ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-96-156

A.V.Sidorov[1], D.B.Stamenov[2]

CONSTRAINTS ON «SECOND-ORDER FIXED POINT» QCD FROM THE CCFR DATA ON DEEP INELASTIC NEUTRINO-NUCLEON SCATTERING

Submitted to «Physics Letters B»

[1]E-mail: sidorov@thsun1.jinr.dubna.su
[2]E-mail: stamenov@bgearn.acad.bg
Institute for Nuclear Research and Nuclear Energy Bulgarian Academy of Sciences
Boul. Tsarigradsko chaussee 72, Sofia 1784, Bulgaria

1996

1. Introduction

The success of perturbative Quantum Chromodynamics (QCD) in the description of the high energy physics of strong interactions is considerable. The QCD predictions are in good quantitative agreement with a great number of data on lepton-hadron and hadron-hadron processes in a large kinematic region (e.g. see reviews [1] and references therein). Despite this success of QCD, we consider that it is useful and reasonable to put the question: Do the present data fully exclude the so-called *fixed point* (FP) theory models [2] ?

We remind that these models are not asymptotically free. The effective coupling constant $\alpha_s(Q^2)$ approaches a constant value $\alpha_0 \neq 0$ as $Q^2 \to \infty$ (the so-called fixed point at which the Callan- Symanzik β-function $\beta(\alpha_0) = 0$). Using the assumption that α_0 is *small*, one can make predictions for the physical quantities in the high energy region, as like in QCD, and confront them to the experimental data. This test of FP theory models has been made [3, 4] by using the data of deep inelastic lepton-nucleon experiments started by the SLAC-MIT group [5] at the end of the sixties and performed in the seventies [6]. It was shown that

i) the predictions of the FP theory models with *scalar* and *non- colored (Abelian) vector* gluons *do not agree* with the data

ii) the data *cannot distinguish* between different forms of scaling violation predicted by QCD and the so-called *Fixed point* QCD (FP-QCD), a theory with *colored vector* gluons, in which the effective coupling constant $\alpha_s(Q^2)$ does not vanish when Q^2 tends to infinity.

We think there are two reasons to discuss again the predictions of FP-QCD. First of all, there is the evidence from the non-perturbative lattice calculations [7] that the β- function in QCD vanishes at a nonzero coupling α_0 that is small. (Note that the structure of the β-function can be studied only by non-perturbative methods.) Secondly, in the last years the

accuracy and the kinematic region of deep inelastic scattering data became large enough, which makes us hope that discrimination between QCD and FP-QCD could be performed.

Recently we have analyzed the CCFR deep inelastic neutrino-nucleon scattering data [8] in the framework of the *Fixed point* QCD. It was demonstrated [9] that the data for the nucleon structure function $xF_3(x,Q^2)$ are in good agreement with the LO predictions of this theory model using the assumption that the β function has a *first–order* ultraviolet zero (fixed point) at small $\alpha = \alpha_0$.

Having in mind that up to now the structure of the fixed point theory is not well known, it seems to us to be useful to make predictions for the physical quantities studying the different hypotheses about the β function behaviour near its fixed point α_0 and confront them to the data.

In this letter, we present a leading–order *Fixed point* QCD analysis of the CCFR data [8] in which an expression for $xF_3(x,Q^2)$ based on the assumption that the Callan-Symanzik β function has a *second–order* ultraviolet zero at $\alpha = \alpha_0$ is used. We remind that the structure function xF_3 is a pure non singlet and the results of the analysis are independent of the assumption on the shape of gluons. As in a previous analysis the method [10] of reconstruction of the structure functions from their Mellin moments is used. This method is based on the Jacobi - polynomial expansion [11] of the structure functions. In [12] this method has been already applied to the QCD analysis of the CCFR data.

2. Method and Results of Analysis.

Let us start with the basic formulas needed for our analysis.

The Mellin moments of the structure function $xF_3(x,Q^2)$ are defined as:

$$M_n^{NS}(Q^2) = \int_0^1 dx x^{n-2} x F_3(x,Q^2) , \tag{1}$$

where $n = 1, 2, 3, 4, \ldots$.

In the case of FP-QCD, the effective coupling constant $\alpha_s(Q^2)$ at large Q^2 takes the form:

$$\alpha_s(Q^2) = \alpha_0 + f(Q^2) , \tag{2}$$

where $f(Q^2) \to 0$ when $Q^2 \to \infty$.

Let us assume that α_0 is a *second–order* ultraviolet fixed point for the β-function, i.e.

$$\beta(\alpha) = -b(\alpha - \alpha_0)^2 , \qquad b > 0 . \tag{3}$$

Then

$$\alpha_s(Q^2) = \alpha_0 + \frac{\alpha_s(Q_0^2) - \alpha_0}{1 + b\,[\alpha_s(Q_0^2) - \alpha_0]\; ln\;(Q^2/Q_0^2)} \tag{4}$$

and for the moments of xF_3 we obtain the following leading–order expression:

$$M_n^{NS}(Q^2) = M_n^{NS}(Q_0^2) \left[\frac{Q_0^2}{Q^2}\right]^{\frac{1}{2}d_n^{NS}} \mathcal{F}_n(Q^2) , \tag{5}$$

where

$$\mathcal{F}_n(Q^2) = \left[\frac{1}{1 + b\,[\alpha_s(Q_0^2) - \alpha_0]\; ln\;(Q^2/Q_0^2)}\right]^{d_n^{NS}/2b\alpha_0} . \tag{6}$$

In (5) and (6)

$$d_n^{NS} = \frac{\alpha_0}{4\pi}\gamma_n^{(0)NS} \tag{7}$$

and

$$\gamma_n^{(0)NS} = \frac{8}{3}[1 - \frac{2}{n(n+1)} + 4\sum_{j=2}^{n}\frac{1}{j}] . \tag{8}$$

The n dependence of $\gamma_n^{(0)NS}$ is exactly the same as in QCD, if we assume that the Perturbative QCD expansion for the anomalous dimensions $\gamma_n^{NS}(\alpha_s)$ is valid in the range of small α_s including the fixed point α_0 too. However, the Q^2 behaviour of the moments is different because of

the different Q^2 behaviour of the effective coupling constant $\alpha_s(Q^2)$ in FP-QCD. In contrast to QCD, the Bjorken scaling for the moments of the structure functions is broken by powers in Q^2 in addition to the usual $ln\, Q^2$ terms. In (6) and (7) α_0 and b are parameters to be determined from the data.

Having at hand the moments (5) and following the method [10, 11], we can write the structure function xF_3 in the form:

$$xF_3^{N_{max}}(x, Q^2) = x^{\alpha}(1 - x)^{\beta} \sum_{n=0}^{N_{max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_{j+2}^{NS}(Q^2) , \tag{9}$$

where $\Theta_n^{\alpha\beta}(x)$ is a set of Jacobi polynomials and $c_j^n(\alpha, \beta)$ are coefficients of the series of $\Theta_n^{\alpha,\beta}(x)$ in powers in x:

$$\Theta_n^{\alpha,\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) x^j . \tag{10}$$

Teh quantities N_{max}, α and β have to be chosen so as to achieve the fastest convergence of the series in the r.h.s. of Eq. (9) and to reconstruct xF_3 with the accuracy required. Following the results of [10] we use $\alpha = 0.12$, $\beta = 2.0$ and $N_{max} = 12$. These numbers guarantee an accuracy better than 10^{-3} .

Finally, we have to parametrize the structure function xF_3 at some fixed value of $Q^2 = Q_0^2$. We choose $xF_3(x, Q^2)$ in the form:

$$xF_3(x, Q_0^2) = Ax^B(1 - x)^C . \tag{11}$$

The parameters A, B and C in Eq. (11) and the FP-QCD parameters α_0 and b are free parameters which are determined by the fit to the data.

In our analysis, the target mass corrections [13] are taken into account. To avoid the influence of higher–twist effects, we have used only the experimental points in the plane (x, Q^2) with $10 < Q^2 \leq 501\ (GeV/c)^2$. This cut corresponds to the x range: $0.015 \leq x \leq 0.65$.

The results of the fit are presented in Table 1. In all the fits only statistical errors are taken into account.

b	$\chi^2_{d.f.}$	α_0	$\alpha_s(M_z^2)_{FP-QCD}$
0.9	83.5/61	0.029 ± 0.014	0.123 ± 0.022
1.0	83.3/61	0.040 ± 0.014	0.126 ± 0.021
1.1	83.1/61	0.048 ± 0.014	0.128 ± 0.021
1.3	82.9/61	0.063 ± 0.013	0.133 ± 0.019
1.5	82.6/61	0.074 ± 0.013	0.136 ± 0.019

Table 1. The results of the LO FP-QCD fit to xF_3 data. $\chi^2_{d.f.}$ is the χ^2-parameter normalized to the degree of freedom $d.f.$.

Summarizing the results in the Table one can conclude:

1. The values of $\chi^2_{d.f.}$ are practically the same as in the case of a first–order ultraviolet fixed point for the β function [9]. They are slightly smaller than those obtained in the LO QCD analysis [12] of the CCFR data by the same method and indicate a *good description* of the data.

2. It is seen from the Table that α_0 increases with increasing b. The values of b, for which the asymptotic coupling α_0 is acceptable, are found to range in the following interval:

$$0 < b \leq 1.5 . \qquad (12)$$

For the values of $b > 1.5$, the corresponding theoretical values for $\alpha(M_z^2)$ obtained by Eq.(4) are larger than $\alpha_s(M_z^2)$ determined from the LEP experiments [14]:

$$\alpha_s(M_z^2) = 0.125 \pm 0.005 . \qquad (13)$$

For the values of b smaller than 0.9 α_0, cannot be determined from CCFR data. The errors in α_0 exceed the mean values of this parameter.

3. The accuracy of determination of α_0 is not good enough. The accuracy increases with increasing b.

4. The values of $\alpha_0 = 0.029, 0.040, 0.048$ corresponding to $b =$ 0.9, 1.0, 1.1 are preferable to other values of α_0 determined from the data.

5. The values of $\alpha_s(M_z^2)$ corresponding to the values of b from the range (12) are in agreement within one standard deviation with $\alpha_s(M_z^2)$ determined from the LEP experiments.

6. The values of parameters A, B and C are in agreement with the results of [9] and [12]. For illustration, here we present the values of these parameters at $Q_0^2 = 3\ (GeV/c)^2$ for $b = 1.0$:

$$A = 7.07 \pm 0.20 , \quad B = 0.865 \pm 0.013 , \quad C = 3.43 \pm 0.004 .$$

They are found to be independent of b and α_0. We have found also that multiplying the r.h.s. of (11) by term $(1+\gamma x)$ one cannot improve the fit.

Summary

The CCFR deep inelastic neutrino-nucleon scattering data have been analyzed in the framework of the *Fixed point* QCD. It has been demonstrated that the data for the nucleon structure function $xF_3(x, Q^2)$ are in good agreement with the LO predictions of this quantum field theory model using the assumption that α_0 is a *second–order ultraviolet fixed point* of the β function and α_0 is *small*. Some constraints on the behaviour of the β function near α_0 have been found from the data. The values for α_0

$$0.029 \leq \alpha_0 \leq 0.048 ,$$

corresponding to the β function parameter b in the range

$$0.9 \leq b \leq 1.1$$

are preferable to the other ones determined from the data.

In conclusion, taking also into account our previous results [9] in the case of β function with a *first-order* fixed point, we find that the CCFR data, the most precise data on deep inelastic scattering at present, *do not eliminate* the FP-QCD and therefore other tests have to be made in order to distinguish between QCD and FP-QCD.

Acknowledgments

We are grateful to A.L. Kataev and C.V. Mikhailov for useful discussions and remarks. One of us (D.S) would also like to thank I.T. Todorov for calling his attention to the papers by A. Patrascioiu and E. Seiler, in which the problem on vanishing the β function in QCD at nonzero coupling is discussed.

This research was partly supported by INTAS (International Association for the Promotion of Cooperation with Scientists from the Independent States of the Former Soviet Union) under Contract nb 93-1180, by the Russian Foundation for Fundamental Research, Grant N 96-02-17435-a, and by the Bulgarian Science Foundation under Contract Ph 510.

References

[1] G. Altarelli, in Proc. of the "QCD-20 Years Later" Conf. 9-13 June 1992, Aachen; World Scientific 1993, v. 1., p. 172; Ed. by P. M. Zerwas and H. A. Kastrup.
S. Bethke. Proc. QCD-94 Conference, Montpelier, France, July 1994.

[2] A. M. Polyakov, *ZHETF* **59** (1970) 542. G. Mack, *Nucl. Phys.* **B35** (1971) 592; A. V. Efremov and I.F. Ginzburg, *Phys. Lett.* **B36** (1972) 371; D. Bailin and A. Love, *Nucl. Phys.* **B75** (1974) 159.

[3] M. Glück and E.Reya, *Phys. Rev.* **D.16** (1977) 3242; *Nucl. Phys.* **B156** (1979) 456.

[4] S. I. Bilenkaya and D. B. Stamenov, *Sov. J. of Nucl. Phys.* **31** (1980) 122.

[5] D.H.Coward et al., *Phys. Rev. Lett.* **20** (1968) 292; E.D. Bloom et al., *Phys. Rev. Lett.* **23** (1969) 930; H. Breidenbach et al., *Phys. Rev. Lett.* **23** (1969) 935.

[6] R.G. Roberts and M.R. Whalley, *J. Phys. G.; Nucl. Part. Phys.* **17** (1991) D1-D151.

[7] A. Patrascioiu and E. Seiler, *Expected Deviations from Perturbative QCD at 1 TeV or Less*, preprint MPI-Ph/92-18; *Scaling, Asymptotic Scaling and Improved Perturbation Theory,* preprint MPI-Ph/93-34; J. Finberg, U. Heller and F. Karsh, *Nucl. Phys.* **B392** (1993) 493, *Phys. Rev. Lett.* **74** (1995) 1924..

[8] CCFR Collab., S. R. Mishra et al., Nevis Preprint N 1459 (1992); CCFR Collab., W. C. Leung et al., *Phys. Lett.* **B317** (1993) 655; CCFR Collab., P. Z. Quintas et al., *Phys. Rev. Lett.* **71** (1993) 1307.

[9] A.V. Sidorov and D. B. Stamenov, *Phys. Lett.* **B357** (1995) 423.

[10] V. G. Krivokhizhin et al., *Z. Phys.* **C36** (1987) 51;
V. G. Krivokhizhin et al., *Z. Phys.* **C48** (1990) 347.

[11] G. Parisi and N. Sourlas, *Nucl. Phys.* **B151** (1979) 421;
I. S. Barker, C. B. Langensiepen and G. Shaw, *Nucl. Phys.* **B186** (1981) 61.

[12] A. L. Kataev and A.V. Sidorov, *Phys. Lett.* **B331** (1994) 179.

[13] O. Nachtmann, *Nucl. Phys.* **B63** (1973) 237;
S. Wandzura, *Nucl. Phys.* **B122** (1977) 412.

[14] G. Altarelli, Talk at the Four Seas Conference, Trieste, Italy, June, 1955.