ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-96-135

M.K.Volkov

VECTOR MESONS IN PIONS AND KAON FORM FACTORS

Submitted to «Ядерная физика»

1996

1. Introduction

The electromagnetic form factors of pions and kaons and their radii have been calculated by many authors with the use of different models (see, for instance,[1]–[9]). We have also calculated the pion form factor (PFF) in the nonlinear chiral model with pion and baryon loops [2] and in the quark model [9].

However, many calculations of meson radii in the one-loop approximation with the q^2-expansion of quark loops have one deficiency. Indeed, if we calculate the PFF in the tree approximation, we get a contribution to the PFF only from the ρ meson pole diagram [9, 10]. If we consider the one-loop approximation with the q^2- expansion of the quark loop diagrams, we get an additional contribution to PFF from the triangle quark diagram describing the $\gamma\pi\pi$ vertex (here γ and π are the photon and pion, respectively). Some authors consider in this approximation only the latter contribution without taking into account the contribution from the ρ-pole diagram [4, 5, 7]. Since the ρ-meson does exist, it is necessary to consider both the diagrams together, similarly to the case of tree approximation. However, then we face the problem of "doubling of the final result", as each of these diagrams separately provides satisfactory values for the meson radii (see below).[1]

In this short note we would like to show that the triangle quark loop diagrams and vector meson pole diagrams can be considered together without "doubling of the final result" if we take into account the vector-meson form factors. We shall demonstrate our method considering the Nambu-Jona-Lasinio model and vector-dominance model in the tree and q^2- approximations.

2. The pion form factor

In refs. [8, 9], it was shown that the Nambu-Jona-Lasinio model (NJL model) [11] can be a common source of the chiral linear sigma model

[1] Within this paper, the notion "doubling of the final result" refers to obtaining twice the experimental value in corresponding calculations for the meson radius.

[12] and the vector dominance model (VDM) [10]. The $U(3) * U(3)$ symmetric NJL Lagrangian with the electromagnetic interactions has the form

$$\mathcal{L}_1 = \bar{q}(i\partial^\nu\gamma_\nu - M^0 - eQA^\nu\gamma_\nu)q + \frac{G_1}{2}[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2] - \frac{G_2}{2}[(\bar{q}\gamma_\nu\lambda^a q)^2 + (\bar{q}\gamma_5\gamma_\nu\lambda^a q)^2], \quad (1)$$

where $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ are the color quark fields, $M^0 = (m_u^0, m_d^0, m_s^0)$ is the current quark mass matrix, λ^a are the Gell-Mann matrices $0 \le a \le 8$, A^ν are the photon fields, e and G_i are the electromagnetic and strong coupling constants and $Q = \frac{1}{2}(\lambda^3 + \frac{\lambda^8}{\sqrt{3}})$ is the quark charge operator.

From (1), using identical transformations, we can get the meson-quark Lagrangian (see [8, 9]). Here we give only part of this Lagrangian to be necessary for the description of the pion and kaon form factors

$$\mathcal{L}_2 = \bar{q}(i\partial^\nu\gamma_\nu - M - eQA^\nu\gamma_\nu + ig_\phi\gamma_5\phi_a\lambda^a + \frac{g_\rho}{2}V_a^\nu\gamma_\nu\lambda^a)q. \quad (2)$$

Here $M = (m_u, m_d, m_s)$ is the constituent quark mass matrix ($m_u \approx m_d = 280\text{MeV}$, $m_s = 460\text{MeV}$, [8]), g_ϕ and g_ρ are the pseudoscalar and vector meson coupling constants, $g_{\phi_{i,j}} = \frac{m_i+m_j}{2F_{ij}}$, where F_{ij} are the decay constants of mesons ($F_{ud} = F_\pi = 93$ MeV, $F_{us} = F_K = 1.2F_\pi$, $g_\rho = 6.14$) and ϕ_a and V_a^ν are the pseudoscalar and vector mesons.

Now we shall consider different approximations for the meson form factors following from the Lagrangian (2). We shall investigate only the one-loop quark diagrams (Hartree approximation of the NJL model).

2.1. Tree approximation for the PFF

Let us consider the PFF in the tree approximation (Fig.1). These diagrams are described by the Lagrangian [8, 9, 10]

$$\mathcal{L}_3 = i(eA_\mu - g_\rho\rho_\mu^0)(\pi^-\partial^\mu\pi^+ - \pi^+\partial^\mu\pi^-) + \frac{e}{2g_\rho}F^{\mu\nu}\rho_{\mu\nu}^0 \quad (3)$$

where A_μ, ρ_μ, $\pi^\pm$ are the photon, ρ–meson and pion fields, respectively, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We can obtain the Lagrangian (3) from

(2) by using the local approximation for the quark loop diagrams of Fig.2.[2] It means that we have to use only the divergent parts of the corresponding integrals and the equations for the coupling constants g_π and g_ρ which are in the NJL model (ref. [8])

$$g_\pi^{-2} = -i\frac{3}{4\pi^4}\int^\Lambda \frac{d^4k}{(m^2-k^2)^2}, \quad g_\rho = \sqrt{6}g_\pi. \quad (4)$$

Then from the amplitude $T_{A\pi^+\pi^-}$

$$T_{A\pi^+\pi^-} = ep^\nu A_\nu F_\pi(q^2), \quad (5)$$

we get the standard result for the PFF [10, 9], using the Lagrangian (3),

$$F_\pi(q^2) = 1 + \frac{q^2}{m_\rho^2 - q^2} \approx 1 + \frac{q^2}{m_\rho^2} \quad (6)$$

where $p = p_1 + p_2$, $q = p_1 - p_2$ and the p_i are the pion momenta.

2.2. q^2-approximation for the quark loops.

Now let us consider the corresponding diagrams with quark loops in all the vertices in the q^2-approximation (see Fig.2). Using the expansion of diagram 2a in the photon momentum we obtain [8, 13] [3]

$$F_\pi^a(q^2) = 1 + \frac{q^2}{a_\pi}, \quad (7)$$

where the notation $a_\pi = 8\pi^2F_\pi^2$ was introduced. The denominator in the second term of (7) is approximately the square of the ρ–meson mass,

$$a_\pi \approx m_\rho^2.$$

Therefore, we could get the previous result only by considering one triangle diagram 2a. If, in addition, we consider diagram 2b, we arrive at the double experimental value for the pion radius.

[2] Note that the NJL model in the tree approximation gives all chiral phenomenological Lagrangians and the vector dominance model (see [8, 10])

[3] This expansion has been outlined in detail in ref. [7].

Indeed from the diagram 2b using the Lagrangian (3) (or (2)) we get

$$F_\pi^b(q^2) = \frac{q^2}{(m_\rho^2 - q^2)} \frac{g_\rho(q^2)}{g_\rho}, \tag{8}$$

where $g_\rho(q^2)$ corresponds to the triangle diagram 4b, but with the ρ-meson off the mass shell, i.e. $q^2 \ll m_\rho^2$. Then if we suppose that $g_\rho = g_\rho(m_\rho) \approx g_\rho(q^2 \ll m_\rho^2)$ (as in the previous section), we get from (8) an additional contribution to PFF, namely,

$$F_\pi^b \approx \frac{q^2}{m_\rho^2}.$$

This gives the double experimental value for the pion radius.

Unfortunately, we cannot calculate the vector meson form factor off the mass shell of the ρ meson within the local NJL model. For that purpose it is necessary to consider the nonlocal version of this model which does not yet exist. However, we can construct this form factor "by hand", using the ambiguity connected with the divergent part of the triangle diagram 2b and the information which we have got from the q^2 expansion of this diagram. A form factor of this type was proposed and used in refs. [8, 13]; it is of the form

$$g_\rho(q^2) = g_\rho(1 + \frac{q^2 - m_\rho^2}{a_\pi}). \tag{9}$$

Here the q^2 coefficient comes from the q^2 expansion of the triangle diagram (see eq.(7)) and the construction of other parts in (9) results in

$$g_\rho(q^2 = m_\rho^2) = g_\rho \quad ; \quad g_\rho(q^2 \ll m_\rho^2) \approx g_\rho \frac{q^2}{a_\pi}. \tag{10}$$

This behaviour of $g_\rho(q^2)$ allows one to describe the decay $\rho \to 2\pi$ and the processes $\omega \to 3\pi$ and $\gamma \to 3\pi$ in accordance with low-energy theorems (see [13]).

In the NJL model the quark loop with photon and $\rho-$ meson legs (see Figs. 2b, 5b and 6a) gives an expression which equals the last term in the Lagrangian (3) (see [8]). As a result, we get

$$F_\pi(q^2) = 1 + \frac{q^2}{a_\pi} + \frac{q^2}{m_\rho^2 - q^2}(1 + \frac{q^2 - m_\rho^2}{a_\pi}) \approx 1 + \frac{q^2}{m_\rho^2}. \tag{11}$$

This is again the standard result. Therefore we can see that the vector form factor in our form permits us to avoid "doubling of the final result" in calculations of the meson radii.[4]

2.3. Tree approximation for PFF in the vector dominance model

Now let us turn to the standard vector dominance model (VDM) [10]. After a transformation of the vector field ρ,

$$\rho = \rho' + \frac{e}{g_\rho} A$$

we obtain, instead of the last term in (3), a nondiagonal term without derivatives (from the mass term of the ρ–meson) [10, 8],

$$\mathcal{L}_4 = \frac{e}{g_\rho} m_\rho^2 \rho'_\mu A^\mu. \tag{12}$$

Then from the diagrams in Fig.3 we obtain in the tree approximation

$$F_\pi(q^2) = 1 - 1 + \frac{m_\rho^2}{m_\rho^2 - q^2} \approx 1 + \frac{q^2}{m_\rho^2}. \tag{13}$$

2.4. q^2-approximation for quark loops in the vector dominance model

Finally, considering the quark loop diagrams of Fig.4, using q^2 expansions of the triangle diagrams (4a,b), and the form factor (9) for the ρ meson, we get

$$F_\pi(q^2) = 1 + \frac{q^2}{a_\pi} - (1 + \frac{q^2 - m_\rho^2}{a_\pi}) + \frac{m_\rho^2}{m_\rho^2 - q^2}(1 - \frac{m_\rho^2 - q^2}{a_\pi}) \approx 1 + \frac{q^2}{m_\rho^2}. \tag{14}$$

Therefore we see that in all cases we can obtain the standard VDM result for the PFF using the form factor of the ρ meson together with q^2-expansions of the quark loops.

All these equations give the following value for the pion radius

$$< r^2 >_\pi = \frac{6}{m_\rho^2} \approx 0.4 \text{fm}^2 \tag{15}$$

[4]It is worth to note that using this form factor we can also describe the processes $\rho \to 2\pi, \omega \to 3\pi, \phi \to 3\pi, \eta \to \pi^+\pi^-\gamma, K_L \to \pi^+\pi^-\gamma$ and $\gamma \to 3\pi$ in accordance with experiment and low-energy theorems (see [13, 14]).

3. The kaon form factors

Now let us show how analogous results can be obtained for the kaon form factors (KFF). To this end, we use the Lagrangian which can be found from (2) (see ref. [8])

$$\mathcal{L}_5 = -i\frac{g_\rho}{2}[(K^-\partial_\nu K^+ - K^+\partial_\nu K^-)(\rho^0+\omega+\sqrt{2}\phi)^\nu + (\bar{K}^0\partial_\nu K^0 - K^0\partial_\nu \bar{K}^0)(-\rho^0+\omega+\sqrt{2}\phi)^\nu], \quad (16)$$

where K, ρ, ω, ϕ are the fields of the kaons, $\rho-, \omega-$ and $\phi-$ mesons.

Consider the diagrams of Fig.5. For diagram 5a we get

$$F^a_{K^+}(q^2) = 1 + \frac{q^2}{a_K} \quad , \quad a_K = 8\pi^2 F_K^2 \quad , \quad F_K = 1.2F_\pi. \quad (17)$$

To calculate diagrams 5b, it is necessary to consider the quark loop diagrams describing the transitions of photons into the ρ^0, ω, ϕ (Fig.6). These diagrams give the nondiagonal Lagrangian [8]

$$\mathcal{L}_6 = \frac{e}{2g_\rho}F^{\mu\nu}(\rho^0_{\mu\nu} + \frac{1}{3}\omega_{\mu\nu} + \frac{\sqrt{2}}{3}\phi_{\mu\nu}) \quad (18)$$

For the contribution to the KFF corresponding to Fig.5b, we have the following expression using the Lagrangians (16) and (18) and the form factor (9) with the exchange $a_\pi \to a_K$

$$F^b_{K^+}(q^2) = \frac{q^2}{2}[\frac{1}{m_\rho^2 - q^2} - \frac{1}{a_K} + \frac{1}{3(m_\omega^2 - q^2)} - \frac{1}{3a_K} + \frac{2}{3(M_\phi^2 - q^2)} - \frac{2}{3a_K}] \approx \frac{q^2}{2}[\frac{1}{m_\rho^2} + \frac{1}{3m_\omega^2} + \frac{2}{3m_\phi^2} - \frac{2}{a_K}]. \quad (19)$$

Summation of (17) and (19) yields

$$F_{K^+}(q^2) = 1 + \frac{q^2}{2}[\frac{1}{m_\rho^2} + \frac{1}{3m_\omega^2} + \frac{2}{3m_\phi^2}]. \quad (20)$$

and

$$< r^2 >_{K^+} = 3[\frac{1}{m_\rho^2} + \frac{1}{3m_\omega^2} + \frac{2}{3m_\phi^2}] = 0.336\text{fm}^2. \quad (21)$$

For the neutral kaons we have only the last diagrams of Fig.5 (5b). These diagrams give the following contributions to the neutral KFF

$$F_{K^0}(q^2) = \frac{q^2}{2}[-\frac{1}{(m_\rho^2 - q^2)} + \frac{1}{(3m_\omega^2 - q^2)} + \frac{2}{3(m_\phi^2 - q^2)} + \frac{3-1-2}{3a_K}] \approx -\frac{q^2}{2}[\frac{1}{m_\rho^2} - \frac{1}{3m_\omega^2} - \frac{2}{3m_\phi^2}], \quad (22)$$

$$< r^2 >_{K^0} = -3[\frac{1}{m_\rho^2} - \frac{1}{3m_\omega^2} - \frac{2}{3m_\phi^2}] = -0.059\text{fm}^2. \quad (23)$$

Therefore we have again arrived at the results coinciding with those found from the standard vector dominance model. These results are in satisfactory agreement with the experimental data (see Table and [15]).

Our calculations show that only using the vector form factor as given above we can obtain the correct results for the meson radii in the q^2-approximation of quark loops.[5]

4. Temperature dependence of the pion form factor

We shall demonstrate the definition of the temperature dependence of meson form factors using the PFF as an example. For this purpose let us consider equation (11). We can see that the contribution of

[5]It is worth to note that even small changes of the vector meson form factor may significantly alter the final result. For instance, if we use some other form factor,

$$g_V(q^2) = g_\rho(1 + \frac{q^2 - m_V^2}{m_V^2}), \quad (24)$$

which satisfies the conditions (10) as well, then (15), (21) and (23) are replaced by

$$< r^2 >_\pi = \frac{3}{(2\pi F_\pi)^2} = 0.34\text{fm}^2, \quad < r^2 >_{K^+} = \frac{3}{(2\pi F_K)^2} = 0.24\text{fm}^2, \quad < |r^2| >_{K^0} = 0. \quad (25)$$

However, the new result for K^0 does not agree with the experimantal data. The physical reason of that is the following: The denominator a_π in eq.(9) comes from the q^2-expansion of the triangle diagram (see eq.(7)) therefore it connects with the inner structure of the meson. $m_\rho \approx a_\pi$, but if we use m_ρ instead of a_π we lose this nonlocal characteristic of the meson. We therefore do not have many convenient expressions for the vector meson form factors to choose from.

diagram 2b with an intermediate ρ meson to the PFF becomes very small if the ρ meson form factor (9) is taken into account,

$$< r^2 >_\pi = (0.34 \text{ fm}^2)_a + (0.05\text{fm}^2)_b. \qquad (26)$$

On the other hand, diagram 2a connects to the electromagnetic structure of the pion more closely. Taking into account that the ρ meson mass changes very slowly with temperature (see [16]), and that the ρ-meson form factor is very phenomenological in character, we suppose that the temperature behaviour of the PFF is defined by the triangle diagram 2a. Thus we obtain [7] [6]

$$< r^2 >_\pi \approx \frac{3}{(2\pi F_\pi)^2}. \qquad (28)$$

The decay constant F_π decreases very quickly with temperature if $T > 180-190MeV$ (see Fig.8, which coincides with Fig.4 from [16], D.Ebert et al.). This means that the pion radius increases very quickly in this temperature domain (see Fig.9, where eq. (28) and $F_\pi(T)$ from Fig.8 was used). If $T > T_c$, where T_c is the critical temperature ($T_c \approx 210-220MeV$ [16]), the pion is no longer a bound quark-antiquark system. Equation (28) satisfactory describes this process in the temperature domain close to T_c.

Here we have considered only the main (Hartree) approximation for the PFF. To next order in the $1/N_c$ expansion we also have to consider the pion loop diagram, Fig.10. Diagrams of this type were estimated in [1, 2, 3]. Their contribution to the PFF is comparable with the contribution of the ρ pole diagram. In [2] we have obtained the following expression

$$< r^2 >_\pi \approx \frac{1}{(4\pi F_\pi)^2} \ln\left(\frac{2\pi F_\pi}{m_\pi}\right)^2 \approx 0.08\text{fm}. \qquad (29)$$

[6]It is interesting to note that in the nonlinear chiral model with baryon loops [2] we have obtained a result very close to (28)

$$< r^2 >_\pi^B = \frac{1.73 g_A^2}{(2\pi F_\pi)^2}, \qquad (27)$$

where $g_A \approx 1.25$ is the renormalization of the axial baryon current and the factor 1.73 describes the contributions of the $SU(3)$ baryons ($P, \Sigma^\pm, \Xi^\pm$, see Fig.7). The factor $1.73 g_A^2 \approx N_c$ (N_c is the colour number).

This contribution also increases with temperature, but near T_c the logarithm may be equal to zero. A similar result was obtained in [7], [7]

$$< r^2 >_\pi \approx \frac{1}{(4\pi F_\pi)^2} \ln\left(\frac{m_\sigma}{m_\pi}\right)^2 \approx 0.08\text{fm}, \qquad (30)$$

where $m_\sigma \approx 2\pi F_\pi$ is the mass of the scalar meson.

5. Conclusion

The above calculations show that the simplest representation for the vertex $\rho\pi\pi$ of the form $g_{\rho\pi\pi} = g_\rho$ ($\frac{g_\rho^2}{4\pi} \approx 3$) can only be used on the tree level (standard VDM, [10]). If we consider the q^2–expansion of the quark loop diagrams which describe the vertices $\gamma\pi\pi$, $\rho\pi\pi$, and $\rho\gamma$, it is necessary to introduce a new representation of the vertex $\rho\pi\pi$ in order to avoid "doubling of the final result" in this approximation. Because the vertices $\rho\pi\pi$ and VKK in the diagrams describing PFF (or KFF, respectively) are off the mass shell for the vector mesons ($q^2 \ll m_V^2$, very far from the vector meson mass), in the framework of the standard local NJL model we cannot exactly calculate these form factors. We therefore have to use some phenomenological expressions for them which have to show, however, a very realistic behaviour in different domains of q^2. One possibility has been discussed here.

To calculate these form factors more exactly, it is necessary to consider a nonlocal version of the NJL model or even some other model (see, for instance, [17]).

The q^2-approximation is more convenient for a careful investigation of the meson inner structure at zero and finite temperature. Our representation for the vector form factor permits one to avoid "doubling of the final result", because only in this case we can neglect the contributions of the vector pole diagrams. [8]

In future we hope to treat this problem more carefully, using the nonlocal version of the NJL model.

[7]Note that in eq. (30) not all meson diagrams corresponding to the $1/N_c$ approximation were taken into account.

[8]Note, that some authors have done this without any explanation (see, for instance, [4, 5, 7]).

TABLE

The theoretical and experimental values of the pion and kaon radii (units are in fm).

	$\sqrt{\langle r^2\rangle_{\pi^\pm}}$	$\sqrt{\langle r^2\rangle_{K^\pm}}$	$\sqrt{\langle r^2\rangle_{K^0}}$
Theor.	0.63	0.58	0.24
Exp. [15]	0.663 ± 0.023	0.53 ± 0.05	0.28 ± 0.09

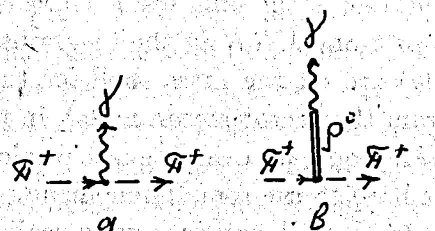

Figure 1: Tree diagrams for PFF.

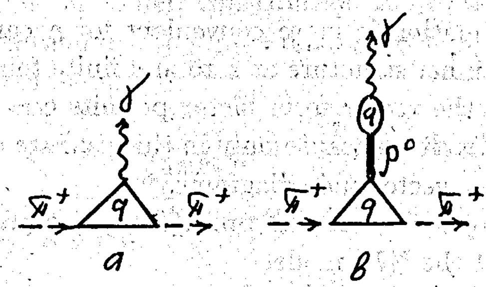

Figure 2: Quark loop diagrams for PFF.

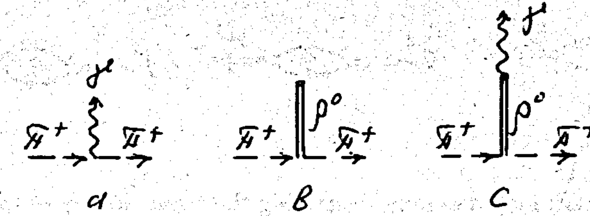

Figure 3: Tree diagrams describing PFF in the VDM.

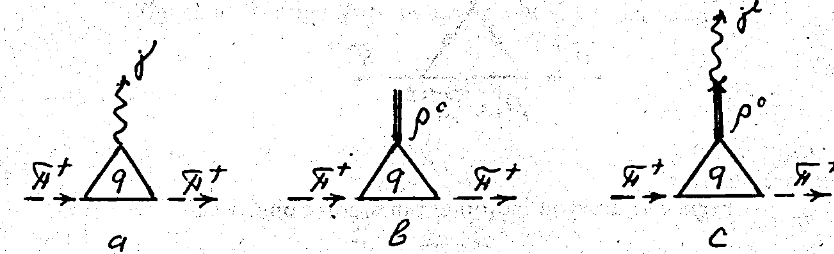

Figure 4: Quark loop diagrams describing PFF in the VDM.

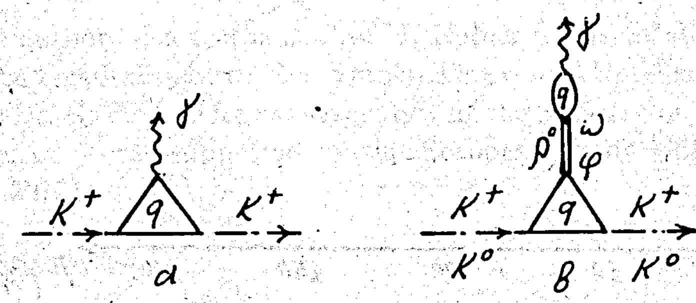

Figure 5: Quark loop diagrams for KFF.

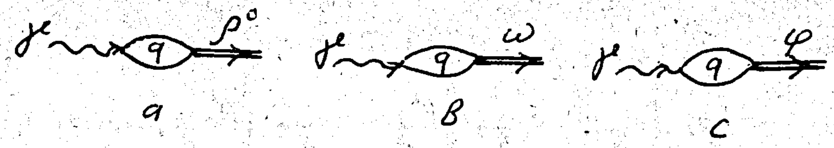
Figure 6: Quark loop diagrams describing the transitions $\gamma \to (\rho, \omega, \phi)$.

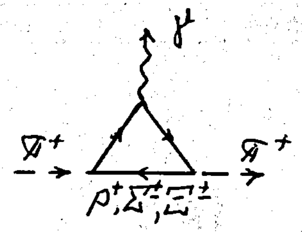
Figure 7: Baryon loop diagrams describing PFF.

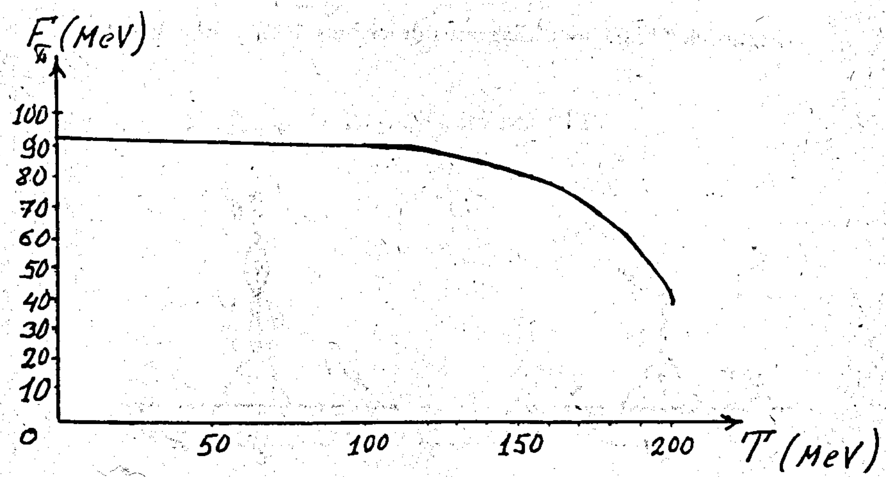

Figure 8: Temperature behaviour of the F_π.

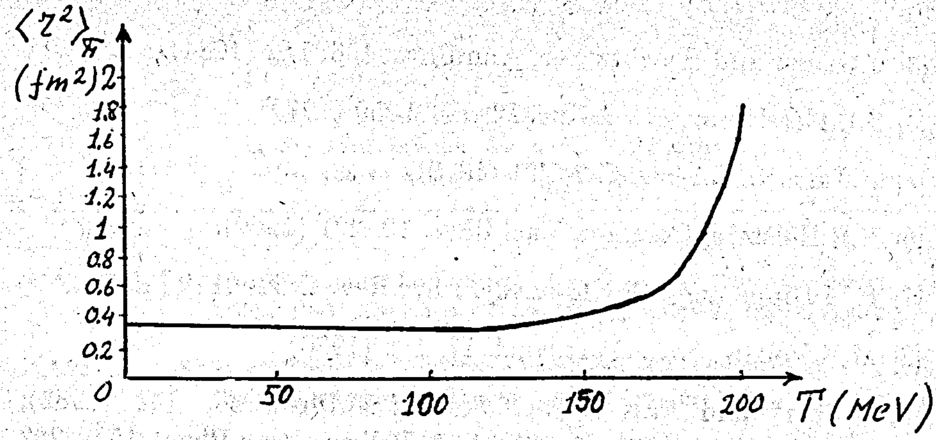

Figure 9: Temperature behaviour of the pion radius.

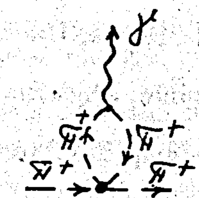
Figure 10: Pion loop diagram for PFF.

Acknowledgments

The author wish to thank Prof. J. Hüfner for useful discussions and warm hospitality at the Institute of Theoretical Physics of Heidelberg and Dr. G. Pätzold for assistance during the preparation of this article. This work was supported by the European Community via INTAS (94-2915).

References

[1] M.A.Beg and A.Zepeda, Phys.Rev. D 6,2912 (1972).

[2] M.K.Volkov and V.N.Pervushin, Phys.Lett. B 51, 356 (1974); Nuovo Cim. 27,277 (1975).

[3] J.Gasser and H.Leutwyler, Ann.Phys. 158, 142 (1984).

[4] S.B.Gerasimov, Sov.J.Nucl.Phys. 29,259 (1979)

[5] R.Tarrach, Z.Phys. C 2, 221 (1979).

[6] B.R.Holstein, Commun.Nucl.Phys. 19, 221 (1990).

[7] H.-J.Hippe and S.P.Klevansky, Phys.Rev. C 52, 2172 (1995).

[8] M.K.Volkov, Sov.J.Part.Nucl. 17, 186 (1986).

[9] D.Ebert and M.K.Volkov, Sov,J,Nucl.Phys. 36, 734 (1982); Z.Phys. C 16, 205 (1983); M.K.Volkov, Ann.Phys. 157, 282 (1984); D.Ebert and H.Reinhardt, Nucl.Phys. B 271, 188 (1986).

[10] J.J.Sakurai, Currents and Mesons, Univ. of Chicago Press, 1969.

[11] Y.Nambu and G.Jona-Lasinio, Phys.Rev. 122,345 (1961).

[12] S.Gasiorowicz and D.A.Geffen, Rev.Mod.Phys. 41, 531 (1969).

[13] M.K.Volkov,A.N.Ivanov,H.Reinhardt and D.Ebert, Phys.Lett. B 182, 193 (1986).

[14] M.K.Volkov and A.A.Osipov, Sov.J.Nucl.Phys. 55, 107 (1992).

[15] O.Dumbrais e.a., Nucl.Phys. B 216, 277 (1983).

[16] U.Vogl and W.Weise, Prog.Part.Nucl.Phys. 27, 195 (1991); S.P.Klevansky, Rev.Mod.Phys. 64, 649 (1992); T.Hatsuda and T.Kunihiro, Phys.Rep. 247, 221 (1994); D.Ebert,Yu.L.Kalinovsky,L.Munchov and M.K.Volkov, Int.J.Mod.Phys. A 8, 1295 (1993).

[17] G.V.Efimov and M.A.Ivanov, Int.J.Mod.Phys. A 4, 2031 (1989);Sov.J.Part.Nucl. 20. 479 (1989).

Received by Publishing Department
on April 16, 1996.