ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-96-148

I.Zborovský[1], Yu.A.Panebratsev[2], M.V.Tokarev[3], G.P.Škoro[4,5]

Z-SCALING IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES

Submitted to «Physics Review D»

[1]Nuclear Physics Institute, Academy of Sciences of the Czech Republic, Řež, Czech Republic
zborovsky@ujf.cas.cz
[2]panebrat@sunhe.jinr.dubna.su
[3]tokarev@sunhe.jinr.dubna.su
[4]goran@sunhe.jinr.dubna.su
[5]INS «Vinča», Belgrade, Yugoslavia

1996

1 Introduction

One of the most important problem in the modern high energy physics is a search for general properties of quark and gluon interactions in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions. Universal approach to description of the processes allows us detail understanding of the physical phenoména underlying the secondary particle production. Numerous experimental data obtained from pp, pA and AA interactions show that the general tendencies can be manifested mostly in the high energy region. They reflect specific characteristics of the elementary constituent interactions. This is especially actual with connection of the commissions of the large accelerators of hadrons and nuclei as RHIC at Brookhaven or LHC at CERN. The main physical goal of the investigations on these colliders is to search for quark-gluon plasma - the hot and superdense phase of the nuclear matter. Therefore it is extremely important to find the main features of pp interaction in order to extract nuclear effects and to study influence of nuclear matter in pA and AA interactions.

Up to date, the investigation of hadron properties in the high energy collisions has revealed widely known scaling laws. From the most popular and famous let us mention the Bjorken scaling observed in deep inelastic scattering (DIS) [1], y-scaling valid in DIS on nuclei [2], limiting fragmentation established for nuclei fragmentation [3], scaling behaviour of the cumulative particle production [4, 5, 6], KNO scaling [7] and others.

The common feature of the mentioned processes indicates the local character of the interactions which leads to the conclusion about dimensionless constituents taking part in the interactions. However, detailed experimental study of the established scaling laws has shown certain violations of these. This can be connected with the dynamics concerning the transition from the perturbative QCD quarks and gluons to the observed hadrons.

The fact that the interaction is local naturally leads to the conclusion of the scale-invariance of the hadron interactions cross sections. The invariance is a special case of the automodelity principle which is an expression of self-similarity [4, 8]. This principle reflects the dropping of certain dimensional quantities or parameters out of the physical picture of the interactions.

In the paper inclusive particle production in $pp/\bar{p}p$ collisions at high energies is considered. Based on automodelity principle the scaling function $H(z)$ expressed via cross section is constructed. The properties of the $H(z)$ function are described. It is shown that the available experimental data confirm the scaling behaviour of $H(z)$. Monte Carlo code HIJING [9, 10] is used to simulate events in pp-collisions and to calculate the inclusive cross section at different energies $\sqrt{s}$ and angles θ of the secondary particles in order to predict $H(z)$ for the corresponding processes.

2 Scaling function H(z) and its general properties

We start with the investigation of the inclusive process

$$M_1 + M_2 \to m_1 + X, \tag{1}$$

where M_1 and M_2 are masses of the colliding nuclei (or hadrons) and m_1 is the mass of inclusive particle. In accordance with Stavinsky's ideas [5] the gross features of the inclusive particle distributions for the reaction (1) at high energies can be described in terms of the corresponding kinematical characteristics of the exclusive subprocess

$$(x_1M_1) + (x_2M_2) \to m_1 + (x_1M_1 + x_2M_2 + m_2). \tag{2}$$

The parameter m_2 is a minimal mass introduced in connection with internal conservation laws (for isospin, baryon number and strangeness). The x_1 and x_2 are the scale-invariant fractions of the incoming 4-momenta P_1 and P_2 of the colliding objects. The centre-of-mass energy of the subprocess (2) is defined as

$$s_x^{1/2} = \sqrt{(x_1P_1 + x_2P_2)^2} \tag{3}$$

and represents the energy of the colliding constituents necessary for the production of the inclusive particle. In accordance with the space-time picture of hadron interactions at the parton level the cross section for the production of the inclusive particle is governed by the minimal energy of colliding partons

$$\sigma \sim 1/s_{min}(x_1, x_2). \tag{4}$$

The fractions x_1 and x_2 which correspond to the minimal value of (2) we find under the additional constraint

$$\frac{\partial \Delta_q(x_1,x_2)}{\partial x_1} = 0, \qquad \frac{\partial \Delta_q(x_1,x_2)}{\partial x_2} = 0, \tag{5}$$

where $\Delta_q(x_1, x_2)$ is given by the equation

$$(x_1P_1 + x_2P_2 - q)^2 = (x_1M_1 + x_2M_2 + m_2)^2 + \Delta_q(x_1, x_2) \tag{6}$$

and q is the 4-momentum of the secondary particle with the mass m_1. So, we determine the fractions x_1 and x_2 in the way to minimize the value of Δ_q, simultaneously fulfilling the symmetry requirement of the problem, i.e. $A_1x_1 = A_2x_2$ for the inclusive particle detected at 90^0 in the corresponding NN centre-of-mass system. This gives

$$x_1 \equiv \frac{\bar{x}_1}{A_1} = \frac{(P_2q) + M_2m_2}{(P_1P_2) - M_1M_2}, \qquad x_2 \equiv \frac{\bar{x}_2}{A_2} = \frac{(P_1q) + M_1m_2}{(P_1P_2) - M_1M_2}. \tag{7}$$

Here A_1 and A_2 are mass numbers and $\bar{x}_1$ and $\bar{x}_2$ are the fractions of the colliding nuclei expressed in units of the nucleon mass. Note, that x_1 and x_2 are therefore less than 1 for all values of q. The minimal value of Δ_q corresponds to the subprocess (2) with the minimal released energy to the away side direction.

In the rest of the paper we will confine our considerations to the inclusive particle production in the $p(\bar{p}) + p \to h + X$ processes. In accordance with the automodelity principle we search for the solution

$$\frac{d\sigma}{dz} \equiv \psi(z), \tag{8}$$

where $\psi(z)$ has to be a scaling function and choose the variable z as a physically meaningful variable which could reflect the self-similarity (scale-invariance) as a general pattern of the hadron production.

The invariant differential cross section for the production of the inclusive particle m_1 depends on two variables, say $q_\perp$ and $q_\parallel$, through $z = z(x_1(q_\perp, q_\parallel), x_2(q_\perp, q_\parallel))$ in the following way:

$$E\frac{d^3\sigma}{dq^3} = -\frac{1}{s\pi}\left(\frac{d\psi(z)}{dz}\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2} + \psi(z)\frac{\partial^2 z}{\partial x_1 \partial x_2}\right). \tag{9}$$

This can be easy shown by partially differentiating using the approximation to the Jacobian of the transformation (7) which at high energies tends to the value $-2q_\perp/(sE)$.

In the first step we use the simple choice

$$z = \frac{s_x^{1/2}}{Q} \to \frac{\sqrt{x_1x_2}\sqrt{s}}{Q}, \tag{10}$$

with Q as a scale which in a first approximation doesn't depend on x_1 and x_2. The the expression (9) at asymptotically high energies reads:

$$Q^2E\frac{d^3\sigma}{dq^3}(q_\perp, q_\parallel) = 4H\left(\frac{s_x^{1/2}}{Q}\right), \tag{11}$$

where the scaling function $H(z)$ is determined by

$$H(z) \equiv -\frac{1}{16\pi}\left(\frac{d\psi(z)}{dz} + \frac{\psi(z)}{z}\right). \tag{12}$$

Now, we introduce a new dynamical scaling hypothesis for the inclusive particle production in $pp/p\bar{p}$ interactions at high energies [11]. We determine the scale Q to be proportional to the dynamical quantity - average multiplicity density $dN(0)/d\eta$ - produced in the central region of the collision at a given energy. We postulate

$$z = \frac{s_x^{1/2}}{\Delta M \cdot dN(0)/d\eta}, \tag{13}$$

where the coefficient ΔM has the dimension of energy and we determine it as 'the reaction energy of the inclusive reaction' or, in other words, as the kinetic energy transmitted from the initial channel to the final channel of the subprocess (2). From the total energy conservation we have

$$\Delta M = 2M - m_1 - (x_1M + x_2M + m_2) = T_f^x - (T_i - E_R) \equiv T_f^x - T_i^x, \tag{14}$$

where T_i, T_f^x and E_R are the initial kinetic energy, the kinetic energy in the final state of the subprocess (3) and the energy consumed on creation of the associate multiplicity, respectively. Inserting (13) into (9) we obtain the expression

$$E\frac{d^3\sigma}{dq^3} = -\frac{1}{16\pi(dN(0)/d\eta)^2M^2}\left(\frac{d\psi(z)}{dz}h_1(x_1,x_2) + \frac{\psi(z)}{z}h_2(x_1,x_2)\right), \tag{15}$$

where the functions h_1 and h_2 are proportional to the partial derivatives in (9).

For the high energy region ($\sqrt{s} > 30$ GeV) we get the expressions (with 2% accuracy according to the exact calculations) for h_1 and h_2

$$h_1 = 4\frac{\delta^2 - (x_1 - x_2)^2}{(\delta - x_1 - x_2)^4}, \qquad h_2 = 4\frac{\delta^2 - (x_1 - x_2)^2 + 4x_1x_2}{(\delta - x_1 - x_2)^4} \tag{16}$$

with $\delta \equiv 2 - (m_1 + m_2)M^{-1}$. The exact expressions for these functions are given in Appendix. We would like to note that $h_1 \simeq h_2$ for $\sqrt{s} > 30$ GeV.

Therefore, according to (15), (16) and (12), we obtain the approximate relation

$$H(z) = \frac{(\delta - x_1 - x_2)^4 M^2 (dN(0)/d\eta)^2}{4(\delta^2 - (x_1 - x_2)^2)} \cdot E\frac{d^3\sigma}{dq^3} \quad (17)$$

in the high energy region. This relation connects the inclusive differential cross section and multiplicity dencity $dN(0)/d\eta$ with the scaling function $H(z)$.

The properties of the scaling functions $\psi(z)$ and $H(z)$ under scale transformations of their argument z can be written in the following forms:

$$z \to \frac{z}{a} \quad (18)$$

$$\psi(z) \to \frac{1}{a} \cdot \psi(\frac{z}{a}) \quad (19)$$

$$H(z) \to \frac{1}{a^2} \cdot H(\frac{z}{a}). \quad (20)$$

These scaling properties are valid also in the general case (15).

We would like to present some qualitative picture, the substantional elements of which are the basic characteristics of the underlying parton subprocess (2) in terms of the scaling proposed. As we have mentioned above the cross section of hadron interactions at the parton level to produce the inclusive particle is governed by the minimal energy of colliding partons $\sigma \sim 1/s_{min}(x_1, x_2)$. The invariant cross section (15) is also proportional to proton structure functions $f_P^q(x_1), f_P^q(x_2)$ and fragmentation function $D^h(z)$

$$E\frac{d^3\sigma}{dq^3} \sim \frac{1}{s_{min}(x_1, x_2)} \cdot f_P^q(x_{1min}) \cdot f_P^q(x_{2min}) \cdot D^h(z). \quad (21)$$

Here x_{1min}, x_{2min} satisfy the condition $min\ s(x_1, x_2) = s_{min}(x_1, x_2)$. We assume that the fragmentation function D^h depends on the relative formation length z/z_{max} of the produced particle m_1 and is independent of x_1 and x_2.

Really, the variable z can be interpreted in terms of parton-parton collision with the subsequent formation of a string stretched by the leading quark out of which the inclusive particle is formed. The minimal energy of the colliding constituents $s_{min}^{1/2}$ is just the energy of the string which connects the two objects in the final state of the subprocess (2). The off-shell behaviour of the subprocess corresponds to a scenario in which the string has the maximal possible space-like virtuality. The string evolves further, splits into pieces decreasing so its virtuality. The resultant number of the string pieces is proportional to number/density of the final hadrons measured in experiment. Therefore, we interpret the ratio

$$\sqrt{s_h} \equiv \sqrt{s_x}/(dN(0)/d\eta) \quad (22)$$

as a quantity proportional to the energy of a string piece $\sqrt{s_h}$ which doesn't split already but during the hadronization converts into the observed hadron. The process of string splitting is self-similar in the sense that the leading piece of a string forgets the string history and its hadronization does not depend on the number and behaviour of other pieces. The factor ΔM in the definition of z is proportional to the kinetic energy of the two objects in the final state of the subprocess (2) and it can be considered therefore as something which reflects the tension of the string. We write therefore

$$\sqrt{s_h} = \Delta M \cdot \lambda, \quad (23)$$

where λ can be considered as the length of the elementary string peace or more precisely the ratio of the length to its characteristic (e.g. average or maximal) value.

The dimensional properties of the scaling function $H(z)$ confirm mentioned above. One can see from the equation (11) that they depend on the dimension of its argument. If we require the argument z to be dimensionless, then $H(z)$ has the dimension of $[fm^2]$. For dimensionless scaling function $H(z)$ we find the argument z to have the dimension of $[fm^1]$. From the transformation properties (18), (20) and from (21) it follows then

$$H(z) \sim D^h(z). \quad (24)$$

So, we interpret the variable z as a quantity proportional to the length of the elementary string, or to the formation length, on which the inclusive hadron is formed from its QCD ancestor. In this picture we interpret the variable z as a hadronization parameter, namely as hadronization length. The scaling function $H(z)$ reflects local properties of the hadronization process.

3 Z-scaling in $pp/\bar{p}p$-collisions

Figure 1 shows the scaling function $H(z)$ as a function of the variable z for charged hadrons emitted at $\theta = 90^0$ cms. in $pp/\bar{p}p$ collisions over a wide energy range [11]. Data on inclusive differential cross sections are taken from [12, 13]. Similar dependencies of $H(z)$ for π^--meson production at $\sqrt{s} = 45, 53$ GeV and $\theta = 2.86^0 - 90^0$ cms. are shown in Figure 2. The function $H(z)$ and the variable z are expressed via the multiplicity density $dN/d\eta$ at $\eta = 0$. It was found strong sensitivity of the scaling behaviour on the energy dependence of $dN(0)/d\eta$. The values of the multiplicity densities of charged particles produced in the central (pseudo)rapidity region in $pp/\bar{p}p$ collisions are shown as a function of the cms. energy $\sqrt{s}$ in Figure 3. The full line represents the fit $dN(0)/d\eta = 0.74s^{0.105}$ to the inelastic data taken from [15]. The non single-diffractive (NSD) data and the parameterization $dN(0)/d\eta = 0.023\ln^2(s) - 0.25\ln(s) + 2.5$ are from [16]. The values of $dN(0)/d\eta$ resulting from the requirement of the z-scaling are shown with crosses.

To study in detail the properties of the scaling proposed, we have used HIJING Monte Carlo model [9, 10]. It was shown in [10] that this model provides a comprehensive explanation of a broad spectrum of data on $pp/\bar{p}p$-collisions in the energy range of $\sqrt{s} = 5$ - 2000 GeV. We applied HIJING code to simulate pp collisions at $\sqrt{s} = 53, 200$ and 1800 GeV and to study systematically the features of the scaling function. In our analysis we have been oriented on the particles which can be experimentally observed. Following the arguments from the previous section, we have chosen the density of the secondary hadrons for $dN(0)/d\eta$ at a given energy. The values of m_1 and m_2 appearing in (2) are summarized in Table 1. The procedure of obtaining m_1 and m_2 is described in [5] and for the case of charged hadrons it is motivated by considerations from [11]. The results of the simulations are shown in Figures 4 to 9.

The dependence of $H(z)$ on z for charged hadrons detected at $\theta = 90^0$ in cms. for three different energies is shown in Figure 4a. The angular dependence of the scaling function is presented in Figure 4b. Figure 4c represents the comparison of the Monte Carlo calculations with the experimental data obtained at $\theta = 90^0$.

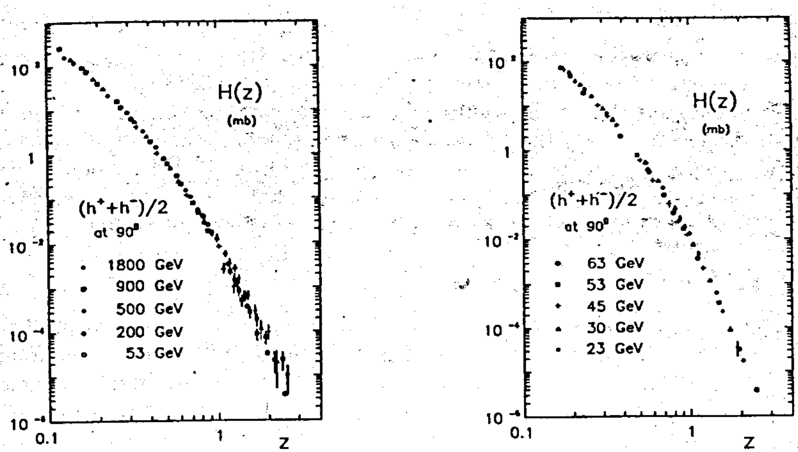


Figure 1. Scaling function $H(z)$ for charged particle production. Experimental data on inclusive differential cross section for charged hadrons in $pp/\bar{p}p$ interactions at $\theta = 90^0$ and $\sqrt{s} = 23 - 1800\ GeV$ are taken from [12, 13].

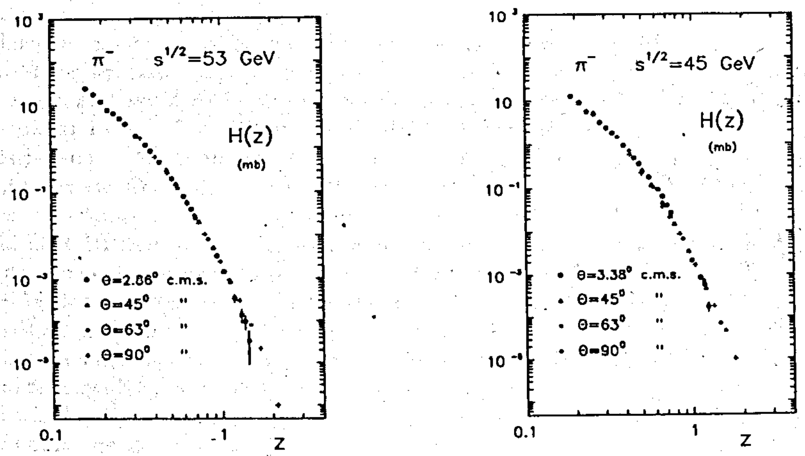


Figure 2. Scaling function $H(z)$ for π^--meson production. Experimental data are taken from [13, 14].

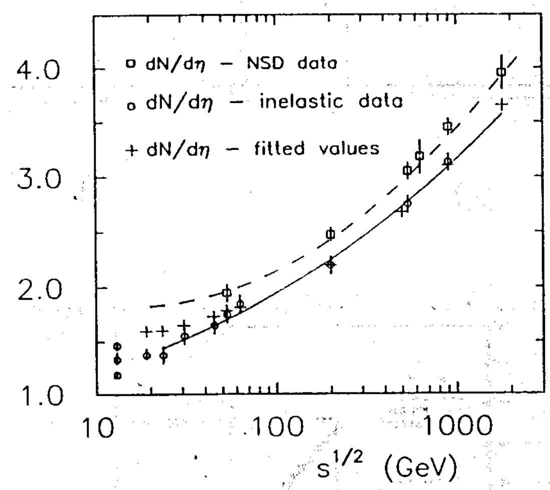
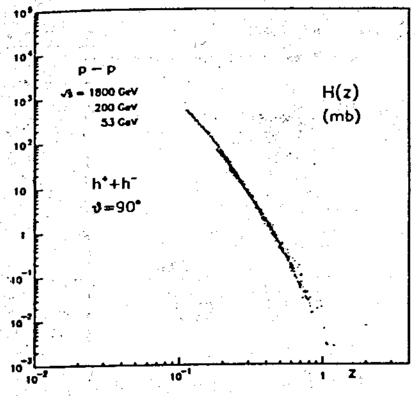
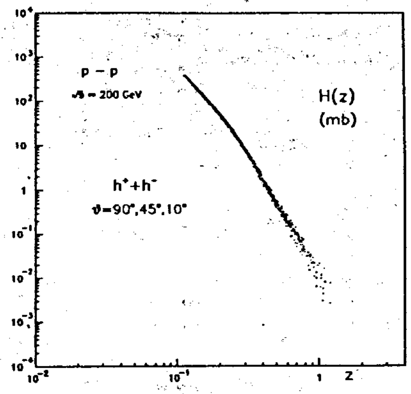


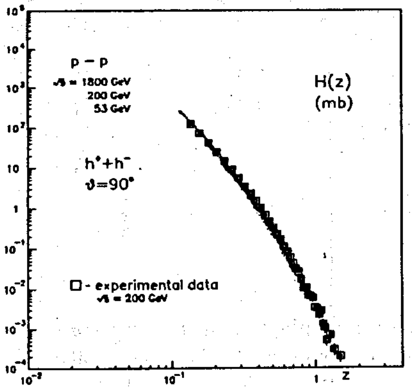
Figure 3. The dependence of multiplicity density $dN(0)/d\eta$ of the produced particles at pseudorapidity $\eta = 0$ on the cms. energy $\sqrt{s}$ for $pp/\bar{p}p$ collisions. Experimental data are taken from [15, 16].



a)

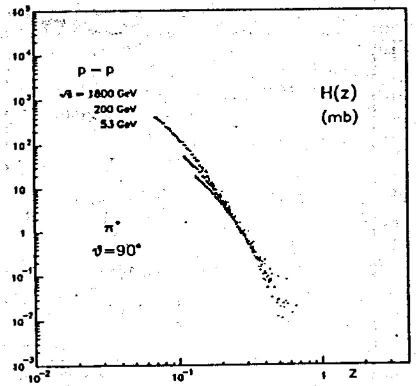


b)

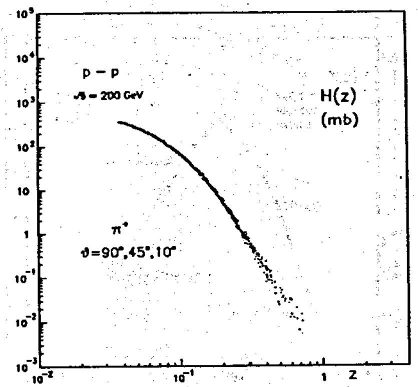


c)

Figure 4. Simulation results of scaling function $H(z)$ for charged particle production and comparison with the experimental data obtained at $\sqrt{s} = 200\ GeV$ and $\theta = 90^0$ cms [12].

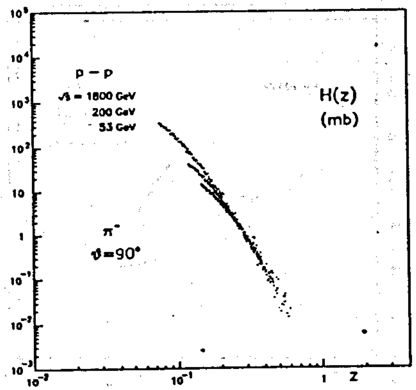


a)

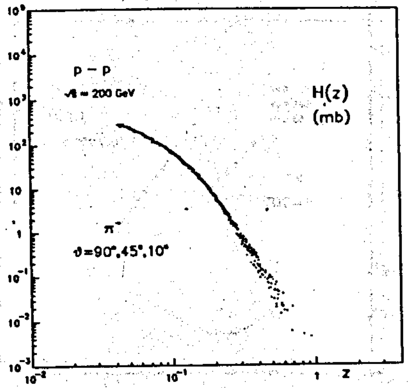


b)

Figure 5. Simulation results of scaling function $H(z)$ for π^+-meson production.

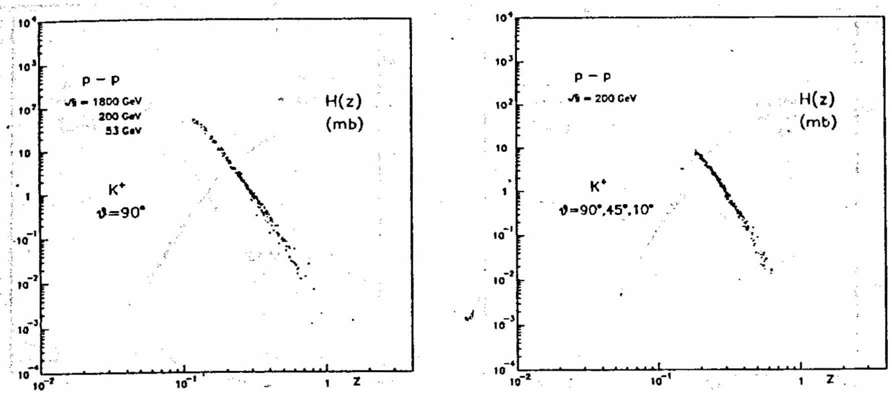


a)



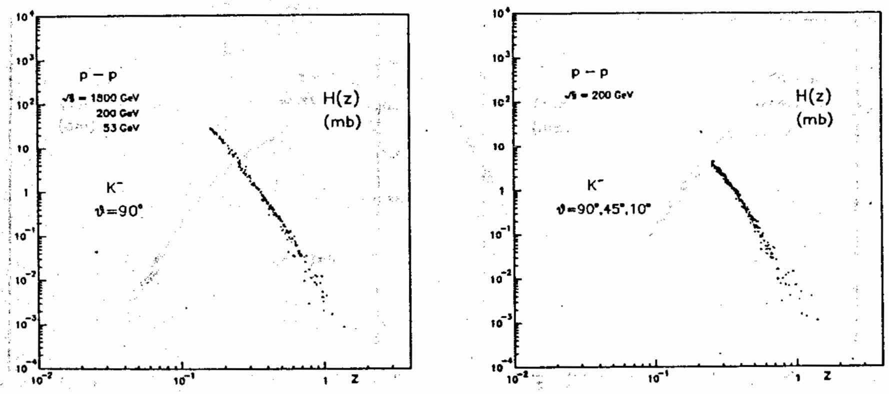
b)

Figure 6. Simulation results of scaling function $H(z)$ for π^--meson production.



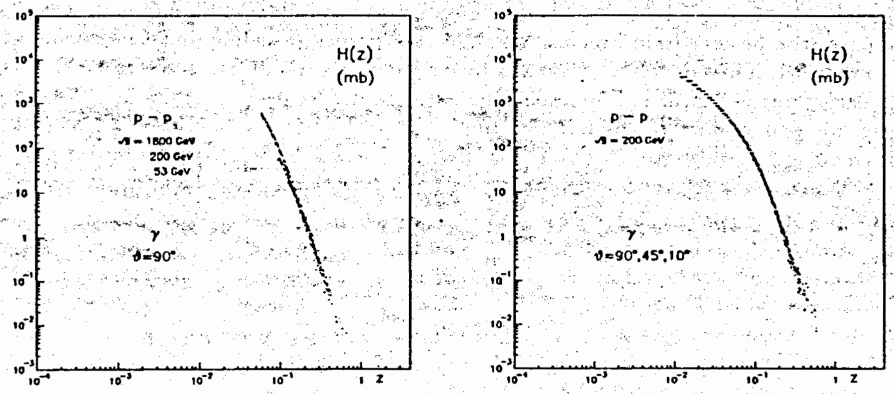
a) b)

Figure 7. Simulation results of scaling function $H(z)$ for K^+-meson production.



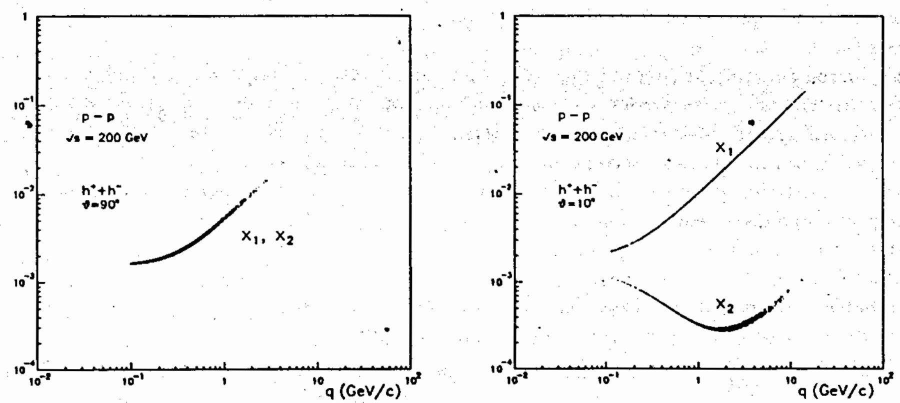
a) b)

Figure 8. Simulation results of scaling function $H(z)$ for K^--meson production.



a) b)

Figure 9. Simulation results of scaling function $H(z)$ for γ production.



a) b)

Figure 10. Dependence of x_1 and x_2 on the secondary particle momentum in the central (a) and beam fragmentation region (b).

The situation in the case of the charged pion inclusive production is ploted in Figures 5 and 6. As can be seen from Figures 5a and 6a, the proposed scaling for pions sets on at value of $z \simeq 0.2$. This roughly corresponds to the secondary pion momenta $q \simeq 0.4$ GeV/c in the energy range considered. It should be noted that there is practically no difference between π^+- and π^--meson scaling properties.

Figures 7 and 8 show the corresponding distributions for $K^\pm$-mesons. For both types of the secondary strange mesons the scaling function $H(z)$ does not depend on energy $\sqrt{s}$ and angle θ.

Table 1. The values of mass parameters m_1 and m_2 used in the analysis of the scaling function $H(z)$

produced particle	m_1 (GeV)	m_2 (GeV)
π^+	m_π	$m_n - m_p$
π^-	m_π	m_π
K^+	m_K	$m_\Lambda - m_K$
K^-	m_K	m_K
γ	0	0
$h^\pm$	0.31	0.31

Figure 9 demonstrates the properties of $H(z)$ for γ's. Because the γ's are, mainly, products of π^0-meson decays this distribution reflects the properties of the π^0-mesons scaling function.

Analyzing the angular dependence of the scaling function, presented in Figures 4b, 5b, 6b, 7b, 8b and 9b, one can see the excellent agreement in different kinematical regions, i.e., the scaling function $H(z)$ does not depend on the emission angle of the secondary particle. The ratios of characteristic (average) formation lengths $a^{h/\pi^+} = \bar{z}^h/\bar{z}^{\pi^+}$ for various secondary particles are given in Table 2. One can see that a^{h/π^+} depends on sort and the mass of the produced hadron h.

Table 2. Relative formation lengths of hadrons produced in $pp/\bar{p}p$-collisions at $\sqrt{s} > 53$ GeV and $q > 0.4$ GeV/c

a^{h/π^+}	a^{π^-/π^+}	a^{K^+/π^+}	a^{K^-/π^+}	a^{γ/π^+}
relative formation lenght	1.05	1.1	1.5	0.83

The obtained results on the ratios among characteristic formation lengths of the different hadrons ($h = \pi^{\pm,0}, K^\pm, h^\pm$) demonstrate z-scaling in a similar way for the fragmentation functions $D^h(z)$ as for the scaling function $H(z)$. Taking into account the symmetry properties of $H(z)$ and (24), we connect the fragmentation functions $D^h(z)$ for different hadrons h_1 and h_2 by the relation

$$D^{h_1}(z) = \frac{1}{(a^{h_2/h_1})^2} \cdot D^{h_2}\Big(\frac{z}{a^{h_2/h_1}}\Big), \qquad (25)$$

and to conclude that the general property of the hadronization process is an universality of the fragmentation function.

4 Results and discussion

Finally, we would like to discuss the qualitative picture resulting from the proposed scheme for various types of secondary particles. The different values of the factors a^{h/π^+}, $h = \pi^-, K^\pm, h^\pm$ represent the relative ratios of characteristic (e.g. average or maximal) formation lengths for various hadrons. The value of the variable z depends also on the factor ΔM, which we interpret as a quantity proportional to the tension of the formated string. Let us look nearer to this aspect of our construction. We consider two kinematical regions: one characterized with high transverse momenta $q_\perp$ which at high energies correspond to low $x_{1,2} < 0.1$ and another with extremely high longitudinal momenta $q_\parallel$ which gives $x_1(x_2) \to 1$, and $x_1 + x_2 \simeq 1$. The first region is the central region of secondary particle production and the second one is the fragmentation region of one of the incoming particle. The dependence of x_1 and x_2 on the secondary particle momentum q is illustrated in Figure 10. Figure 10b shows the clear difference between x_1 and x_2 in the fragmentation region of the incoming particle M_1.

The string tension in the central region is higher than in the fragmentation one. It corresponds to our ideas about the hadronization process in which the produced bare quark dresses itself dragging out some matter (sea $q\bar{q}$ pairs, gluons) from the vacuum forming so a string. The string connects the leading quark of the hadron m_1 with the virtual object with the effective mass $(x_1M_1 + x_2M_2 + m_2)$. The momentum of this object compensates the high momentum of the inclusive particle m_1. The quark dressing in the central region is more intensive than that in the fragmentation region. In our opinion it can be connected with the substantially lower relative velocities of the leading quark to the vacuum in the central region than those in the fragmentation one. For the slowly moving quark it is more easy to obtain an additional mass. Such a quark is strongly decelerated with the string which has the high tension. Consequently, the hadron generated from this quark is formed on smaller formation length.

We study the regime of local parton interactions of incident hadrons at high energies $\sqrt{s}$ and for the secondary particle momenta $q > 0.4$ GeV/c. In this regime the quark distribution functions of the incoming hadrons are separated and therefore the scaling function $H(z)$ describes directly universality of the fragmentation process of secondary partons into the observable hadrons. The universal behaviour of $H(z)$ for different hadrons, taking into account the symmetry properties of the function under the transformation $z \to z/a$, supports the obtained results.

5 Conclusions

The inclusive particle ($\pi^\pm, K^\pm, h^\pm, \gamma$) production in $pp/\bar{p}p$ collisions is considered. Based on automodelity principle the new scaling, z-scaling, is predicted. It was shown that the $H(z)$ scaling function is expressed via two observables - the invariant inclusive cross section $Ed^3\sigma/dq^3$ and the multiplicity density $dN/d\eta$ of particle production at pseudorapidity $\eta = 0$ without any free parameters. We conclude therefore that the predicted z-scaling is model independent and reflects the general properties of $pp/\bar{p}p$ interaction. The available experimental data and results of Monte Carlo simulation for pp-collisions over a wide energy region $\sqrt{s}$ and angle θ of the secondary particles confirm the universal dependence of the $H(z)$ function on the variable z. The $H(z)$ is independent of $\sqrt{s}$ and θ in the central region of particle production. The function $H(z)$ can be connected with the fragmentation function $D^h(z)$ and it is shown that the both have the same z-symmetry properties. The experimental verification of the z-scaling in $pp/\bar{p}p$ collisions at Tevatron(Fermilab), RHIC(BNL) and LHC(CERN) allows to determine general features of hadron-hadron interaction on the parton level and establish the possible mechanism of hadronization [17]. The scaling proposed can be an excellent "instrument" in searching for new phenomena not only in hadron-hadron but in hadron-nucleus, nucleus-nucleus and semi-inclusive deep-inelastic lepton-nucleus interactions.

Acknowledgement

The authors would like to thank A.M.Baldin for his support of the present work. This work has been partially supported by Grant of the Russian Foundation of Fundamental Research No. 95-02-05061.

Appendix

In the appendix we present the results of calculation of the functions h_1 and h_2. Starting from the approximate expression (9) for the invariant cross section $Ed^3\sigma/dq^3$ we define the functions h_1 and h_2 in the following way

$$h_1(x_1,x_2) = 4(dN(0)/d\eta)^2(M_1+M_2)^2 s^{-1}\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2} \qquad (A1)$$

$$h_2(x_1,x_2) = 4(dN(0)/d\eta)^2(M_1+M_2)^2 s^{-1} z\frac{\partial^2 z}{\partial x_1 \partial x_2}. \qquad (A2)$$

The equation (9) we rewrite into the form

$$E\frac{d^3\sigma}{dq^3} = -\frac{1}{4\pi(M_1+M_2)^2\cdot(dN(0)/d\eta)^2}\cdot\left(\frac{d\psi(z)}{dz}\cdot h_1(x_1,x_2)+\frac{\psi(z)}{z}\cdot h_2(x_1,x_2)\right). \qquad (A3)$$

If the scaling variable z is determined by

$$z = \sqrt{s_x}/(\Delta M\cdot dN(0)/d\eta), \quad \Delta M = M_1+M_2-m_1-m_2-x_1M_1-x_2M_2, \qquad (A4)$$

the direct calculation of h_1 and h_2 for $M_1 = M_2 = M$ gives the result

$$h_1 = \{sx_1x_2[\delta^2-(x_1-x_2)^2]+2M^2(x_1-x_2)^2[\delta(\delta-x_1-x_2)+4x_1x_2]$$

$$-4M^4s^{-1}(x_1-x_2)^2[\delta(\delta-2x_1-2x_2)+4x_1x_2]\}\cdot F^{-1} \qquad (A5)$$

$$h_2 = \{sx_1x_2[\delta^2-(x_1-x_2)^2+4x_1x_2]+2M^2\{(\delta-x_1-x_2)[(x_1^2+x_2^2)(x_1+x_2)-2x_1x_2\delta]$$

$$+8x_1x_2(x_1-x_2)^2\}+8M^4s^{-1}(x_1-x_2)^4\}\cdot F^{-1}, \qquad (A6)$$

where $F = [x_1x_2s+(x_1-x_2)^2M^2](\delta-x_1-x_2)^4/4$ and $\delta = 2-(m_1+m_2)/M$, $s=(P_1+P_2)^2$. In the case of different masses of the incoming objects $M_1 \neq M_2$ these expresions become too clumsy and we present them therefore in an approximative form here. This can be obtained writting for z the expression (A4) with $\sqrt{s_x}\sim\sqrt{x_1x_2s}$. Consequently, we get the relations

$$h_1 = \frac{\Delta^2-x_{eff}^2+4x_1x_2M_1M_2/(M_1+M_2)^2}{(\Delta-x_{eff})^4} \qquad (A7)$$

$$h_2 = \frac{\Delta^2-x_{eff}^2+8x_1x_2M_1M_2/(M_1+M_2)^2}{(\Delta-x_{eff})^4}, \qquad (A8)$$

where $\Delta = 1-(m_1+m_2)/(M_1+M_2)$ and $x_{eff} = (x_1M_1+x_2M_2)/(M_1+M_2)$.

References

[1] J.D. Bjorken, Phys.Rev. **179** (1969) 1547;
J.D. Bjorken, E.A.Paschanos, Phys.Rev. **185** (1969) 1975.

[2] P. Bosted et al., Phys. Rev. Lett. **49** (1982) 1380.

[3] J.Benecke et al., Phys.Rev. **188** (1969) 2159.

[4] A.M. Baldin, Particles and Nuclei **8** (1977) 429.

[5] V.S. Stavinsky, Particles and Nuclei **10** (1979) 949.

[6] G.A. Leksin, Preprint ITEP-147, Moscow 1976, 12p.;
G.A. Leksin, Proceedings of the XVIII International Conference on High Energy Physics, Tbilisi, 1976, JINR D1,2-10400, Dubna, 1977, A6-3.

[7] Z. Koba, H.B. Nielsen, P.Olesen, Nucl. Phys. **B40** (1972) 317.

[8] V.A.Matveev, R.M.Muradyan, A.N.Tavkhelidze, Lett. Nuov. Cim.**5** (1972) 907;
V.A.Matveev, R.M.Muradyan, A.N.Tavkhelidze, Particles and Nuclei **2** (1971) 7.
S.Brodsky, G.Farrar, Phys. Rev. Lett. **31** (1973) 1153;
S.Brodsky, G.Farrar, Phys. Rev. **D11** (1975) 1309.

[9] X. N. Wang and M. Gyulassy, Phys. Rev. **D44** (1991) 3501.

[10] X. N. Wang and M. Gyulassy, Phys. Rev. **D45** (1992) 844.

[11] I.Zborovsky, JINR preprint, E2-94-504, Dubna, 1994.

[12] F. Abe et al., Phys. Rev. Lett.**61** (1988) 1819;
C. Albajar et al., CERN-EP/89-85, July 12th, 1989.

[13] B. Alper et al., Nucl. Phys. **B87** (1975) 19;
B. Alper et al., Nucl. Phys. **B100** (1975) 237.

[14] M.G.Albrow et al., Nucl. Phys. **B56** (1973) 19.

[15] D.R. Ward, CERN-EP/87-178, September 29th, 1987.

[16] F. Abe et al., Phys. Rev.**D41** (1990) 2330.

[17] R.P. Feynman, Photon - Hadron Interaction. N.Y.: Benjamin, 1972.

Received by Publishing Department
on April 25, 1996.