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A CHIRAL MODEL FOR EXCITED PIONS

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1. Introduction

The study of radial excitations of light mesons is currently of great interest in hadronic physics. During the next years, facilities at CEBAF and IHEP (Protvino) are going to provide improved experimental information, e.g., on the π' meson. The π' is thought to have mass of $\sim 1.4\,\mathrm{GeV}$. However, recent results suggest that the mass of the π' may be below $1\,\mathrm{GeV}$ [1].

The theoretical description of radially excited mesons poses some interesting challenges. From the point of view of effective meson theories, the introduction of the excited degrees of freedom should not spoil the low–energy theorems for pions which are a consequence of spontaneous chiral symmetry breaking and PCAC. In other words, the pion field must decouple from the "hard" degrees of freedom in the chiral limit in order to describe a Goldstone boson. This requirement restricts the form of the interaction of the newly introduced fields for excited states with the usual pion field.

At quark level, the spontaneous breaking of chiral symmetry and PCAC are concisely described by the Nambu–Jona-Lasinio model, which employs a local four–quark interaction [2, 3]. The bosonization of this model and the momentum expansion of the resulting fermion determinant reproduce the Lagrangian of the linear sigma model, which embodies the physics of soft pions. The NJL model also affords a reasonable description of the massive vector mesons.

When extending the NJL model to describe radial excitations of mesons, it is clear that one has to introduce some degree of non-locality in the four–quark interaction. This non-locality has two related aspects. First, it makes possible the occurrence of excited states "orthogonal" to the ground state. Second, it provides a form factor in the meson–quark–antiquark interaction for the ground state meson as well. Thus, it seems impossible to introduce excited states without, to some extent, modifying the successful results of the usual NJL model.

Many non-local generalizations of the NJL model have been proposed, using either instantaneous [4, 5] or covariant–euclidean [6] effective quark interactions. While reproducing the properties of pions as following from dynamical chiral symmetry breaking, these momentum–dependent interactions allow for considerable leeway in the description of excited states. In this type of schemes the main technical advantage of the NJL model is lost, namely the gap equation is solved by a constant constituent quark mass. A very interesting method of describing the excited meson states in the effective quark model was also proposed in [8]. It would be desirable to have a model on hand which would allow one to include excited states while keeping a constant quark mass.

In this paper, we present a simple extension of the usual NJL model, which describes radial excitations of $\pi-$ and $\sigma-$mesons with a minimum number of additional parameters. In particular, the gap equation of the NJL model remains unchanged. By momentum expansion we obtain the effective Lagrangian of the π–π' system which, after diagonalization, describes the decoupling of the pion in the chiral limit and the vanishing of the π' leptonic decay constant, as expected on general grounds. For finite current quark masses, modifications of pion properties due to the presence of the excited degrees of freedom are seen to be small. Within this approach we evaluate the π' decay constant as a function of the π' mass.

2. Effective quark model with separable interactions

In the usual NJL model, the spontaneous breaking of chiral symmetry is described by a local (current–current) effective quark interaction. This model is defined by the action

$$
\begin{aligned}
S_{\mathrm{NJL}}[\bar\psi, \psi] &= S^0 + S_{\mathrm{int}} , && (1)\\
S^0 &= \int d^4x \bar\psi(x) \left(i\partial\!\!\!/ - m^0 \right) \psi(x) , && (2)\\
S_{\mathrm{int}} &= \frac{g}{2} \int d^4x \left[j_\sigma(x) j_\sigma(x) + j_\pi^a(x) j_\pi^a(x) \right] , && (3)
\end{aligned}
$$

where $j_{\sigma,\pi}(x)$ denote the scalar and pseudoscalar densities of the quark field,

$$
j_\sigma(x) = \bar\psi(x)\psi(x) , \qquad j_\pi^a(x) = \bar\psi(x) i\gamma_5 \tau^a \psi(x) . \qquad (4)
$$

This model can be bosonized by introducing scalar and pseudoscalar meson fields in the standard way. Since the interaction eq.(3) is of current–current form, the bosonization can be achieved through *local* meson fields. This property is of great practical importance in extracting the physical content of this model. It ensures that the resulting effective meson theory, which is obtained by integrating over the quark fields, is formulated in terms of local meson fields. By expanding in the number of derivatives of these local meson fields, one derives an effective meson Lagrangian, which concisely summarizes all low–energy information contained in this model.

The effective meson Lagrangian derived from the NJL model describes only ground–state mesons, *i.e.*, it does not include radial excitations. To include excited states in this picture, one has to use effective quark interactions with a finite range. In general, such interactions require bilocal meson fields for bosonization. A simple possibility which avoids this complication is the use of a separable interaction which is still of current–current form, eq.(3), but which allows for form factors in the definition of the interacting quark currents, eq.(4),

$$
\begin{aligned}
S_{\mathrm{int,sep}} &= \frac{g}{2} \sum_{k=1}^{n} \left(j_\sigma^{(k)2}(x) + j_\pi^{(k)2}(x) \right) , && (5)\\
j_{\sigma,\pi}^{(k)}(x) &= \int d^4x_1 \int d^4x_2 \, \bar\psi(x_1) F_{\sigma,\pi}^{(k)}(x; x_1, x_2) \psi(x_2) . && (6)
\end{aligned}
$$

Here, $F_{\sigma,\pi}^{(k)}(x; x_1, x_2)$ $(k = 1, \ldots n)$ denote a set of form factors, the precise form of which will be specified below. Upon bosonization, eq.(6) leads to an action

$$
S_{\mathrm{sep}} = \int d^4x_1 \int d^4x_2 \, \bar\psi(x_1) \left[(i\partial\!\!\!/_{x_2} - m^0) \delta(x_1 - x_2) \right.
$$

$$+ \int d^4x \sum_{k=1}^{n} \left(\sigma_k(x)F_\sigma^{(k)}(x;x_1,x_2) + \pi_k(x)F_\pi^{(k)}(x;x_1,x_2)\right)\Big]\,\psi(x_2)$$
$$- \frac{1}{2g}\int d^4x \sum_{k=1}^{n} \left(\sigma_k^2(x) + \pi_k^2(x)\right). \qquad (7)$$

This action describes a system of local meson fields, $\sigma_k(x), \pi_k(x)$, interacting with the quarks through form factors.

We define the form factors of eq.(6) in the momentum representation. Due to translational invariance,

$$F_{\sigma,\pi}^{(k)}(x) = \int \frac{d^4P}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}\exp i\left[\tfrac{1}{2}(P+q)\cdot(x-x_1)+\right.$$
$$\left.+\tfrac{1}{2}(P-q)\cdot(x-x_2)\right] F_{\sigma,\pi}^{(k)}(q|P). \qquad (8)$$

Here, q and P denote, respectively, the relative and total momentum of the quark–antiquark pair. We choose the form factor to depend only on the part of the relative momentum transverse to the total momentum,

$$F_{\sigma,\pi}^{(k)}(q|P) \;\equiv\; F_{\sigma,\pi}^{(k)}(q_\perp|P), \qquad q_\perp \;=\; q - \frac{P\cdot q}{P^2}\,P\,. \qquad (9)$$

Eq.(9) is the covariant generalization of the condition of instantaneity of the interaction in the rest frame of the meson, i.e., the frame in which $P = (P_0,0,0,0)$. This ensures the absence of spurious (relative–time) excitations and allows one to interpret the resulting on–shell meson amplitudes as ordinary 3–dimensional bound state amplitudes in the rest frame[1] [10]. This choice leads to a consistent description of excited states. In particular, it allows one to use the concept of a 3–dimensional "excited state" wave function when modelling the form factors.

Our aim is to construct a generalized NJL model for scalar (0^+) and pseudoscalar (0^-) mesons. We therefore choose form factors $F_{\sigma,\pi}^{(k)}$ of the form

$$F_\sigma^{(k)}(q_\perp|P) \;=\; f^{(k)}(q_\perp^2)\,\mathbf{1} \;+\; f_P^{(k)}(q_\perp^2)\,\not{P}\,, \qquad (10)$$
$$F_{\pi^a}^{(k)}(q_\perp|P) \;=\; f^{(k)}(q_\perp^2)\,\gamma_5\,\tau^a \;+\; f_P^{(k)}(q_\perp^2)\,\gamma_5\not{P}\,\tau^a\,. \qquad (11)$$

In the usual NJL model, the first term in the RHS of eqs.(10), (11) corresponds to the standard sigma and π vertices, while the second term is known as the induced vector and axial vector component. We note that the most general form factor could include also the structures $\not{q}_\perp$, $\not{P}\not{q}_\perp$ or $\gamma_5\not{q}_\perp$, $\gamma_5\not{P}\not{q}_\perp$, respectively, which describe bound states with the orbital angular momentum $L = 1$. We shall not consider these components here.

The functions $f^{(k)}, f_P^{(k)}$ in eqs.(10, 11) are scalar functions of $k_\perp^2$ and P^2 which we define for the case where $P = (P_0,0,0,0)$ and $k_\perp^2 = \mathbf{k}^2$. For simplicity, we first consider the simplest case without vector couplings, $f_P^{(k)} \equiv 0$; the role of vector couplings will be investigated subsequently. We want to describe the ground and first excited states of the mesons and choose the form factors as

$$f^{(1,2)}(\mathbf{k}^2) \;=\; \Theta(\Lambda_3^2 - \mathbf{k}^2)\times\begin{cases}1\\ f(\mathbf{k})\end{cases} \qquad (12)$$
$$f(\mathbf{k}) \;=\; a + b\,\mathbf{k}^2 \;\equiv c(1+d\,\mathbf{k}^2). \qquad (13)$$

Note that for $d < 0$ eq.(13) has the form of an excited state wave function with one radial mode.

3. Effective Lagrangian for π and π' mesons

We now construct the effective Lagrangian describing π and π' mesons

$$\mathcal{L} \;=-\frac{1}{2g}\int d^4x\,(\sigma_1^2+\pi_1^2+\sigma_2^2+\pi_2^2) -$$
$$-\; i\,N_c\,\mathrm{tr}\,\log[\,i\not{\partial} - m^0 + \sigma_1 + i\gamma_5\tau^a\pi_1^a + (\sigma_2 + i\gamma_5\tau^a\pi_2^a)f\,]. (14)$$

Here N_c is the colour number ($N_c = 3$). In the usual mean–field approximation, the vacuum of the meson action, eq.(14), is determined

[1] In bilocal field theory, this requirement is usually stated in the form of the so–called Markov–Yukawa condition of covariant instanteneity of the bound state amplitude [5]. An interaction of the form eq.(9) automatically leads to bound state amplitudes satisfying this condition.

by the set of equations ($\pi_i = 0$)

$$\frac{\delta S}{\delta \sigma_1} = -i\, N_c \,\mathrm{tr} \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{1}{\not k - m^0 + \sigma_1 + \sigma_2 f(\mathbf{k})} - \frac{\sigma_1}{g} = 0, \quad (15)$$

$$\frac{\delta S}{\delta \sigma_2} = -i\, N_c \,\mathrm{tr} \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{f(\mathbf{k})}{\not k - m^0 + \sigma_1 + \sigma_2 f(\mathbf{k})} - \frac{\sigma_2}{g} = 0. \quad (16)$$

(Here, we have omitted the trivial dependence of σ_1, σ_2 on the external meson momentum.) In general, the solution of eqs.(15), (16) would have $\sigma_2 \neq 0$, in this case the quark mass becomes momentum–dependent. However, if we choose the form factor, $f(\mathbf{k})$, such that

$$I_1^f \equiv -iN_C \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{f(\mathbf{k})}{m^2 - k^2} = 0, \quad (17)$$

eqs.(15) and (16) admit a solution with constant quark mass, *i.e.*, with $\sigma_2 \equiv 0$ and $\sigma_1 - m^0 \equiv -m$. In this case, eq.(15) reduces to the usual gap equation of the NJL model,

$$-8mI_1 \equiv -8miN_C \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2} = \frac{m^0 - m}{g}. \quad (18)$$

Obviously, the condition, eq.(17), fixes the parameter d in eq.(13), for given values of Λ_3 and m. Eq.(17) expresses the orthogonality of the σ_2–variation to the usual NJL vacuum $\sigma_1 =$ const.. In the following, we will consider the vacuum as defined by eqs.(17) and (18).

We now construct the effective Lagrangian describing the π–π' system. Expanding the action to quadratic order in the fields $\pi_{1,2}$,

$$\mathcal{L} = \int \frac{d^4p}{(2\pi)^4} \sum_{n\geq 2} \mathcal{L}^{(n)}, \qquad \mathcal{L}^{(2)} = \frac{1}{2} \sum_{i,j=1}^{2} \pi_i(p) K_{ij}(p) \pi_j(p) \quad (19)$$

we obtain in leading order momentum expansion

$$K_{11} = Z_1(p^2 - m_1^2), \qquad K_{22} = Z_2(p^2 - m_2^2),$$

$$K_{12} = K_{21} = \gamma p^2, \quad (20)$$

where

$$\begin{aligned} Z_1 &= 4I_2, \qquad Z_2 = 4I_2^{ff}, \\ m_1^2 &= Z_1^{-1}(-8I_1 + g^{-1}) = \frac{m^0}{Z_1 g m}, \\ m_2^2 &= Z_2^{-1}(-8I_1^{ff} + g^{-1}), \\ \gamma &= 4I_2^f. \end{aligned} \quad (21)$$

Here, I_n, I_n^f and I_n^{ff} denote the usual loop integrals arising in the momentum expansion of the NJL quark determinant, but now without, one or two form factors f in the numerator (*cf.* eq.(17)),

$$I_n^{f..f} = -i\, N_c \int_{\Lambda_3} \frac{d^4k}{(2\pi)^4} \frac{f(\mathbf{k})..f(\mathbf{k})}{(m^2 - k^2)^n}. \quad (22)$$

The evaluation of these integrals with a 3–dimensional cutoff is discussed in ref.[9].

Note that a mixing between the π_1– and π_2–fields occurs only in the kinetic (p^2) terms of eq.(19), but not in the mass terms. This is a direct consequence of the orthogonality condition, eq.(17), which ensures that the quark loop with one form factor has no p^2–independent part. This "softness" of the π_1–π_2 mixing has the important consequence that for $p^2 \to 0$ the π_1–field decouples.

The masses of the physical π and π' states are found as zeros of the determinant of $K_{ij}(p^2)$,

$$\Delta(p^2) = Z_1 Z_2 (p^2 - m_1^2)(p^2 - m_2^2) - \gamma^2 p^4 = 0. \quad (23)$$

Expanding over $m_1^2 \propto (m^0)^2$, one finds

$$m_\pi^2 = m_1^2\,(1 - \Gamma^2) + \mathcal{O}(m_1^4), \quad (24)$$

$$m_{\pi'}^2 = m_2^2 + m_1^2\,\Gamma^2 + \mathcal{O}(m_1^4). \quad (25)$$

Here $\Gamma = \gamma/\sqrt{Z_1 Z_2}$.

Thus, for $m^0 \to 0$ the Lagrangian eq.(19) describes a massless Goldstone pion, while the π' remains massive. (Here and in the following, when discussing the dependence of quantities on the current quark mass, m^0, we keep the constituent quark mass fixed and assume the coupling constant, g, to be changed in accordance with m^0, such that the gap equation, eq.(18), remains fulfilled exactly. In this way, the loop integrals and eq.(17) remain unaffected by changes of the current quark mass.)

After renormalization of the pion fields

$$\pi_i^r = \sqrt{Z_i}\,\pi_i \tag{26}$$

we can rewrite $\mathcal{L}^{(2)}$ in the form

$$\mathcal{L}^{(2)} = \frac{1}{2}[(p^2 - m_1^2)\,\pi_1^{r2} + 2\,\Gamma\,p^2\,\pi_1^r\pi_2^r + (p^2 - m_2^2)\,\pi_2^{r2}]. \tag{27}$$

The kinetic part of the Lagrangian $\mathcal{L}^{(2)}$ can be diagonalized by transformation of the pion fields

$$\pi = \frac{1}{\sqrt{2}}(\pi_1^r + \pi_2^r),\ \pi' = \frac{1}{\sqrt{2}}(-\pi_1^r + \pi_2^r). \tag{28}$$

Then, the Lagrangian $\mathcal{L}^{(2)}$ can be rewritten in the form

$$\begin{aligned}\mathcal{L}^{(2)} &= \frac{1}{2}[(p^2 - \frac{m_1^2 + m_2^2}{2(1+\Gamma)})\,\pi^{r2} + (p^2 - \frac{m_1^2 + m_2^2}{2(1-\Gamma)})\,\pi'^{r2} - \\ &\quad - \frac{m_2^2 - m_1^2}{\sqrt{(1-\Gamma^2)}}\,\pi^r\pi'^r],\end{aligned} \tag{29}$$

where

$$\pi^r = \sqrt{1+\Gamma}\,\pi,\ \pi'^r = \sqrt{1-\Gamma}\,\pi'. \tag{30}$$

The Lagrangian (29) can be diagonalized by the additional transformation of the pion fields

$$\bar{\pi} = \pi^r \sin\alpha - \pi'^r \cos\alpha,\ \bar{\pi}' = -\pi^r\cos\alpha - \pi'^r \sin\alpha. \tag{31}$$

As a result, we obtain the final form for the Lagrangian $\mathcal{L}^{(2)}$

$$\mathcal{L}^{(2)} = \frac{1}{2}(p^2 - m_{\bar{\pi}}^2)\,\bar{\pi}^2 + \frac{1}{2}(p^2 - m_{\bar{\pi}'}^2)\,\bar{\pi}'^2. \tag{32}$$

Here

$$m_{\bar{\pi},\bar{\pi}'}^2 = \frac{1}{2(1-\Gamma^2)}\left[m_1^2 + m_2^2 \mp \sqrt{(m_1^2 - m_2^2)^2 + (2m_1m_2\Gamma)^2}\right]. \tag{33}$$

Expanding over $m_1^2 \propto (m^0)^2$, one finds (compare with (24) and (25)).

$$m_{\bar{\pi}}^2 = m_1^2 + \mathcal{O}(m_1^4), \tag{34}$$

$$m_{\bar{\pi}'}^2 = \frac{m_2^2 + m_1^2\Gamma^2}{1-\Gamma^2} + \mathcal{O}(m_1^4). \tag{35}$$

the mixing angle α is obtained as

$$\begin{aligned}\tan\alpha &= \frac{1}{\sqrt{1-\Gamma^2}}\left[\frac{m_2^2 + m_1^2}{m_2^2 - m_1^2}\Gamma + \sqrt{1 - \left(\frac{2m_1m_2\Gamma}{m_2^2 - m_1^2}\right)^2}\right] = \\ &= \sqrt{\frac{1+\Gamma}{1-\Gamma}} + \mathcal{O}(m_1^2).\end{aligned} \tag{36}$$

4. The weak decay constants of the $\bar{\pi}$ and $\bar{\pi}'$

We can now evaluate the weak decay constants of the $\bar{\pi}$ and $\bar{\pi}'$. They are defined through the matrix element of the divergence of the axial current between meson states and the vacuum,

$$\langle 0|\partial_\mu A^\mu|\bar{\pi}\rangle = m_{\bar{\pi}}^2\,f_{\bar{\pi}}, \tag{37}$$

$$\langle 0|\partial_\mu A^\mu|\bar{\pi}'\rangle = m_{\bar{\pi}'}^2\,f_{\bar{\pi}'}. \tag{38}$$

By using the usual local quark weak current of the NJL model for the axial current operator, the decay constants are given by the divergent loop integrals. Taking into account the two renormalizations ((26) and (30)) and the two transformations ((28) and (31)) of the pion fields, we obtain

$$\begin{aligned}f_{\bar{\pi}} &= \sin\alpha\frac{m}{\sqrt{2(1+\Gamma)}}\left(Z_1^{-\frac{1}{2}}Z_1 + Z_2^{-\frac{1}{2}}\gamma\right) - \\ &\quad - \cos\alpha\frac{m}{\sqrt{2(1-\Gamma)}}\left(-Z_1^{-\frac{1}{2}}Z_1 + Z_2^{-\frac{1}{2}}\gamma\right) = \\ &= \frac{m\sqrt{Z_1}}{\sqrt{2}}\left(\sin\alpha\,\sqrt{1+\Gamma} + \cos\alpha\,\sqrt{1-\Gamma}\right),\end{aligned} \tag{39}$$

$$
\begin{aligned}
f_{\bar\pi'} &= -\cos\alpha\frac{m}{\sqrt{2(1+\Gamma)}}\left(Z_1^{-\frac{1}{2}}Z_1+Z_2^{-\frac{1}{2}}\gamma\right)- \\
&- \sin\alpha\frac{m}{\sqrt{2(1-\Gamma)}}\left(-Z_1^{-\frac{1}{2}}Z_1+Z_2^{-\frac{1}{2}}\gamma\right)= \\
&= \frac{m\sqrt{Z_1}}{\sqrt{2}}\left(\sin\alpha\sqrt{1-\Gamma}-\cos\alpha\sqrt{1+\Gamma}\right).
\end{aligned}
\tag{40}
$$

Expanding over $m_1^2 \propto (m^0)^2$, one finds

$$\sin\alpha = \sqrt{\frac{1+\Gamma}{2}}+\mathcal{O}(m_1^2), \tag{41}$$

$$\cos\alpha = \sqrt{\frac{1-\Gamma}{2}}+\mathcal{O}(m_1^2), \tag{42}$$

$$f_{\bar\pi} = m\sqrt{Z_1}+\mathcal{O}(m_1^2), \tag{43}$$

$$f_{\bar\pi'} = \mathcal{O}(m_1^2). \tag{44}$$

The $f_{\bar\pi}$ is very close to the value following from the Goldberger-Treiman identity, and it coincides with one for the case $m^0=0$. On the other hand, the $\bar\pi'$ decay constant vanishes in the chiral limit $(m^0)^2 \sim m_1^2 \to 0$, as expected.

5. Numerical estimates and Conclusions

In summary, the Lagrangian eq.(19) illustrates in a compact way the two possible ways in which the axial current is conserved for vanishing quark mass. Both matrix elements of $\partial_\mu A^\mu$, eqs.(37) and (38), must vanish for $m^0=0$. The pion matrix element, eq.(37), does so as $m_{\bar\pi}^2 \to 0$, with $f_{\bar\pi}$ remaining finite, while for the excited pion matrix element the opposite occurs, $f_{\bar\pi'} \to 0$ with $m_{\bar\pi'}$ finite. We remark that this behaviour has previously been seen in more elaborate models describing chiral symmetry breaking by non-local interactions [4, 11].

We can now estimate $f_{\bar\pi'}$ in this model. We take a constituent quark mass of $m = 300\,\mathrm{MeV}$ and fix the cutoff at $\Lambda_3 = 671\,\mathrm{MeV}$ by fitting the physical pion decay constant $f_{\bar\pi} = 93 MeV$ in the chiral limit, as in the usual NJL model without excited states. Using these parameters we obtain for the quark condensate the standard value $<\bar qq>_0 = -(254\ MeV)^3$, and $g = 9.1\ GeV^{-2}$, $m^0 = 5\ MeV$. Let us also give the values of the integrals $I_n^{f..f}$ (see (22)).

$$
\begin{aligned}
I_1 = 0.15\ m^2, \quad & I_1^f = 0, \quad I_1^{ff} = 0.038\ m^2\ c^2, \\
I_2 = 0.024, \quad & I_2^f = 0.0055\ c, \quad I_2^{ff} = 0.0075\ c^2
\end{aligned}
\tag{45}
$$

and

$$\Gamma = 0.41. \tag{46}$$

From eq.(17) we find $d = -4.06\ GeV^{-2}$. Using eq. (33) we can obtain the equation

$$m_{1,2}^2 = \frac{1-\Gamma^2}{2}\left[m_{\bar\pi}^2+m_{\bar\pi'}^2 \mp \sqrt{(m_{\bar\pi}^2+m_{\bar\pi'}^2)^2-\frac{(2m_{\bar\pi}m_{\bar\pi'})^2}{1-\Gamma^2}}\right] \tag{47}$$

and then calculate c, using eq. (21).

Let us consider the two possible values of the $m_{\bar\pi'}$: 1) $m_{\bar\pi'} = 750\ MeV$ [1] and
2) $m_{\bar\pi'} = 1300\ MeV$ [12]. In the first case, we obtain

$$
\begin{aligned}
m_1 = 139MeV,\ m_2 = 682MeV,\ c = 1.64,\ \alpha = 57.5^o, \\
f_{\bar\pi} = 92.9MeV,\ f_{\bar\pi'} = 0.65MeV.
\end{aligned}
\tag{48}
$$

For the second case we get

$$
\begin{aligned}
m_1 = 142MeV,\ m_2 = 1180MeV,\ c = 1.41,\ \alpha = 57.3^o, \\
f_{\bar\pi} = 92.9MeV,\ f_{\bar\pi'} = 0.32MeV.
\end{aligned}
\tag{49}
$$

We can see that the modifications of $f_{\bar\pi}$ due to the excited states turn out to be very small. The ratios $f_{\bar\pi'}/f_{\bar\pi}$ are of the same order as the ones found in models with bilocal interactions [11].

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