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A.V.Belitsky

QUARK DISTRIBUTION IN THE PION FROM QCD SUM RULES WITH NONLOCAL CONDENSATES

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1. One of the most important achievements of Quantum Chromodynamics is the determination of Q^2-evolution law for the structure functions $F_i(x, Q^2)$ of deep inelastic scattering. It allows one to calculate the magnitudes of observable $F_i(x, Q^2)$ at some scale Q^2 starting from its value at another one Q_0^2. The theoretical basis for this application is provided by factorization theorems which give a possibility to express the physical cross section as a "hard" parton subprocess convoluted with a "soft" parton distribution function. While the former can be treated perturbatively due to the celebrated asymptotical freedom, the latter is governed by strong interaction dynamics at large distances and, therefore, it cannot be evaluated within perturbative QCD. On the other hand, the deep inelastic cross section can also be computed by using the operator product expansion (OPE). This gives structure functions in terms of certain coefficients multiplied by the target matrix elements of local quark and gluon operators of definite twist. Combining the two approaches allows one to express the parton distributions in terms of quark and gluon correlation functions on the light cone. Following Collins and Soper [1], we can write for the twist-2 valence quark distribution in a hadron

$$\langle h(p)|\mathcal{O}\left(\frac{\lambda}{2}n, -\frac{\lambda}{2}n\right)|h(p)\rangle = 4\int_0^1 \cos(\lambda x) u_h(x), \tag{1}$$

where[1]

$$\mathcal{O}\left(\frac{\lambda}{2}n, -\frac{\lambda}{2}n\right) = \bar{u}\left(\frac{\lambda}{2}n\right)\gamma_+ \Phi\left[\frac{\lambda}{2}n, -\frac{\lambda}{2}n\right] u\left(-\frac{\lambda}{2}n\right) + (\lambda \to -\lambda) \tag{2}$$

and Φ is a path ordered exponential in the fundamental representation of the colour group along the straight line which insures the gauge invariance of the parton distribution

$$\Phi[\mathrm{x}, \mathrm{y}] = P\exp\left(ig(\mathrm{x}-\mathrm{y})_\mu \int_0^1 d\sigma t^a B^a_\mu(\mathrm{y} + \sigma(\mathrm{x}-\mathrm{y}))\right). \tag{3}$$

It should be noted that the light-cone position representation is useful to make contact with the OPE approach while the light-cone fraction representation is appropriate for establishing the parton language.

The determination of parton distributions is, up to now, reserved to experimental studies but as a final goal they are expected to be evaluated from the first principles of the theory. In the lack of complete understanding of the yet unclear confinement mechanism they provide a challenging task for nonperturbative methods presently available. Among the approaches which account for nonperturbative effects the most close to QCD perturbation theory are the QCD sum rules [2]. In the last decade they were applied with

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[1]Throughout the paper + subscript means the convolution of the corresponding Lorentz index with light-cone vector n_μ, such that $n^2 = 0$, $(np) = 1$, $(nq) = 0$ and q is t-channel momentum introduced below.

moderate success to determine nucleon and photon structure functions in the region of intermediate values of the Bjorken variable [3, 4] and, recently, in the small λ-region for the light-cone position representation [5].

While the nucleon structure functions are now well defined by the analyses of the precise experimental data and are attacked theoretically, much less is known about the parton distribution of other hadrons, in particular of π-meson. Being of interest in their own right they provide good testing ground for predictions of the QCD sum rule method which will be used in the following for the determination of the leading twist pionic valence quark distribution.

2. In order to evaluate the quark distribution in the pion by means of the QCD sum rules method, we consider an appropriate three-point correlation function of two axial currents that have non-zero projection onto the pion state being proportional to the pion decay constant $\langle 0|j^5_\mu|\pi(p)\rangle = i f_\pi p_\mu$ and the nonlocal string operator $\mathcal{O}$ on the light cone defined by eq. (2)

$$W_{\mu\nu}(p_1,p_2,q) = i^2 \int d^4x d^4y e^{ip_1x+iqy}\langle 0|T\left\{ j^5_\mu(x), \mathcal{O}\left(y+\frac{\lambda}{2}n, y-\frac{\lambda}{2}n\right), j^{5\dagger}_\nu(0)\right\}|0\rangle. \quad (4)$$

The usual strategy is to use the duality between the hadronic and partonic representation for the correlator under investigation.

On the one hand, we should consider the dispersion relation for the latter and extract the contribution due to the low lying hadron, namely, due to π-meson, approximating the higher state contribution by perturbative spectral density

$$W_{++}(p_1^2,p_2^2,Q^2=0) = \frac{4f_\pi^2}{(p_1^2-m_\pi^2)(p_2^2-m_\pi^2)}\int_0^1 dx\cos(\lambda x)u_\pi(x)$$
$$+\frac{1}{\pi^2}\int_0^\infty\int_0^\infty ds_1 ds_2 \frac{\rho_{pert}(s_1,s_2,\lambda)}{(s_1-p_1^2)(s_2-p_2^2)}(1-\theta(s_0-s_1)\theta(s_0-s_2)), \quad (5)$$

with parameter s_0 characterizing the beginning of the continuum. Note that projecting the Lorentz indices of the pion interpolating fields on the direction picked by vector n_μ, we extract the leading tensor structure in the infinite momentum frame. We omit the subtraction polynomials in p_1^2 and p_2^2 because they disappear after the Borel procedure has been applied. The latter leads to exponential suppression of the excited state contribution in the phenomenological side of the sum rule and gives factorial improvement of the OPE series at the theoretical one. We perform the double Borel transformation and put the parameters equal $M_1^2 = M_2^2 = 2M^2$ in order not to introduce the asymmetry between the initial and final pion states and to make contact with two-point sum rules for the pion decay constant.

On the other hand, we consider the OPE for the same quantity. Of course, the QCD sum rules with local condensates are inappropriate here because the usual local power corrections produce δ-type contribution to the distribution function. It is not surprising since some propagators are substituted by constant factors that do not allow the momentum to flow and the whole hadron momentum be carried by a single quark. The probability density of this configuration in the phase space is $\delta(1-x)$. Higher condensates produce even more singular terms. However, this singular contribution can be smeared over the whole region of the momentum fraction from zero to unity by avoiding the Taylor expansion of the generic nonlocal objects which are the starting point of all QCD sum rule calculations and introducing the concept of nonlocal condensate [6, 7] which assumes the finite correlation length for the vacuum fluctuations.

At the two-loop level, to which we restrict our analysis, we need the bilocal quark and gluon condensates, trilocal quark-gluon condensates and four-quark condensates. The latter will be factorized into the product of bilocal scalar quark condensates via the vacuum dominance hypothesis. For explicit calculations, it is convenient to parametrize the bilocal condensates in the form of the well-known α-representation for propagators [7]

$$\langle 0|\bar\psi(0)\Phi[0,x]\psi(x)|0\rangle = \langle\bar\psi\psi\rangle\int_0^\infty d\alpha f_S(\alpha)e^{\alpha x^2/4},$$
$$\langle 0|\bar\psi(0)\Phi[0,x]\gamma_\mu\psi(x)|0\rangle = -ix_\mu\frac{2}{81}\pi\alpha_s\langle\bar\psi\psi\rangle^2\int_0^\infty d\alpha f_V(\alpha)e^{\alpha x^2/4}. \quad (6)$$

One comment concerning eqs. (6) is that in deriving a QCD sum rule one can always perform a Wick rotation $x_0 \to ix_0$ and treat all the coordinates as Euclidean, $x^2 < 0$. We use the following ansatz for the distribution of vacuum quarks in the virtuality α [8]

$$f_S(\alpha) = \frac{\sqrt{\gamma}}{2\Lambda K_1(2\Lambda\sqrt{\gamma})}\exp\left(-\frac{\Lambda^2}{\alpha}-\alpha\gamma\right), \quad (7)$$

which gives the exponential fall-off for the coordinate dependence of the condensates found on a lattice [9]. Here $\Lambda^2 = 0.2GeV^2$, and γ is fixed from the lowest nontrivial moment of the distribution function f_S that is related to the value of the average virtuality of the vacuum quarks λ_q^2 [10].

3. Conventional calculations of the perturbative diagram (see fig. 1(a)) with the light quark masses neglected result in

$$\Psi_{pert}(M^2,Q^2,\lambda) = \frac{3}{2\pi^2}M^2\int_0^1 dx\cos(\lambda x)x\bar{x}\exp\left(-\frac{\bar{x}}{x}\frac{Q^2}{4M^2}\right). \quad (8)$$

Note that we have kept the t-channel momentum transferred to be nonzero. If we expand the cosine in the Taylor series and integrate over x, we find out that each moment possesses

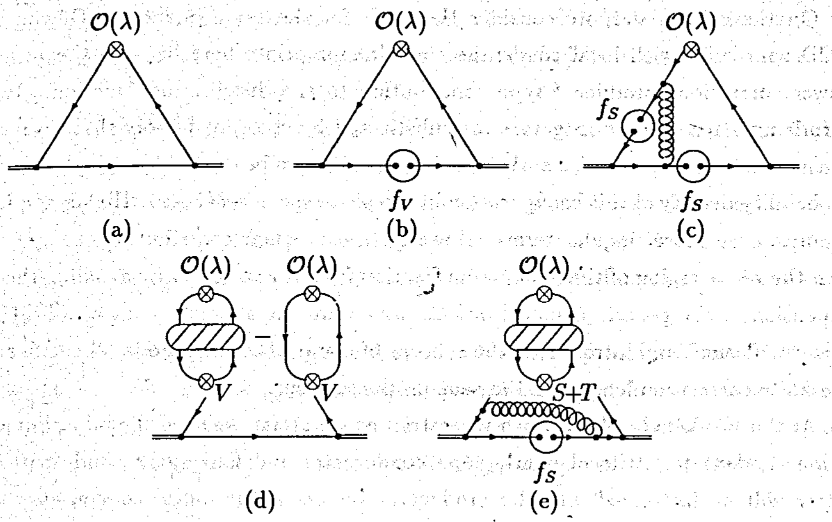

Figure 1: Diagrams contributing to the operator product expansion of the correlation function (3): the first line display ordinary power corrections, while the second one — contribution due to the bilocal correlators.

logarithmic non-analyticities of the type $(Q^2)^n \ln Q^2$. These terms come from the small-x region, where the spectator quark carries almost the whole momentum of the pion, so that the struck quark becomes wee and can propagate over large distances in the t-channel. Therefore, we have to perform additional factorization for separation of small and large distances in the corresponding invariant amplitude; this will lead to the appearance of additional terms in the OPE for the three-point correlation function which correct the small-x dependence of the parton density.

The simplest nonperturbative correction comes from the vector condensate (fig. 1(b))

$$\Psi_V(M^2,\lambda) = \frac{8}{81}\pi\alpha_s\langle\bar{u}u\rangle^2 \int_0^1 dx \cos(\lambda x) x f_V(\bar{x}M^2). \tag{9}$$

The dominant contribution is due to the four-quark condensate. For calculation of two loop diagrams appearing in the consideration (fig. 1(c)) it is very convenient to use the following method which is an extension of the calculation technique developed in ref. [11] for two-point correlators. The main ingredient is a construction of a more general object, namely, the current in the vertex opposite to the gluon propagator should be replaced by the nonlocal one with light-like separation. The advantage of this substitution results in appearing of extra δ-function and introducing through this replacement a set of variables

which give the simplest integration. At the end, we put the nonlocallity parameter to equal zero. Performing straightforward calculations we obtain

$$\Psi_S(M^2,\lambda) = \frac{32}{9}\pi\alpha_s\langle\bar{u}u\rangle^2 \int_0^1 dx \cos(\lambda x) \\ \times \int_0^1 dy \int_0^1 d\xi \int_{\frac{1}{2}}^1 d\zeta f_S\left(\frac{\bar{x}}{\xi}M^2\right) f_S\left(\frac{y}{\bar{\zeta}}M^2\right) \theta\left(\frac{\xi-\zeta}{x-y}\right) \frac{x\bar{y}}{|\bar{x}y\bar{\xi}\zeta - x\bar{y}\xi\bar{\zeta}|}. \tag{10}$$

The gluon as well as trilocal quark-gluon condensate contributions are numerically much less important than the power correction we accounted for; therefore, we neglect them in what follows.

It is well known that there exists a parton sum rule that implies that the pion contains one u-quark. Summing the calculated contributions and taking the formal limit $Q^2 \to 0$ in the perturbative term we can convince ourselves, comparing the result with the sum rule for the pion decay constant[2], that the normalization condition is broken. The reason for this has already been mentioned earlier and we elaborate this point below.

4. Now we derive a Ward identity (WI) [5] and show that the parton sum rule should be exact in QCD. Of course, from the fact that $\mathcal{O}$ is a point-splitted vector current it follows that in the limit $\lambda \to 0$ the correlator (4) is related to the derivative of the two-point correlation function of two axial currents. However, a more general WI (for arbitrary λ) will be useful in the following for discussion of the condensate contribution omitted.

Noticing that we are interested in the limit $Q^2 = 0$, we choose vectors n_μ and q_μ to be proportional. Then, integrating by parts in eq. (4) and using the equation for the complete variation of the phase factor with respect to the smooth variation of the path $\Gamma \to \Gamma' : \mathrm{x}_\mu(\tau) \to \mathrm{x}'_\mu(\tau) = \mathrm{x}_\mu(\tau) + \delta\mathrm{x}_\mu(\tau)$ [12]

$$\delta\Phi_\Gamma[\mathrm{x},\mathrm{y}] = \Phi_{\Gamma'}[\mathrm{x}',\mathrm{y}'] - \Phi_\Gamma[\mathrm{x},\mathrm{y}] = igt^a B^a_\mu(\mathrm{x})\delta\mathrm{x}_\mu(1)\Phi_\Gamma[\mathrm{x},\mathrm{y}] - ig\Phi_\Gamma[\mathrm{x},\mathrm{y}]t^a B^a_\mu(\mathrm{y})\delta\mathrm{x}_\mu(0) \\ +ig\int_0^1 d\tau \Phi_\Gamma[\mathrm{x},\mathrm{x}(\tau)]t^a G^a_{\mu\nu}(\mathrm{x}(\tau))\delta\mathrm{x}_\mu(\tau)\frac{d\mathrm{x}_\nu(\tau)}{d\tau}\Phi_\Gamma[\mathrm{x}(\tau),\mathrm{y}] \tag{11}$$

with $\mathrm{x}_\mu(1) = \mathrm{x}_\mu$, $\mathrm{x}_\mu(0) = \mathrm{y}_\mu$, we obtain

$$W_{\mu\nu}(p_1,p_2,q) \\ = i\int d^4\mathrm{x} e^{ip_1\mathrm{x}}\frac{\mathrm{x}_+}{(q\mathrm{x})}\left[e^{iq\mathrm{x}}\langle 0|T\left\{j^5_\mu(\mathrm{x},\mathrm{x}-\lambda n), j^{5\dagger}_\nu(0)\right\}|0\rangle - \langle 0|T\left\{j^5_\mu(\mathrm{x}), j^{5\dagger}_\nu(0,\lambda n)\right\}|0\rangle\right] \\ +\frac{\lambda}{2}\int_{-1}^1 d\tau \int d^4\mathrm{x} d^4\mathrm{y} e^{ip_1\mathrm{x}+iq\mathrm{y}}\frac{\mathrm{x}_+}{(q\mathrm{x})}\langle 0|T\left\{j^5_\mu(\mathrm{x}), \mathcal{G}\left(\mathrm{y}+\frac{\lambda}{2}n, \mathrm{y}-\frac{\lambda}{2}n, \tau\right), j^{5\dagger}_\nu(0)\right\}|0\rangle \\ +(\lambda \to -\lambda). \tag{12}$$

[2]Of course, the comparison should be made with the sum rule by accounting for nonlocal condensates given in ref. [7].

Here

$$\mathcal{G}\left(y+\frac{\lambda}{2}n, y-\frac{\lambda}{2}n, \tau\right)$$
$$=\bar{u}\left(y+\frac{\lambda}{2}n\right)\Phi\left[y+\frac{\lambda}{2}n, y+\tau\frac{\lambda}{2}n\right]\gamma_\rho g t^a G^a_{\rho+}\left(y+\tau\frac{\lambda}{2}n\right)\Phi\left[y+\tau\frac{\lambda}{2}n, y-\frac{\lambda}{2}n\right]u\left(y-\frac{\lambda}{2}n\right) \tag{13}$$

and

$$j^5_\mu(x, x-\lambda n) = \bar{d}(x)\gamma_\mu\gamma_5\Phi[x, x-\lambda n]u(x-\lambda n) \tag{14}$$

is a point-splitted operator which when sandwiched between the pion state and that of the vacuum, and convoluted with the light-like vector n_μ defines the leading twist-2 pion wave function. From this WI it follows that the normalization of u-quark distribution is exact in QCD, provided it is not spoiled by continuum subtraction

$$\int_0^1 dx u_\pi(x) = 1. \tag{15}$$

5. As we have seen, in the limit $Q^2 \to 0$ the perturbative term though finite contains the logarithmic non-analyticities at this point. This is a typical example of the mass singularities in the QCD sum rules framework [13, 14, 15]. In order to get rid of this perturbative behaviour and replace it by a physical one, it is necessary to modify the original OPE. For the form factor type problem a two-fold structure of the modified OPE has been realized in refs. [16, 17] being of the following schematic form:

$$W(p_1^2, p_2^2, q^2) = \sum_d C^{(d)}(p_1^2, p_2^2, q^2)\langle O_d\rangle + \sum_i \int d^4x e^{ip_1x}\mathcal{C}^{(i)}(x)\mathcal{W}^i(q, x, \lambda). \tag{16}$$

An additional second term determines the contribution due to the long-distance propagation of quarks in the t-channel. Here $\mathcal{W}^i$ are the two-point correlators

$$\mathcal{W}^i(q, x, \lambda) = \int d^4y e^{iqy}\langle 0|T\left\{\mathcal{O}_i(x, 0), \mathcal{O}\left(y+\frac{\lambda}{2}n, y-\frac{\lambda}{2}n\right)\right\}|0\rangle \tag{17}$$

of the operator in question and some nonlocal string operator of a definite twist [15] which arises from the OPE of T-product of pion interpolating fields

$$T\left\{j^5_\mu(x), j^{5\dagger}_\nu(0)\right\} = \sum_i \mathcal{C}^{(i)}(x)\mathcal{O}_i(x, 0). \tag{18}$$

The coefficients $C^{(d)}(p_1^2, p_2^2, q^2)$ in eq. (16) are free from non-analyticities or singularities in Q^2 because they are defined as the difference between the original diagram and its factorized expression which is the perturbative analogue of the corresponding bilocal

correlator. The bilocals cannot be directly calculated in perturbation theory but we can write down the dispersion relation for them

$$\mathcal{W}^i(q, x, \lambda) = \frac{1}{\pi}\int_0^\infty ds\frac{\rho_i(s, (xq), x^2, \lambda)}{s-q^2}, \tag{19}$$

accepting the conventional spectral density model: "low-lying hadron plus continuum". The parameters of the model could be found from auxiliary sum rules. There is no need in additional subtractions in eq. (19) because one always deals with the difference between the "exact" bilocal and its perturbative part; so due to the coincidence of their UV behaviours the subtraction terms cancel in this difference.

6. The simplest bilocal power correction is given by the following convolution:

$$W^{(1)}_{BL}(p_1^2, p_2^2, Q^2, \lambda) = \int d^4x e^{ip_1x}\mathcal{C}^{(1)}_+(x)\mathcal{W}^V_{++}(q, x, \lambda n), \tag{20}$$

where the coefficient function is expressed through the quark propagator $\mathcal{C}^{(1)}(x) = 2S_+(x)$ and

$$\mathcal{W}^V_{++}(q, x, \lambda n) = i\int d^4y e^{iqy}\langle 0|T\left\{\bar{u}(0)\gamma_+\Phi[0, x]u(x), \mathcal{O}\left(y+\frac{\lambda}{2}n, y-\frac{\lambda}{2}n\right)\right\}|0\rangle. \tag{21}$$

We extract the contact term [17, 18] due to the vector condensate from this correlator and saturate the remaining part[3] by the contributions of the mesons of increasing spin; these are ρ^0, g states and so on. The fact that we are interested in the C-odd distribution (valence quark) results in contribution of spin-odd states in the t-channel. It is very convenient to parametrize the appearing matrix elements via the wave functions describing the light-cone momentum fraction distribution of quarks inside mesons. To the leading twist accuracy we can write

$$\begin{aligned}\langle 0|\bar{\psi}(0)\Phi[0, x]\gamma_\mu\psi(x)|M^J(q, \eta)\rangle &= \epsilon^{(\eta)}_{\mu\mu_1\mu_2\ldots\mu_{J-1}}x_{\mu_1}x_{\mu_2}\ldots x_{\mu_{J-1}}(m_M)^J f^{(1)}_J\phi^{(1)}_J(xq) \\ &- iq_\mu\epsilon^{(\eta)}_{\mu_1\mu_2\ldots\mu_J}x_{\mu_1}x_{\mu_2}\ldots x_{\mu_J}(m_M)^J f^{(2)}_J\phi^{(2)}_J(xq),\end{aligned} \tag{22}$$

where J is a spin of the meson, η its polarization and $\epsilon^{(\eta)}_{\mu\mu_1\mu_2\ldots\mu_{J-1}}$ is a polarization tensor. Inspired by our knowledge that in some cases the asymptotical wave functions turn out to be rather close to "exact" ones, we take in our estimation the former in the following forms that are governed by conformal arguments ($\bar{\beta} \equiv 1-\beta$)

$$\varphi^{(1)}_J(\beta) = \frac{\Gamma(2J+2)}{\Gamma^2(J+1)}\left(\beta\bar{\beta}\right)^J, \quad \varphi^{(2)}_J(\beta) = \frac{\Gamma(2J+4)}{\Gamma(J+1)\Gamma(J+2)}\left(\beta-\bar{\beta}\right)\left(\beta\bar{\beta}\right)^J. \tag{23}$$

[3]Although for the present problem the calculation of this part is only of academic interest, as it vanishes in the forward limit being proportional to Q^2, we nevertheless evaluate it in order to demonstrate the difficulties one faces when the contact-type contribution is absent and the estimation of the correlator is carried out saturating it by contribution of physical states.

In our model for the bilocal correlator we can achieve this result if we assume the duality of the meson resonances to the bare quark loop. In general, this quite severe assumption turns out to be reasonable for the case at hand, at least for the mesons of the lowest spins J. It is known experimentally that the physical cross section averaged over the ρ-meson peak coincides with the quark one. Local duality for the low lying states is a nontrivial dynamical property and is not realized in all channels [19]. For the problem at hand it can be explained by the specific interaction of the classical vector mesons with the quark and gluon condensates [2]. The power correction for them even at $M^2 \approx m_\rho^2$ does not exceed $10-20\%$ of the main perturbative term. So, ρ is predicted to be dual to the quark loop with the duality interval about $s_\rho \approx 2m_\rho^2$. However, the local duality for the higher spin mesons can be broken [20].

The net result for the difference between the "exact" bilocal and its perturbative part reads

$$\begin{aligned}
&\mathcal{W}^V_{++}(q,x,\lambda n) - \mathcal{W}^{V(pert)}_{++}(q,x,\lambda n) \\
&= i x_+ \int_0^1 d\tau e^{i\tau(qx)} \left\{ \langle 0|\bar u\left(\frac{\lambda}{2}n\right)\gamma_+ \Phi\left[\frac{\lambda}{2}n, x-\frac{\lambda}{2}n\right] u\left(x-\frac{\lambda}{2}n\right)|0\rangle + (\lambda \to -\lambda)\right\} \\
&+ 2Q^2(x_+)^2 \int_0^1 d\tau \int_0^1 d\beta\beta e^{i\tau\beta(qx)} \sum_{J=1,3,\dots}^{\infty} \varphi_J^{(2)}(\beta)(-\lambda x_+)^{J-1} \\
&\quad \times \left\{ \frac{3}{8\pi^2}\frac{\Gamma(J+2)}{2^{J-1}\Gamma(J)\Gamma(2J+4)} \int_0^{\sigma_J^0} ds \frac{s^J}{s+Q^2} + (-1)^J \frac{(m_M^2)^J f_J^{(1)} f_J^{(2)}}{m_M^2+Q^2} \right\}, \qquad (24)
\end{aligned}$$

where σ_J^0 is the continuum threshold and m_M is the mass of the lowest meson state in the channel of given spin J. Substituting this expression into eq. (20) and performing simple calculations we obtain (diagrammatical representation is given in fig. 1(d))

$$\begin{aligned}
\Psi^{(1)}_{BL}(M^2,\lambda) &= \frac{8}{81}\pi\alpha_s\langle \bar u u\rangle^2 \int_0^1 dx \cos(\lambda x)\bar x f_V(xM^2) + Q^2 e^{\frac{Q^2}{4M^2}} \sum_{J=1,3,\dots}^{\infty} (i\lambda)^{J-1} \\
&\times \left\{ \frac{3}{8\pi^2}\frac{1}{\Gamma(J)\Gamma(J+2)} \int_0^{z_J^0} dz \frac{z^J}{z+\frac{Q^2}{4M^2}} + (-2)^J \left(\frac{m_M^2}{M^2}\right)^J \frac{f_J^{(1)} f_J^{(2)}}{m_M^2+Q^2} \int_0^1 d\beta \varphi_J^{(2)}\left(\frac{1+\beta}{2}\right) \right\},
\end{aligned} \qquad (25)$$

where $z_J^0 = \sigma_J^0/4M^2$. The former term is a contact-type contribution due to the vector condensate. The first one in the curly brackets is the difference between the perturbative analogue of the bilocal correlator and the continuum contribution into the "exact" one. This part cancels the logarithmic non-analyticities in the perturbative diagram (eq. (8)) corresponding to the leading twist-2 operator in the OPE of pion currents. The tower of the next-to-leading non-analyticities can be subtracted in a similar way by accounting for

the twist-4 operator. The last term displays the physical contribution to the correlation function that possesses the correct behaviour in the "momentum transferred Q^2". Requiring that in the limit of large Q^2 the expression in the braces should be zero, we come to the local duality relation for the overlaps

$$(m_M^2)^J f_J^{(1)} f_J^{(2)} = (-1)^{J-1}\frac{3}{8\pi^2}\frac{J(\sigma_J^0)^{J+1}}{2^{J-1}\Gamma(2J+4)}. \qquad (26)$$

The sum of eqs. (8) and (25) is an analytical function in Q^2 as all singularities are replaced by the combination $Q^2+\sigma_J^0$ which is safe in the limit $Q^2 \to 0$. Due to the presence of the non-analyticities in each moment of the distribution function, we need an infinite number of parameters to be found from additional sum rules. Obviously, this is an impossible task. Safely, for the problem at hand, this part vanishes in the forward limit and the sum rule is dominated by the contact terms.

The result of eq. (25), as concerns the Q^2-independent part, can be seen from the WI. The contact term contribution contained in the bilocal correlator is effectively transformed into the power correction due to vector condensate which arises together with eq. (8) from the two-point correlation function in the WI (first two terms of eq. (12)). The latter was investigated in connection with the pion wave function in the same framework [7]. However, it is not so for the most important bilocal part. In this respect the WI is useless as it transforms the bilocals which can be reduced to the condensates not accompanied by the strong coupling constant.

The dominant contribution comes from the bilocal correlator convoluted with a three-propagator coefficient function (see fig. 1(e)) which looks like

$$W^{(2)}_{BL}(p_1^2,p_2^2,Q^2,\lambda) = \int d^4x e^{ip_1x}\mathcal{C}^{(2)}_+(x)\mathcal{W}^S_+(q,x,\lambda n), \qquad (27)$$

(the mirror conjugated contribution can trivially be added) where $\mathcal{C}^{(2)}_\alpha = Ax_\alpha + Bn_\alpha + Cp_{1\alpha}$ and we will not specify the coefficients in this decomposition because of their complexity. All nonperturbative information is accounted for in the correlator

$$\mathcal{W}^S_+(q,x,\lambda n) = i\int d^4y e^{iqy}\langle 0|T\left\{\bar u(0)\Phi[0,x]u(x),\, \mathcal{O}\left(y+\frac{\lambda}{2}n, y-\frac{\lambda}{2}n\right)\right\}|0\rangle. \qquad (28)$$

In order to extract the contact term we make the following decomposition:

$$\mathcal{W}^S_\mu(q,x,\lambda n) = x_\mu\mathcal{P}^S_{(1)} + q_\mu\mathcal{P}^S_{(2)} + \lambda n_\mu\mathcal{P}^S_{(3)} \qquad (29)$$

and convolute this expression with the vector q_μ. Performing the same steps as in the derivation of the Ward identity (12) we find

$$(qx)\mathcal{P}^S_{(1)} = i(qx)\int_0^1 d\tau e^{i\tau(qx)}\langle 0|\bar u\left(\frac{\lambda}{2}\right)\Phi\left[\frac{\lambda}{2}, x-\frac{\lambda}{2}\right]u\left(x-\frac{\lambda}{2}\right)|0\rangle$$

$$
-i\frac{\lambda}{2}\int_{-1}^{1}d\tau\int d^4 y e^{iqy}\langle 0|T\left\{\bar{u}(0)\Phi[0,\mathrm{x}]u(\mathrm{x}),\mathcal{G}\left(\mathrm{y}+\frac{\lambda}{2}n,\mathrm{y}-\frac{\lambda}{2}n,\tau\right)\right\}|0\rangle
$$
$$
+(\lambda\to-\lambda)+Q^2\mathcal{P}^{S}_{(2)}. \tag{30}
$$

The last term in the second line vanishes in the limit of zero Q^2, that manifests the absence of the massless particles in the corresponding channels. Within the accuracy we are limited to, we are left with the first term only because the second one contains an extra power of $gG_{\mu\nu}$, and thus the corresponding OPE starts from the higher orders in the coupling constant and the dimension of the operators. Performing the integration by using the method outlined at the beginning of the paper, we obtain the following contribution to the structure function:

$$
\Psi^{(2)}_{BL}(M^2,\lambda) = \frac{32}{9}\pi\alpha_s\langle\bar{u}u\rangle^2\int_0^1 dx\cos(\lambda x)
$$
$$
\times\int_0^1 dy\int_0^1 d\xi\int_0^{\frac{1}{2}} d\zeta\, f_S\left(\frac{\bar{y}}{\xi}M^2\right)f_S\left(\frac{x}{\bar{\zeta}}M^2\right)\theta\left(\frac{\zeta-\xi}{x-y}\right)\frac{y\bar{x}}{|\bar{x}y\xi\bar{\zeta}-x\bar{y}\bar{\xi}\zeta|}. \tag{31}
$$

Now, having accounted for additional terms in OPE, we can easily check that the normalization condition for the quark distribution in the pion is restored.

7. For zero Q^2 the perturbative spectral density is concentrated on the line $s_1 = s_2$, so that there is no transition between the states with different masses. We collect all contributions and make the continuum subtraction that results in the substitution $M^2 \to M^2(1-\exp(-s_0/M^2))$ in the perturbative term. We have found good stability of the distribution function with respect to the variation of the Borel parameter in the region $0.5 \leq M^2 \leq 0.8$ for the standard value of the continuum threshold $s_0 = 0.7GeV^2$. The normalization point of the OPE is $\mu^2 \sim 0.5GeV^2$; therefore, the function obtained can be regarded as an "input" quark distribution at this low energy scale. In fig. 2, we present the curves for the valence quark distribution in the pion for $M^2 = 0.6GeV^2$: the solid and long-dashed lines correspond to the values of the average virtuality of vacuum quarks $\lambda_q^2 = 0.6GeV^2$ ($\gamma^{-1} = 0.154GeV^2$) and $\lambda_q^2 = 0.4GeV^2$ ($\gamma^{-1} = 0.087GeV^2$), respectively. In the large-x, region the corrections due to the quark condensate do not exceed 30% of the perturbative term. However, in the small-x region at $x = 0.2$ the ratio of the contact term to the main one comprises 50% for $\lambda_q^2 = 0.6GeV^2$ and 70% for $\lambda_q^2 = 0.4GeV^2$. Below this point the nonperturbative contribution increases and reaches 100% at $x = 0.13$ for $\lambda_q^2 = 0.4GeV^2$ (for $\lambda_q^2 = 0.6GeV^2$ it still amounts 50%). So, for x as small as 0.2 we could not trust x-dependence of our result. Of course, there is no possibility to reproduce the correct $x \to 0$ behaviour of the parton density in the present approach as it is determined by the exchanges of the Regge trajectories.

Now we can comment on the contribution of the nonlocal gluon condensate to our sum

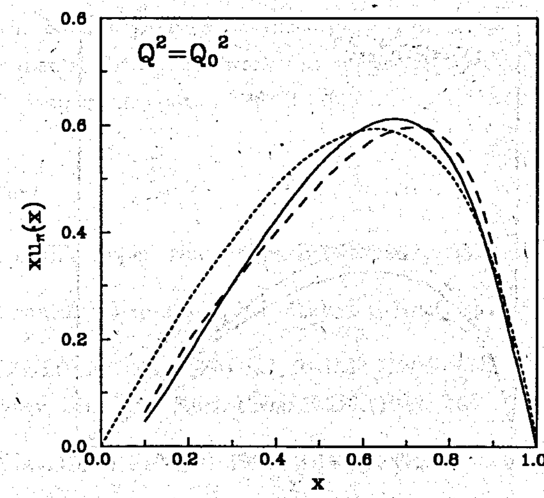

Figure 2: Quark distribution in the pion at the low energy scale $\mu^2 \sim 0.5GeV^2$ calculated from the QCD sum rule for different values of the average virtuality of vacuum quarks: solid and long-dashed curves correspond to $\lambda_q^2 = 0.6GeV^2$ and $\lambda_q^2 = 0.4GeV^2$, respectively. Short-dashed curve is the u-quark density found in the NJL model [22].

rule. As can be easily seen from the WI some part of this contribution is concentrated in the two-point correlation function, which has been studied in ref. [21]. Being numerically rather small, it contains terms not vanishing for $x \to 1$ as distinguished from nonlocal quark condensates that do not spoil the $(1-x)$-behaviour as $x \to 1$, but only renormalize the slope. Therefore, the nonlocal gluon condensate limits the validity of the present approach from the large-x values. This conclusion is made discarding additional terms appearing from the three-point correlator in the WI which can somewhat change the situation. This problem, as well as a particular value of x in the large-x region, where the approach becomes invalid, deserves further investigation and only smallness of the gluon condensate contribution favours our decision to disregard it in the present study.

Since our result is valid only in the limited region of Bjorken variable, we could not evolve it to the experimentally accessible energies. In fig. 2, we compare our calculation with the distribution obtained in the NJL model [22] at the same normalization point and find reasonable agreement between two approaches in a wide region of the momentum

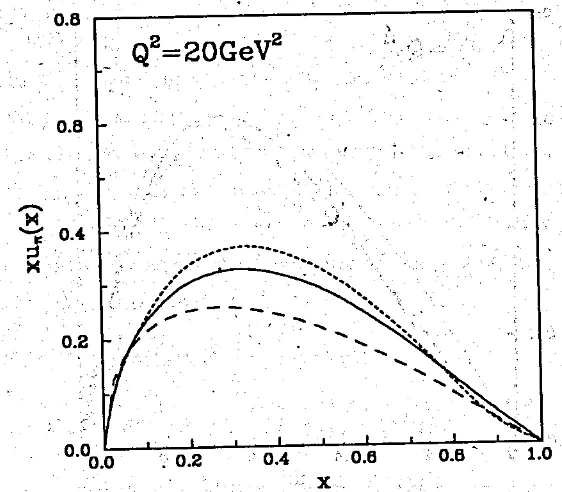

Figure 3: The experimental fits of the valence u-quark distribution in the pion at $Q^2 = 20GeV^2$: SMRS result [24] is depicted by solid curve, GRV analysis [25] is shown by long-dashed one. Short-dashed curve is the u-quark density calculated in the NJL model [22] evolved up to $Q^2 = 20GeV^2$.

fraction. In fig. 3, the latter evolved up to $Q^2 = 20GeV^2$ (short-dashed curve) is compared with the presently available fits of experimental data [23]. It shows good agreement with the result of the analysis of Sutton, Martin, Roberts and Stirling (solid curve) [24], which is consistent with all present Drell-Yan and prompt photon πN data. We also present the (long-dashed) curve due to Glück, Reya and Vogt [25]; however, their result does not agree with E615 experiment [23] which requires the valence distribution to be larger by 20%. Similarly enhanced distribution has been obtained in ref. [26]. If the GRV curve is renormalized within a factor of $1.2 - 1.3$ in the central region, there will be no disagreement between the different analyses.

In conclusion, we have calculated the pionic parton density at low momentum scale in QCD sum rules with nonlocal condensates. It is shown that the parton sum rule is fulfilled only after the bilocal power corrections are accounted for. We have found good agreement with the u-quark distribution function computed in the NJL model which when evolved up to the experimental scales is well comparable with data.

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