\partial L}{\partial \dot{q}}}, \qquad \delta\dot{q}(t) = \frac{d}{dt}\delta q(t). \tag{31}$$

3 Examples

In this section we illustrate our results by a number of examples in both finite- and infinite-dimensional cases.

1. Consider the Lagrangian [14]

$$L = (\dot{q}_1 + \dot{q}_2)q_3 + \frac{1}{2}\dot{q}_3^2 - \frac{1}{2}q_2^2. \tag{32}$$

The generalized momenta are of the form: $p_1 = q_3$, $p_2 = q_3$, $p_3 = \dot{q}_3$. Therefore we have two primary constraints:

$$\phi_1^1 = p_1 - q_3, \qquad \phi_2^1 = p_2 - q_3. \tag{33}$$

The total Hamiltonian gets the form:

$$H_T = \frac{1}{2}(p_3{}^2 + q_2{}^2) + u_1\phi_1^1 + u_2\phi_2^1. \qquad (34)$$

The self-consistency conditions of theory give

$$\dot\phi_1^1 = \{\phi_1^1, H_T\} = -p_3, \qquad \dot\phi_2^1 = \{\phi_2^1, H_T\} = -q_2 - p_3,$$

i.e. two secondary constraints

$$\phi_1^2 = p_3, \qquad \phi_2^2 = p_3 - q_2, \qquad (35)$$

and

$$\dot\phi_1^2 = \{\phi_1^2, H_T\} = u_1 + u_2 = 0, \qquad \dot\phi_2^1 = \{\phi_2^1, H_T\} = u_2 + u_1 + u_2 = 0,$$

that means $u_1 = u_2 = 0$. Two last equations serve for determining the Lagrangian multipliers u_1 and u_2, and there no longer arise constraints. Let us calculate the matrix $\mathbf{W} = \|K^{m_\alpha m_\beta}\| = \left\|\{\phi_\alpha^{m_\alpha}, \phi_\beta^{m_\beta}\}\right\|$:

$$\mathbf{W} = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & -2 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \end{pmatrix}. \qquad (36)$$

We see that $\mathrm{rank}\,\mathbf{W} = 4$, i.e. all constraints are of second class, therefore $\mathbf{W}$ have a quasidiagonal (antisymmetric) form. Performing our procedure we shall pass to the equivalent canonical set of constraints $\Psi_a^{m_a}$ according to the formula (57) of paper I:

$$\begin{pmatrix} \Psi_1^1 \\ \Psi_2^1 \\ \Psi_1^2 \\ \Psi_2^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} \phi_1^1 \\ \phi_2^1 \\ \phi_1^2 \\ \phi_2^2 \end{pmatrix} = \begin{pmatrix} p_1 - q_3 \\ p_1 - p_2 \\ p_3 \\ -q_2 \end{pmatrix}. \qquad (37)$$

For the last set of constraints the quasidiagonal form of $\mathbf{W}$ will have a canonical structure:

$$\mathbf{W}' = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \qquad (38)$$

Now for quasi-invariance of the action with respect to transformations (3) with generator (4)

$$G = \eta_1^1\,\Psi_1^1 + \eta_2^1\,\Psi_2^1 + \eta_1^2\,\Psi_1^2 + \eta_2^2\,\Psi_2^2,$$

it is necessary to realize the condition (7) of conservation of the primary-constraint surface under these transformations

$$\{G, \Psi_a^1\} \overset{\Sigma_1}{=} 0, \qquad a = 1, 2. \qquad (39)$$

From (39) we obtain $\eta_1^2 = \eta_2^2 = 0$. Next from (21) we establish $\eta_1^1 = \eta_2^1 = 0$, i.e. the second-class constraints of system (37) generate the transformations of neither local symmetry nor global one.

2. Consider the Lagrangian [14]

$$L = \dot q_1 q_2 - \dot q_2 q_1 - (q_1 - q_2)q_3. \qquad (40)$$

Then passing to the Hamiltonian formalism we obtain the generalized momenta $p_1 = q_2$, $p_2 = -q_1$, $p_3 = 0$ and, thus, three primary constraints:

$$\phi_1^1 = p_1 - q_2, \qquad \phi_2^1 = p_2 + q_1, \qquad \phi_3^1 = p_3. \qquad (41)$$

The total Hamiltonian gets the form:

$$H_T = (q_1 - q_2)q_3 + u_1\phi_1^1 + u_2\phi_2^1 + u_3\phi_3^1. \qquad (42)$$

From the self-consistency conditions of the theory we obtain

$$\dot\phi_1^1 = -q_3 - 2u_2 = 0, \quad \dot\phi_2^1 = q_3 + 2u_1 = 0, \quad \dot\phi_3^1 = -q_1 + q_2 = 0. \qquad (43)$$

Two first equations (43) serve for determining the Lagrangian multipliers: $u_1 = u_2 = -\frac{1}{2}q_3$. The last relation (43) gives the secondary constraint

$$\phi_3^2 = q_2 - q_1,$$

and there no longer arise constraints. Let us calculate the matrix $\mathbf{W} = \left\|\{\phi_\alpha^{m_\alpha}, \phi_\beta^{m_\beta}\}\right\|$:

$$\mathbf{W} = \begin{pmatrix} 0 & -2 & 0 & 1 \\ 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}. \qquad (44)$$

From rank $\mathbf{W} = 2$ we conclude that two constraints are of second class and two ones are of first class. With the help of our procedure we separate constraints into those of first and second class. For this purpose, by means of the equivalence transformation we pass to the canonical set of constraints according to the formula (57) of paper I:

$$\begin{pmatrix} \Psi_1^1 \\ \Psi_2^1 \\ \Phi_1^1 \\ \Phi_1^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} \phi_1^1 \\ \phi_2^1 \\ \phi_3^1 \\ \phi_3^2 \end{pmatrix} = \begin{pmatrix} p_1 - q_2 \\ p_2 + q_1 \\ p_3 \\ p_1 + p_2 + q_2 - q_1 \end{pmatrix}. \quad (45)$$

For the last set of constraints the matrix $\mathbf{W}$ acquires the canonical form:

$$\mathbf{W}' = \begin{pmatrix} 0 & -2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (46)$$

Further we look for the generator G in the form (4):

$$G = \eta_1^1\, \Psi_1^1 + \eta_2^1\, \Psi_2^1 + \varepsilon_1^1\, \Phi_1^1 + \varepsilon_1^2\, \Phi_1^2. \quad (47)$$

From the second condition (7) of conservation of the primary-constraint surface Σ_1 under transformations (3) we derive $\eta_1^1 = \eta_2^1 = 0$, i.e. the second-class constraints of the system do not contribute to the generator G. The first condition (8) of conservation of Σ_1 is realized because

$$\{\Phi_1^1, \Phi_1^2\} = 0.$$

If we take into account that $g_1^{1\,2} = \frac{1}{2}$ and $g_1^{2\,2} = 0$ in (9), equation (26) becomes

$$\dot{\varepsilon}_1^2 + \frac{1}{2}\varepsilon_1^1 = 0.$$

Denoting $\varepsilon_1^2 \equiv \varepsilon$, we obtain $\varepsilon_1^1 = -2\dot{\varepsilon}$ and, therefore,

$$G = -2\dot{\varepsilon}p_3 + \varepsilon(p_1 + p_2 + q_2 - q_1),$$

which gives $\delta q_1 = \varepsilon$, $\delta q_2 = \varepsilon$, $\delta q_3 = -2\dot{\varepsilon}$, $\delta p_1 = \varepsilon$, $\delta p_2 = -\varepsilon$, $\delta p_3 = 0$. In the $(q,\dot{q})$-space the local-symmetry transformations are established with the help of formulas (31). It is easy to verify that the action is invariant with respect to the transformations generated by G. This is a consequence of the constraints being linear in the momentum variables.

3. We now look at the infinite-dimensional cases. We consider first a Chern–Simons theory. Theories of such type describe, e.g., the fractional quantum Hall effect and other phenomena.

The Lagrangian density for a complex field ϕ interacting with an Abelian Chern–Simons field is [26]

$$\mathcal{L} = (\partial_\mu + iA_\mu)\varphi^*(\partial^\mu - iA^\mu)\varphi + \frac{\alpha}{4\pi}\varepsilon_{ij}\Big(A_0\partial_i A_j + \dot{A}_i A_j + A_i\partial_j A_0\Big), \quad (48)$$

where $i,j = 1,2$ and $\mu = 0,1,2$. The generalized momenta are

$$\pi_0 = \frac{\partial\mathcal{L}}{\partial\dot{A}_0} = 0, \qquad \pi_i = \frac{\partial\mathcal{L}}{\partial\dot{A}_i} = \frac{\alpha}{4\pi}\varepsilon_{ij}A_j,$$

$$\pi_\varphi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = (\partial_0 + iA_0)\varphi^*, \qquad \pi_{\varphi^*} = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}^*} = (\partial_0 - iA_0)\varphi.$$

Therefore, in the phase space we have three primary constraints:

$$\phi_i^1 = \pi_i - \frac{\alpha}{4\pi}\varepsilon_{ij}A_j, \quad i,j = 1,2, \qquad \phi_3^1 = \pi_0 \quad (49)$$

and the canonical Hamiltonian:

$$\begin{aligned} H_c = {} & \int d^2x\Big[\pi_\varphi(x)\pi_{\varphi^*}(x) + (\partial_i + iA_i(x))\varphi^*(x)(\partial_i - iA_i(x))\varphi(x) \\ & + A_0(x)j_\varphi - \frac{\alpha}{4\pi}\varepsilon_{ij}\Big(A_0(x)\partial_i A_j(x) + A_i(x)\partial_j A_0(x)\Big)\Big], \quad (50) \end{aligned}$$

where $j_\varphi = i\big(\varphi(x)\pi_\varphi(x) - \varphi^*(x)\pi_{\varphi^*}(x)\big)$.

Among the conditions of the time conservation of constraints $\dot{\phi}_i^1 = 0$ $(i = 1,2)$ and $\dot{\phi}_3^1 = 0$ two first ones serve for determining the Lagrangian multipliers u_1 and u_2:

$$u^1 = \frac{4\pi}{\alpha}\big[i(\varphi\partial_2\varphi^* - \varphi^*\partial_2\varphi) - 2\varphi^*\varphi A_2\big] - 2\partial_1 A_0,$$

$$u^2 = \frac{4\pi}{\alpha}\big[i(\varphi^*\partial_1\varphi - \varphi\partial_1\varphi^*) + 2\varphi^*\varphi A_1\big] - 2\partial_2 A_0.$$

From the condition of conservation for ϕ_3^1 we obtain the secondary constraint

$$\phi_3^2 = j_\varphi - \frac{\alpha}{2\pi}\varepsilon_{ij}\partial_i A_j, \quad (51)$$

and there do not arise more constraints.

The only nonvanishing Poisson brackets among the constraints are

$$\{\phi_i^1(x), \phi_j^1(y)\} = -\frac{\alpha}{2\pi}\varepsilon_{ij}\delta(x-y), \quad \{\phi_i^1(x), \phi_3^2(y)\} = \frac{\alpha}{2\pi}\varepsilon_{mi}\partial_m\delta(x-y).$$

Therefore, the matrix $\mathbf{W} = \left\|\{\phi_\alpha^{m_\alpha}, \phi_\beta^{m_\beta}\}\right\|$ takes the form:

$$\mathbf{W} = \frac{\alpha}{2\pi}\begin{pmatrix} 0 & -1 & 0 & -\partial_2 \\ 1 & 0 & 0 & \partial_1 \\ 0 & 0 & 0 & 0 \\ \partial_2 & -\partial_1 & 0 & 0 \end{pmatrix}\delta(x-y). \tag{52}$$

From $\text{rank}\mathbf{W} = 2$ we conclude that two constraints are of second class and the two ones are of first class.

With the help of the transformation

$$\bar{\phi}_3^2 = \phi_3^2 + c_1\phi_1^1 + c_2\phi_2^1$$

we shall satisfy the equality $\{\bar{\phi}_3^2, \phi_i^1\} = 0$ if $c_i = -\partial_i$.

Thus, we obtain the canonical set of constraints: $\Psi_1^1 = \phi_1^1$, $\Psi_2^1 = \phi_2^1$, $\Phi_1^1 = \phi_3^1$, $\Phi_1^2 = \bar{\phi}_3^2 = j_\varphi$, separated into the ones of first and second class, since now the matrix $\mathbf{W}$ has the form:

$$\mathbf{W}' = \frac{\alpha}{2\pi}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\delta(x-y).$$

Further, we seek the generator G in the form

$$G = \int d^2x\left[\eta_1^1\,\Psi_1^1 + \eta_2^1\,\Psi_2^1 + \varepsilon_1^1\,\Phi_1^1 + \varepsilon_1^2\,\Phi_1^2\right]. \tag{53}$$

From the second condition (7) of conservation of Σ_1 under transformations (3) we derive $\eta_1^1 = \eta_2^1 = 0$, i.e. the constraints of second class do not contribute to G. The first condition (8) of conservation of Σ_1 is realized because

$$\{\Phi_1^1, \Phi_1^2\} = 0.$$

Since $g_{1\ 1}^{1\ 2} = 1$ and $g_{1\ 1}^{2\ 2} = 0$ in (9), eq.(26) accepts the form:

$$\dot{\varepsilon}_1^2 + \varepsilon_1^1 = 0,$$

i.e. $\varepsilon_1^1 = -\dot{\epsilon}(x)$ where $\epsilon(x) \equiv \varepsilon_1^2$. Therefore we obtain

$$G = \int d^2x\left\{-\dot{\epsilon}\pi_0 + \epsilon\left[\,i(\varphi\pi_\varphi - \varphi^*\pi_{\varphi^*}) - \partial_i\pi_i\right]\right\}, \tag{54}$$

from which it is easily to derive the local-symmetry transformations in the phase space:

$$\begin{array}{ll} \delta\varphi(x) = i\epsilon(x)\varphi(x), & \delta\pi_\varphi(x) = -i\epsilon(x)\pi_\varphi(x), \\ \delta\varphi^*(x) = -i\epsilon(x)\varphi^*(x), & \delta\pi_{\varphi^*}(x) = i\epsilon(x)\pi_{\varphi^*}(x), \\ \delta A_0(x) = \dot{\epsilon}(x), & \delta\pi_0(x) = 0, \\ \delta A_i(x) = \partial_i\epsilon(x), & \delta\pi_i(x) = 0. \end{array} \tag{55}$$

With the help of (31) it is easily to write the local-symmetry transformations in the $(q, \dot{q})$-space and to obtain that $\delta\mathcal{L} = \partial_\mu\left[\frac{\alpha}{4\pi}\varepsilon^{\mu\nu\lambda}\epsilon(x)\partial_\nu A_\lambda\right]$, i.e. the theory is quasi-invariant under obtained transformations.

4. Now we consider the well-known case of spinor electrodynamics:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma_\mu(\partial_\mu - ieA_\mu)\psi - m\overline{\psi}\psi \tag{56}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Here $A_\mu, \psi, \overline{\psi}$ play the role of the generalized coordinates. The generalized momenta are

$$\pi_\mu = \frac{\partial L}{\partial \dot{A}_\mu} = F_{0\mu}, \quad p_\psi = \frac{\partial L}{\partial \dot{\psi}} = i\overline{\psi}\gamma_0, \quad p_{\overline{\psi}} = \frac{\partial L}{\partial \dot{\overline{\psi}}} = 0,$$

from which we have three primary constraints:

$$\phi_1^1 = \pi_0, \qquad \phi_2^1 = p_\psi - i\overline{\psi}\gamma_0, \qquad \phi_3^1 = p_{\overline{\psi}} \tag{57}$$

and the total Hamiltonian:

$$H_T = \int d^3x\left[\frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}\pi^i\pi^i + \pi_i\partial_iA_0 + iep_\psi A_0\psi \right.$$
$$\left. + i\overline{\psi}\gamma_i(\partial_i - ieA_i)\psi + m\overline{\psi}\psi + u_1\phi_1^1 + u_2\phi_2^1 + u_3\phi_3^1\right]. \tag{58}$$

Among the conditions of the constraint conservation in time $\dot{\psi}_i^1 = 0$ ($i = 1, 2, 3$) the two last ones serve for determining the Lagrangian multipliers u_2 and u_3. From the first condition we obtain one secondary constraint

$$\phi_1^2 = \partial_i\pi^i - iep_\psi\psi,$$

and there do not arise more constraints. Calculating the matrix $\mathbf{W} = \left\| \{ \phi_\alpha^{m_\alpha}, \phi_\beta^{m_\beta} \} \right\|$:

$$\mathbf{W} = \delta(x - x') \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & i\gamma_0 & -iep_\psi \\ 0 & i\gamma_0 & 0 & -e\gamma_0\psi \\ 0 & iep_\psi & e\gamma_0\psi & 0 \end{pmatrix}, \tag{59}$$

we see that $\mathrm{rank}\,\mathbf{W} = 2$; therefore, two constraints are of second class and the two ones are of first class. Now implementing our procedure, we shall pass to the canonical set of constraints by the equivalence transformation:

$$\begin{pmatrix} \Psi_1^1 \\ \Psi_2^1 \\ \Phi_1^1 \\ \Phi_1^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ ie\psi & -ie\overline{\psi} & 0 & 1 \end{pmatrix} \begin{pmatrix} \phi_2^1 \\ \phi_3^1 \\ \phi_1^1 \\ \phi_1^2 \end{pmatrix} = \begin{pmatrix} p_\psi - i\overline{\psi}\gamma_0 \\ p_{\overline{\psi}} \\ \pi_0 \\ \partial_i \pi^i - ie(p_\psi\psi + \overline{\psi}p_{\overline{\psi}}) \end{pmatrix}$$

where the constraints are already separated into the ones of first and second class, since now the matrix $\mathbf{W}$ has the form:

$$\mathbf{W}' = \delta(x - x') \begin{pmatrix} 0 & i\gamma_0 & 0 & 0 \\ i\gamma_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Further, we look for the generator G in the form

$$G = \int d^3x \left[\eta_1^1\,\Psi_1^1 + \eta_2^1\,\Psi_2^1 + \varepsilon_1^1\,\Phi_1^1 + \varepsilon_1^2\,\Phi_1^2 \right]. \tag{60}$$

From the second condition (7) of conservation of the primary-constraints surface Σ_1 under transformations (3) we derive $\eta_1^1 = \eta_2^1 = 0$, i.e. the constraints of second class do not contribute to G. The first condition (8) of conservation of Σ_1 is realized because

$$\{ \Phi_1^1, \Phi_1^2 \} = 0.$$

Taking into account that $g_1^{1\,2} = -1$ and $g_1^{2\,2} = 0$ in (9), eq.(26) accepts the form:

$$\dot{\varepsilon}_1^2 - \varepsilon_1^1 = 0,$$

i.e. $\varepsilon_1^1 = \dot{\varepsilon}$ where $\varepsilon \equiv \varepsilon_1^2$. Therefore we have

$$G = \int d^3x \left\{ \dot{\varepsilon}\pi_0 + \varepsilon \left[\partial_i\pi^i - ie(p_\psi\psi + \overline{\psi}p_{\overline{\psi}}) \right] \right\},$$

from which it is easily to obtain the gauge transformations in the phase space and well-known transformation rule:

$$\delta A_\mu = \partial_\mu\varepsilon, \quad \delta\psi = ie\varepsilon\psi, \quad \delta\overline{\psi} = -ie\varepsilon\overline{\psi}.$$

4 Conclusion

Constrained special-form theories with first- and second-class constraints, when the first-class primary constraints are the ideal of quasi-algebra of all the first-class constraints, are considered. One must say that this restriction on the algebra of constraints is fulfilled in most of the physically interesting theories, e.g., in electrodynamics, in the Yang – Mills theories, etc., and it has been used by us in previous works [10] in the case of dynamical systems only with the first-class constraints and also by other authors in obtaining gauge transformations on the basis of different approaches [3, 4, 15, 24, 25].

Here in the framework of generalized Hamiltonian formalism by Dirac for systems with first- and second-class constraints we have suggested the method of constructing the generator of local-symmetry transformations in both phase and configuration space. The generator is derived from the requirement of quasi-invariance (within a surface term) of the action functional (in the phase space) under desired transformations which must be supplemented by the demand on the primary-constraints surface Σ_1 to be conserved at these transformations. Necessity of second requirement can be seen from following reasoning. Because Σ_1 is whole $(q, \dot{q})$-space image in the phase space and under operation of the local-symmetry transformation group the $(q, \dot{q})$-space is being mapped into itself in a one-to-one manner, then one-to-one mapping of Σ_1 into itself corresponds to this in the phase space.

Note that the condition of the Σ_1 conservation actually is not the additional restriction on the properties of the local-symmetry transformation generator. It naturally follows from definition of the symmetry group of the action functional (see the explanation after relation (8)).

We have proved, that Dirac's hypothesis [1] (see Introduction) holds true in general case also at presence of second-class constraints and these mentioned constraints do not contribute to the local-symmetry transformation law and do not generate global transformations in lack of first-class constraints.

The corresponding transformations of local symmetry in the $(q, \dot{q})$-space are determined with the help of formulae (31).

When deriving the local-symmetry transformation generator the employment of obtained equation system (26) is important, the solution of which manifests a mechanism of appearance of higher derivatives of coordinates and group parameters in the Noether transformation law in the configuration space, the highest possible order of coordinate derivatives being determined by the structure of the first-class constraint algebra, and the order of the highest derivative of group parameters in the transformation law being by unity smaller than the number of stages in deriving secondary constraints of first class by the Dirac procedure. The arising problem of canonicity of transformations in the phase space in the presence of higher derivatives of coordinates and momenta will be considered in our subsequent paper.

So, we can state in the case of special-form theories with first- and second-class constraints that the necessary and sufficient condition for certain quantity G to be the local-symmetry transformation generator is the representation of G as the linear combination of all the first-class constraints (and only of them) with the coefficients determined by the system of equations (26).

Obtained generator (29) satisfies the group property (30). The amount of group parameters, which determine a rank of quasigroup of these transformations, equals to the number of primary constraints of first class.

As it is known, gauge-invariant theories belong to the class of degenerate theories. In this paper we have shown that the degeneracy of special-form theories with the first- and second-class constraints is due to their quasi-invariance under local-symmetry transformations.

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Читая Н.П., Гогилидзе С.А., Суровцев Ю.С.   E2-96-234
Связи второго рода и локальные симметрии

В обобщенном гамильтоновом формализме Дирака исследуются локальные симметрии систем со связями первого и второго рода. Предложен метод конструирования генератора преобразований локальной симметрии для теорий с алгеброй связей специального вида (к которым относится большинство физически интересных теорий) из требования квазиинвариантности действия относительно этих преобразований. Доказано, что связи второго рода не дают вклада в закон преобразований локальной симметрии, который полностью определяется всеми связями первого рода. Тем самым показано, что вырожденность теорий специального вида со связями первого и второго рода обусловлена их инвариантностью относительно преобразований локальной симметрии.

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Chitaia N.P., Gogilidze S.A., Surovtsev Yu.S.   E2-96-234
Second-Class Constraints and Local Symmetries

In the framework of the generalized Hamiltonian formalism by Dirac, the local symmetries of dynamical systems with first- and second-class constraints are investigated. For theories with an algebra of constraints of special form (to which a majority of the physically interesting theories belongs) the method of constructing the generator of local-symmetry transformations is obtained from the requirement of the quasi-invariance of an action. It is proved that second-class constraints do not contribute to the transformation law of the local symmetry which entirely is stipulated by all the first-class constraints. It is thereby shown that degeneracy of special form theories with the first- and second-class constraints is due to their quasi-invariance under local-symmetry transformations.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.