ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-96-108

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PROBLEMS WITH NONPERTURBATIVE EFFECTS IN DEEP INELASTIC SCATTERING

Submitted to «Physics Letters B»

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1996

1. The present theory of deep inelastic scattering (DIS) is based on two approaches which are the complement of one another. In the first approach (see e.g. ref. [1] and references therein) one assumes that only Feynman diagrams from a certain class dominate in DIS, and in the second approach DIS is considered in the framework of the operator product expansion (OPE) [2]. Although the assumptions used in the both approaches are natural, the problem of their substantiation remains since we do not know how to work with QCD beyond perturbation theory. In particular, the OPE has been proved only in perturbation theory [3] and its validity beyond that theory is problematic (see the discussion in ref. [4] and references therein).

In the present paper we show that an important information about the structure of the (electromagnetic or weak) current operator in DIS can be obtained from the investigation of restrictions imposed on this operator by its commutation relations with the representation operators of the Poincare group.

2. If $J(x)$ is the electromagnetic or weak current operator and q is the momentum transfer then the DIS cross-section is fully defined by the hadronic tensor

$$W^{\mu\nu} = \frac{1}{4\pi} \int e^{\imath qx} \langle P'|J^{\mu}(x)J^{\nu}(0)|P'\rangle d^4x \qquad (1)$$

where $|P'\rangle$ is the state of the initial nucleon with the four-momentum P' and we use $\mu, \nu = 0, 1, 2, 3$ to denote the components of the operator $J(x)$.

Translational invariance of the current operator implies that

$$J(x) = exp(\imath Px)J(0)exp(-\imath Px), \qquad (2)$$

where P is the four-momentum operator, and Lorentz invariance implies that

$$[M^{\mu\nu}, J^{\rho}(0)] = -\imath(g^{\mu\rho}J^{\nu}(0) - g^{\nu\rho}J^{\mu}(0)) \qquad (3)$$

where $M^{\mu\nu}$ are the Lorentz group generators and $g^{\mu\nu}$ is the Minkowski tensor.

In turn, the state $|P'\rangle$ is the eigenstate of the operator P with the eigenvalue P' and the eigenstate of the spin operators $\mathbf{S}^2$ and S^z which are constructed from $M^{\mu\nu}$. In particular, $P^2|P'\rangle = m^2|P'\rangle$ where m is the nucleon mass. Therefore the four-momentum operator necessarily depends on the soft part of the interaction which is responsible for binding of quarks and gluons in the nucleon. The Lorentz transformations of the

nucleon state are described by the operators $M^{\mu\nu}$ and therefore in the general case they also depend on the soft part.

It is important to note that the same operators $(P^{\mu}, M^{\mu\nu})$ describe the transformations of both the operator $J(x)$ and the state $|P'\rangle$, and this guaranties that $W^{\mu\nu}$ has the correct transformation properties.

We see that the relation between the current operator and the state of the initial nucleon is highly nontrivial. Meanwhile in the present theory they are considered separately. In the framework of the approach based on Feynman diagrams the possibility of the separate consideration follows from the factorization theorem [5] which asserts in particular that the amplitude of the lepton-parton interaction entering into diagrams dominating in DIS depend only on the hard part of this interaction. Moreover, in leading order in $1/Q$, where $Q = |q^2|^{1/2}$, one obtains the parton model up to anomalous dimensions and perturbative QCD corrections which depend on $\alpha_s(Q^2)$ where α_s is the QCD running coupling constant.

It is well-known that the parton model is equivalent to impulse approximation (IA) in the infinite momentum frame (IMF). This fact is in agreement with our experience in conventional nuclear and atomic physics according to which in processes with high momentum transfer the effect of binding is not important and the current operator can be taken in IA. However this experience is based on the nonrelativistic quantum mechanics where only the Hamiltonian is interaction dependent and the other nine generators of the Galilei group are free. Note also that in the nonrelativistic case the kinetic energies and the interaction operators in question are much smaller than the masses of the constituents.

The usual motivation of the parton model is that, as a consequence of asymptotic freedom (i.e. the fact that $\alpha_s(Q^2) \to 0$ when $Q^2 \to \infty$), the partons in the IMF are almost free and therefore, at least in leading order in $1/Q$, the soft part of $J(x)$ is not important. We will consider a bit later whether this property can be substantiated in the framework of the OPE but first we consider some consequences of Eqs. (2) and (3).

3. As noted by Dirac [6], the operators $(P^\mu, M^{\mu\nu})$ can be realized in different representations, or, in Dirac's terminology, in different forms of dynamics. Suppose that the Hamiltonian P^0 contains the soft part and consider the well-known relation $[M^{0i}, P^k] = -\imath\delta_{ik}P^0$ $(i,k = 1,2,3)$. Then it is obvious that if all the operators P^k are free then all the operators M^{0i} inevitably contain the soft part and *vice versa*, if all the operators M^{0i} are free then all the operators P^k inevitably contain this part. According to the Dirac classification [6], in the instant form the Hamiltonian P^0 and the operators M^{0i} are interaction dependent and the other six generators of the Poincare group are free, while in the point form all the components P^μ are interaction dependent and all the operators $M^{\mu\nu}$ are free. In the front form the operators P^- and M^{-j} $(j = 1,2, p^\pm = p^0 \pm p^z)$ are interaction dependent and the other seven generators are free. The fact that if P^- is the only dynamical component of P then all the M^{-j} inevitably contain interaction terms follows from the relation $[M^{-j}, P^l] = -\imath\delta_{jl}P^-$. Of course, the physical results should not depend on the choice of the form of dynamics and in the general case all ten generators can be interaction dependent.

The usual form of the electromagnetic current operator is $J^\mu(x) = \mathcal{N}\{\bar\psi(x)\gamma^\mu\psi(x)\}$ and in particular $J^\mu(0) = \mathcal{N}\{\bar\psi(0)\gamma^\mu\psi(0)\}$, where $\mathcal{N}$ stands for the normal product and for simplicity we do not write flavor operators and color and flavor indices. However such a definition ignores the fact that the product of two field operators at coinciding points is not a well-defined operator (strictly speaking, the operator $\psi(0)$ also is not defined since $\psi(x)$ is the operator-valued distribution; for a more detailed discussion see ref. [7]). The reader thinking that it is not reasonable to worry about the mathematical rigor will be confronted with the following contradiction.

The canonical quantization on the hyperplane $x^0 = 0$ or on the light cone $x^+ = 0$ (which leads to the instant and front forms respectively [6]) implies that the operator $\psi(0)$ is free since the Heisenberg and Schrodinger pictures coincide at $x = 0$. Then $J(0)$ is free too and, as follows from Eq. (3), the interaction terms in $M^{\mu\nu}$ should commute with $J^\rho(0)$. If the operators $M^{\mu\nu}$ are constructed by means of canonical quantization then in QED the interaction terms and their commutators with $J^\rho(0)$ can be readily calculated. The commutators are expressed in terms of the Schwinger terms [8] which cannot be equal to zero (the corresponding calculation is given in ref. [7]). Therefore the conclusion that all the components of $J(0)$ are free is incorrect and some components of $J(0)$ are inevitably interaction dependent.

Moreover, it can be shown that if the field operators are quantized, for example, on the hyperplane $x^0 = 0$ then the operator $\mathbf{J}(0)$ in QED is necessarily interaction dependent. Indeed, the generator of the gauge transformations is $div\mathbf{E}(\mathbf{x}) - J^0(\mathbf{x})$, and if $\mathbf{J}(0)$ is gauge invariant then $[div\mathbf{E}(\mathbf{x}) - J^0(\mathbf{x}), \mathbf{J}(0)] = 0$. The commutator $[J^0(\mathbf{x}), \mathbf{J}(0)]$ cannot be

equal to zero [8] and therefore $\mathbf{J}(0)$ does not commute with $div\mathbf{E}(\mathbf{x})$ while the free operator $\mathbf{J}(0)$ commutes with $div\mathbf{E}(\mathbf{x})$.

The above examples illustrate the well-known fact that formal manipulations with local operators in quantum field theory can lead to incorrect results. For this reason we prefer to rely only upon algebraic considerations according to which all the components of $J(0)$ cannot be free simply because there is no reason for the interaction terms in $M^{\mu\nu}$ to commute with the free operators $J^{\rho}(0)$ (see Eq. (3)). Therefore in the instant and front forms some of the operators $J^{\rho}(0)$ depend on the soft part. On the other hand, as follows from Eq. (3), if the operator $J(0)$ is free in the point form, this does not contradict Lorentz invariance but, as follows from Eq. (2), the operator $J(x)$ in that form necessarily contains the soft part.

The problem of the correct definition of the product of two local operators at coinciding points is known as the problem of constructing the composite operators (see e.g. ref. [9]). So far this problem has been solved only in the framework of perturbation theory for special models. When perturbation theory does not apply the usual prescriptions are to separate the arguments of the operators in question and to define the composite operator as a limit of nonlocal operators when the separation goes to zero (see e.g. ref. [10] and references therein). Since we do not know how to work with quantum field theory beyond perturbation theory, we do not know what is the correct prescription. Moreover, it is not clear at all whether it is possible to define local interaction dependent operators in QCD. Indeed, the dependence of an operator on the soft part implies that the operator depends on the integrals from the quark and gluon field operators over the region of large distances where

the QCD running coupling constant α_s is large. It is obvious that such an operator cannot be local. In particular it is not clear whether in QCD it is possible to construct local electromagnetic and weak current operators beyond perturbation theory.

4. In the framework of the OPE the product of the currents entering into Eq. (1) can be written symbolically as

$$J(x)J(0) = \sum_i C_i(x^2) x_{\mu_1} \cdots x_{\mu_n} O_i^{\mu_1 \cdots \mu_n} \qquad (4)$$

where $C_i(x^2)$ are the c-number Wilson coefficients while the operators $O_i^{\mu_1 \cdots \mu_n}$ depend only on field operators and their covariant derivatives at the origin of Minkowski space and have the same form as in perturbation theory. The basis for twist two operators contains in particular

$$O_V^{\mu} = \mathcal{N}\{\bar{\psi}(0)\gamma^{\mu}\psi(0)\}, \quad O_A^{\mu} = \mathcal{N}\{\bar{\psi}(0)\gamma^5\gamma^{\mu}\psi(0)\} \qquad (5)$$

As noted above, the operator $J(x)$ necessarily depends on the soft part while Eq. (4) has been proved only in the framework of perturbation theory. Therefore if we use Eq. (4) in DIS we have to assume that either nonperturbative effects are not important to some orders in $1/Q$ and then we can use Eqs. (1) and (4) only to these orders (see e.g. ref. [11]) or it is possible to use Eq. (4) beyond perturbation theory. The question also arises whether Eq. (4) is valid in all the forms of dynamics (as it should be if it is an exact operator equality) or only in some forms.

In the point form all the components of P depend on the soft part and therefore, in view of Eq. (2), it is not clear why there is no soft part in the x dependence of the right hand

side of Eq. (4), or if it is possible to include the soft part only into the operators O_i then why they have the same form as in perturbation theory.

One might think that in the front form the $C_i(x^2)$ will be the same as in perturbation theory due to the following reasons. The value of q^- in DIS is very large and therefore only a small vicinity of the light cone $x^+ = 0$ contributes to the integral (1). The only dynamical component of P is P^- which enters into Eq. (4) only in the combination P^-x^+. Therefore the dependence of P^- on the soft part is of no importance. These considerations are not convincing since the integrand is a singular function and the operator $J(0)$ depends on the soft part in the front form, but nevertheless we assume that Eq. (4) in the front form is valid.

If we assume as usual that there is no problem with the convergence of the OPE series then experiment makes it possible to measure each matrix element $\langle P'|O_i^{\mu_1\cdots\mu_n}|P'\rangle$. Let us consider, for example, the matrix element $\langle P'|O_V^\mu|P'\rangle$. It transforms as a four-vector if the Lorentz transformations of O_V^μ are described by the operators $M^{\mu\nu}$ describing the transformations of $|P'\rangle$, or in other words, by analogy with Eq. (3)

$$[M^{\mu\nu}, O_V^\rho] = -\imath(g^{\mu\rho}O_V^\nu - g^{\nu\rho}O_V^\mu) \qquad (6)$$

It is also clear that Eq. (6) follows from Eqs. (2-4). Since the M^{-j} in the front form depend on the soft part, we can conclude by analogy with the above consideration that at least some components O_V^μ, and analogously some components $O_i^{\mu_1\cdots\mu_n}$, also depend on the soft part. Since Eq. (6) does not depend on Q, this conclusion has nothing to do with asymptotic freedom and is valid even in leading order in $1/Q$ (in contrast with the statement of the factorization theorem [5]). Since the struck quark is not free but interacts nonperturbatively with the rest of the target then, in terminology of ref. [1], not only "handbag" diagrams dominate in DIS but some "cat ears" diagrams or their sums are also important (in other words, even the notion of struck quark is questionable).

Since the operators $O_i^{\mu_1\cdots\mu_n}$ depend on the soft part then by analogy with the considerations in subsection 3 we conclude that the operators in Eq. (5) are ill-defined and the correct expressions for them involve integrals from the field operators over large distances where the QCD coupling constant is large. Therefore the Taylor expansion at $x = 0$ is questionable, and, even if it is valid, the expressions for $O_i^{\mu_1\cdots\mu_n}$ will depend on higher twist operators which contribute even in leading order in $1/Q$.

5. Let us now discuss our results. First we have shown that the current operator nontrivially depends on the nonperturbative part of the interaction responsible for binding of quarks and gluons in the nucleon. Then the problem arises whether this part contributes to DIS. Our consideration shows that the dependence of $J(x)$ on the nonperturbative part of the interaction makes the OPE problematic. Nevertheless we assume that Eq. (4) is valid beyond perturbation theory but no form of the operators $O_i^{\mu_1\cdots\mu_n}$ is prescribed. Then we come to conclusion that the nonperturbative part contributes to DIS even in leading order in $1/Q$.

To understand whether the OPE is valid beyond perturbation theory several authors (see e.g. ref. [4] and references therein) investigated some two-dimensional models and came to different conclusions. We will not discuss the arguments of

these authors but note that the Lie algebra of the Poincare group for 1+1 space-time is much simpler than for 3+1 one. In particular, the Lorentz group is one-dimensional and in the front form the operator M^{+-} is free. Therefore Eqs. (3) and (6) in the "1+1 front form" do not make it possible to conclude that the operators $J^\rho(0)$ and O_V^ρ should depend on the nonperturbative part of the quark-gluon interaction.

Since the operators $O_i^{\mu_1\cdots\mu_n}$ in Eq. (4) should depend on the nonperturbative part of the quark-gluon interaction then, as noted above, there is no reason to think that these operators are local but even if they are then twist (dimension minus spin) no longer determines in which order in $1/Q$ the corresponding operator contributes to DIS. This is clear from the fact that the dependence on the nonperturbative part implies that we have an additional parameter Λ with the dimension of momentum where Λ is the characteristic momentum at which $\alpha_s(\Lambda^2)$ is large.

Nevertheless if we assume that (for some reasons) Eq. (4) is still valid and consider only the q^2 evolution of the structure functions then all the standard results remain. Indeed the only information about the operators $O_i^{\mu_1\cdots\mu_n}$ we need is their tensor structure since we should correctly parametrize the matrix elements $\langle P'|O_i^{\mu_1\cdots\mu_n}|P'\rangle$. However the derivation of sum rules in DIS requires additional assumptions.

Let us consider sum rules in DIS in more details. It is well-known that they are derived with different extent of rigor. For example, the Gottfried and Ellis-Jaffe sum rules [12] are essentially based on model assumptions, the sum rule [13] was originally derived in the framework of current algebra for the time component of the current operator while the sum rules

[14] also involve the space components. As noted in subsection 3, the operator $\mathbf{J}(0)$ is necessarily interaction dependent; on the other hand there exist models in which $J^0(0)$ is free (see e.g. calculations in scalar QED in ref. [7]). Therefore in the framework of current algebra the sum rule [13] is substantiated in greater extent than the sum rules [14] (for a detailed discussion see refs. [15, 10]). Now the sum rules [13, 14] are usually considered in the framework of the OPE and they have the status of fundamental relations which in fact unambiguously follow from QCD. However the important assumption in deriving the sum rules is that the expression for O_V^μ coincides with $J^\mu(0)$, the expression for O_A^μ coincides with the axial current operator $J_A^\mu(0)$ etc. (see Eq. (5)). Our results show that this assumption has no physical ground. Therefore although (for some reasons) there may exist sum rules which are satisfied with a good accuracy, the statement that the sum rules [13, 14] unambiguously follow from QCD is not substantiated.

For comparing the theoretical predictions for the sum rules with experimental data it is also very important to calculate effects in next-to-leading order in $1/Q$. As shown in ref. [16] there exist serious difficulties in calculating such effects in the framework of the OPE, and the authors of ref. [16] are very pessimistic about the possibility to overcome these difficulties (while in our approach problems exist even in the leading order).

The current operator satisfying Eqs. (2) and (3) can be explicitly constructed for systems with a fixed number of interacting relativistic particles [17]. In such models it is clear when the corresponding results and the results in IA are similar and when they considerably differ [18].

We conclude that the present theory of DIS based on perturbative QCD does not take into account the dependence of the current operator on the nonperturbative part of the quark-gluon interaction which cannot be neglected even in leading order in $1/Q$. On the other hand, the present theory has proven rather successful in describing many experimental data. It is very important to understand why this situation takes place.

Acknowledgments. The author is grateful to G.Salme and E.Pace for the idea to write this paper, constructive criticism and valuable discussions. The author has also greatly benefited from the discussions with B.L.G.Bakker, R. Van Dantzig, A.V.Efremov, L.A.Kondratyuk, B.Z.Kopeliovich, M.P.Locher, M.Marinov, S.Mikhailov, P.Muelders, N.N.Nikolaev, R.Petronzio, R.Rosenfelder, O.Yu.Shevchenko, I.L.Solovtsov and H.J.Weber. This work is supported by grant No. 96-02-16126a from the Russian Foundation for Fundamental Research.

References

[1] P.V.Landshoff and J.C.Polkinghorne, Phys. Rep. 5 (1972) 1; F.Close, *An Introduction to Quark and Partons* (Academic Press, London - New York - San Francisco, 1979).

[2] K.G.Wilson, Phys.Rev. 179 (1969) 1499; Phys. Rev. D 3 (1971) 1818.

[3] R.Brandt, Fortschr. Phys. 18 (1970) 249; W.Zimmerman, Annals of Physics 77 (1973) 570.

[4] V.A.Novikov, M.A.Shifman, A.I.Vainshtein and V.Zakharov, Nucl. Phys. B 249 (1985) 445; M.Soldate, Annals of Physics 158 (1984) 433; F.David, Nucl. Phys. B 263 (1986) 637.

[5] A.V.Efremov and A.V:Radyushkin, Rivista Nuovo Cimento 3, No 2, (1980); G.Curci, W.Furmanski and R.Petronzio, Nucl. Phys. B 175 (1982) 27; R.K.Ellis, W.Furmanski and R.Petronzio, Nucl. Phys. B 212 (1983) 29.

[6] P.A.M.Dirac, Rev. Mod. Phys. 21 (1949) 392.

[7] F.M.Lev, hep-ph 9512010.

[8] J.Schwinger, Phys. Rev. Lett. 3 (1959) 296.

[9] W.Zimmerman, Annals of Physics 77 (1973) 536; O.I.Zavialov. *Renormalized Feynman Diagrams* (Nauka, Moscow 1979).

[10] R.Jackiw, in *Lectures on Current Algebra and its Applications*, S.Treiman, R.Jackiw and D.Gross, eds. (Princeton University Press, Princeton NJ 1972).

[11] R.L.Jaffe and M.Soldate, Phys. Lett. B 105 (1981) 467.

[12] K.Gottfried, Phys. Rev. Lett. 18 (1967) 1174; J.Ellis and R.L.Jaffe, Phys. Rev. D 9 (1974) 1444, D 10 (1974) 1669.

[13] S.L.Adler, Phys. Rev. 143 (1966) 1144.

[14] J.D.Bjorken, Phys. Rev. 148 (1966) 1467, 163 (1967) 1767; D.J.Gross and C.H.Llewellyn Smith, Nucl. Phys. B 14 (1969) 337.

[15] M.Gell-Mann, Physics 1 (1964) 63.

[16] G.Martinelli and C.T.Sachrajda, hep-ph 9605336.

[17] F.M.Lev, Annals of Physics 237 (1995) 355.

[18] F.M.Lev, hep-ph 9507421, 9511406.

Received by Publishing Department
on July 11, 1996.