ОБЪЕДИНЕННЫЙ
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96-10

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ELECTRODYNAMICS IN SPACE WITH MULTI-DIMENSIONAL TIME

Talk given at the «Workshop on Macroscopic Electrodynamics», Cracow, 5—11 November, 1995

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1996

Introduction

The Einsteinean theory of relativity was the first to reveal a symmetry in properties of space and time, the subsequent generalizations introducing a proper time for each particle [1] and for each space point **x** [2] equalized space and time in right still more. Though these generalizations by himself did not discover any new physical effects they improved essentially the theory and allowed to formulate a condition of compatibility for motion equations forbidding an exchange of faster-than-light signals and to develop a simple renormalizatiom procedure. It is interesting to take a next step on this way and to consider a more consistent from relativistic viewpoint theory with the equal number of space and time co-ordinates.

The most general philosophical sense of the categories of space and time express, respectively, structural correlations of a heterogeneity, co-existence of material objects and a property of their changeability, unsteadiness [3]. One can speak about time only in the case when some quantitative or qualitative changes occur. We grasp time not immediately but only by means of comparison of several "photographs" of the events happening around us. In an utterly immutable world there is no time and in this sense one can say that time is created by changes.

Though physical theories prompt that at level of ultra-small scales space, apparently, possesses supplementary dimensions, experience convinces us that at macroscopic distances and in a region of up to now accessible microscopic lengths ($\geq 5.10^{-16}$ cm) it is three-dimensional. An assumption of space extra-dimensions results at once in a violation of the habitual for us matter and energy conservation laws because in this case a motion and an information transfer along extra dimensions become possible what would be treated as a created "from nothing" or a disappearance "into nowhere". We and our physical devices do not fix such phenomena in Nature. And what is more, the simple calculations convince that an existance of a stable atomes becomes at all impossible in thise case.

The experimental status of the time dimensionality is not so evident. For the space of centuries a linear sequence of the following one after the other moments when the order of priority of any event with respect to all others is determined by a parameter which we just name as time was considered as the only one property of the time. A more late theories introduced an idea about a space-time curvature and trained us to a thought about possible time discretedness at level of very small scales [3, 4]. Nevertheless, until recently, right up to an appearance of string and supersymmetric generalizations, neither theory made seriously an encroachment upon the time one-dimensionality. It is "fused" into the structure of our attitude. Some philosophers and physiologists incline even to suppose that the time one-dimensionality has an effect on our brain structure and therefore has to be considered as a priory one.

To what extent such an "evidence" does come true? The time multi-dimensionality is equivalent to a splitting of the one-dimensional events into seeresses of "more particular" events whose order of priority is characterized by supplementary parameters t_2, t_3 and so on. But why to put in order any, even very complicated developments it is enough

always only one parameter? The space diversity of our world is multi-dimensional, but its time is for some reason one-dimensional.

In order that we may size up the essence of time, it is necessary to investigate various generalizing models because only in that way one can clear up a "degree of stability" of its properties.

Multi-dimensional vector of time

Dorling in 1970 and Demers in 1975 were, probably, the first who investigated a possibility of introducing into physical theory an idea of multi-dimensional, in general case even infinite-dimensional, time [5, 6]. Comparing the geometrical properties of space- and time-like intervals in one- and multi-dimensional spaces, both authors drew a conclusion that the introducion of multi-dimensional time violates the energy conservation law. Owing to that some experimentally non-observable decays of particles become possible and vacuum loses its stability with respect to a creation of unrestricted amount of matter. These peculiarity of any multi-time approach seems quite inadmissible, so the authors content oneself only by short report about results of their investigations without any attempts to sidestep the revealed difficulties.

Irrespective of these results, a theory of multi-dimensional time was investigated by N.Kalizin [7]. Formally, space and time components of a world vector x_μ distinguish in his theory, like the papers [5, 6], only by the signs which they have in the expression of a square "length"

$$ds^2 = x_\mu x^\mu = \sum_{i=1}^{\infty} c_i^2 dt_i^2 - \sum_{i=1}^{3} dx_i^2.$$

Besides, N.Kalizin supposed that each next coefficient c_i characterizing the light velocity in the corresponding time dimension excels a previous one: $c_i > c_{i-1}$. Because the variable t_i passes (c_i/c_{i-1}) times slower than t_{i-1}, then if $(c_i/c_{i-1} << 1)$ we pay no heel to the "time diversity". Unfortunately, N.Kalizin had no time to analyze consequences of his theory and the latter was left on a level of initial hypothesis which, however, stimulated subsequent investigations in this field [1].

In the following the idea of multi-time worlds, in general air, was popularized by A.D.Sacharov [8]. However, its concrete realizations were discussed, mainly, in connection with the attempts to avoid the difficulties appearing in theories with faster-than-light particles by generalization of Lorentz transformations beyond the light cone boundaries where due to the unequal number of space and time dimensions $(3+1)$ the light wave from a point source loses its spherical shape and the principle of relativity is violated [9 – 11]. For that purpose it was enough to consider a six-dimensional space-time with the some light velocity along all three time directions.

In several papers (see, for example, [12 – 15]) it was supposed that an evolution of events along each of three space axes x_i is determined by its proper time independent

[1] I am deeply indebted to late Nicola Kalizin for numerous discussions of properties of multi-time theories during his visits to Dubna.

of two others x-axis, i. e. the six-dimensional world was considered as a simple product of three two-dimensional ones:

$$\{x_\mu, \mu = 1, ..., 6\} = (x, t_x) \otimes (y, t_y) \otimes (z, t_z).$$

In this case for each pair (x_i, t_i) one can use the usual Lorentz transformation. However, such an approach is unsatisfactory because, particularly, it doesn't confine Tomas precession and the transverse Doppler effect [16, 17].

One can find in literature also other, more refined multi-time models. Paying homage to an resourcefulness of the authors we, nevertheless, will not discuss these models. Be founded on the famous Ockham principle we confine oneself for the present to a minimal number of hypothesis and, following the papers [18 – 21], shall consider space and time co-ordinates as utterly equal in right quite independent one from an other components of a six-dimensional vector

$$(\hat{\mathbf{x}})_\mu = (-\mathbf{x}, \hat{t})^\mu = (\mathbf{x}.\hat{t})^{T\mu}.$$

(Here and in what follows the superscript "T" denotes the transpose, three-dimensional vectors in x- and t-subspaces will be denoted, respectively, by bold symbols and by a hat, six-dimensional vectors we shall denote by bold symbols with a hat; the latin and greek indices take values $k = 1, ..., 3, \mu = 1, ..., 6$). Several papers contain somewhat less convenient but mathematically equivalent three-dimensional complex vector expression $\mathbf{X} = \mathbf{x} + ic\mathbf{t}$ is used [3, 13].

We shall consider the multi-time generalization by using as an example the classical (non-quantum) electrodynamics. It will be shown that if we take into account the causality principle, one can avoid the mentioned above difficulties with vacuum instability. As in the usual one-time theory all created waves (particles in quantum case) have positive energies.

Generalization of Maxwell equations

Let us consider a six-dimensional vector-potential

$$(\hat{\mathbf{A}})_\mu = (-\mathbf{A}, \hat{A})^T_\mu, \qquad (\hat{\mathbf{A}})^\mu = (\mathbf{A}, \hat{A})^{T\mu}$$

and electromagnetic 6×6 tensor $\mathcal{F}_{\mu\nu} = \partial A_\mu/\partial x^\nu - \partial A_\nu/\partial x^\mu$ The latter can be written in a matrix form

$$\hat{\mathbf{F}} = \begin{pmatrix} -\hat{\mathbf{H}} & \hat{\mathbf{E}} \\ -\hat{\mathbf{E}}^T & \hat{\mathbf{G}} \end{pmatrix};$$

where

$$\mathbf{H} = \nabla \times \mathbf{A} = \begin{pmatrix} 0 & -H_3 & H_2 \\ H_3 & 0 & -H_1 \\ -H_2 & H_1 & 0 \end{pmatrix}, \qquad (2a)$$

$$\hat{G} = -\hat{\nabla} \times \hat{A} = \begin{pmatrix} 0 & -G_3 & G_2 \\ G_3 & 0 & -G_1 \\ -G_2 & G_1 & 0 \end{pmatrix} \qquad (2b)$$

are the magnetic and "time-magnetic" 3×3 tensors,

$$\hat{\mathbf{E}} = \mathbf{A}\hat{\nabla} - \nabla\hat{A} \qquad (3)$$

is the 9-component electric field [2]. The del operators $\hat{\boldsymbol{\nabla}}_\mu = (-\boldsymbol{\nabla}, \hat{\nabla})_\mu$ and $\hat{\nabla}_i = -\partial/\partial t_i$.

[2] The physical meaning of the tensor $\hat{\mathbf{E}}$ becomes more clear if we take into account that the tensor value on a fixed time trajectory $\mathbf{E} \equiv \hat{\mathbf{E}}\hat{\tau} = -\boldsymbol{\nabla}\varphi - \partial\mathbf{A}/\partial\hat{\tau}$, where $\varphi = \hat{A}\hat{\tau}$. This is similar to Maxwell expression $\mathbf{E} = -\boldsymbol{\nabla}\varphi - \partial\mathbf{A}/\partial t$.

It is easy to prove that the tensor $\mathcal{F}^\mu_{\ \nu} = g^{\mu\alpha}\mathcal{F}_{\alpha\nu}$, where $\hat{\mathbf{g}} = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$ and I is the unity matrix, can be obtained from (1) by the replacements $\hat{\mathbf{H}} \to (-\hat{\mathbf{H}})$, $\hat{\mathbf{E}} \to (-\hat{\mathbf{E}})$. The contravariant tensor $\mathcal{F}^{\mu\nu} = g^{\nu\beta}\mathcal{F}_{\alpha\beta}$ can be obtained from (1), if $\hat{\mathbf{E}} \to (-\hat{\mathbf{E}})$ and $\hat{\mathbf{E}}^T \to (-\hat{\mathbf{E}}^T)$. The magnetic field $\hat{\mathbf{H}}$ and the new field $\hat{G}$ are vectors, respectively, in x- and t-spaces. From this point of view the generalized electric field $\hat{\mathbf{E}}_{ik}$ takes an intermediate place. Its indices i and k correspond to both subspaces : i to x- and k to t-spaces.

From action principle $dS = 0$ where

$$S = \frac{1}{c^2}\int \hat{\mathbf{A}}\hat{\mathbf{J}}d^4x + \frac{1}{16\pi c}\int \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}d^4x,$$

$\hat{\mathbf{J}}_\mu = (-\mathbf{J}, c\hat{\rho})$, $\mathbf{J} = c\rho(\hat{\mathbf{x}})$, $\hat{\rho} = \rho(\hat{\mathbf{x}})\hat{\tau}$, $\hat{\mathbf{v}} = (-\mathbf{v}, c\hat{\tau})^T$, $\mathbf{v}$ is the three-dimensional velocity in x-subspace, $\hat{\tau} = d\hat{t}/dt$ is the analogous velocity in t-subspace, t is a proper time along the considered time trajectory $\hat{t}$ and the integration is performed over all x-subspace along the trajectory $\hat{t}$ ($d^4x = dV dt$), one can get the equation

$$\hat{\boldsymbol{\nabla}}\hat{\mathbf{F}} = 4\pi c^{-1}\hat{\mathbf{J}},$$

which can be split into two three-dimensional ones:

$$(\boldsymbol{\nabla} \times \mathbf{H})_i + \hat{\boldsymbol{\nabla}}_k E_{ik} = 4\pi c^{-1}\rho(\hat{\mathbf{x}})v_i \qquad (4)$$

$$(\hat{\nabla} \times \hat{G})_i + \boldsymbol{\nabla}_k E_{ki} = 4\pi\rho(\hat{\mathbf{x}})\tau_i \qquad (5)$$

The two other generalized equations can be derived. if we multiply the equality (3) at first time by the operator $\boldsymbol{\nabla}$, at second time by the $\hat{\nabla}$, take into account the identities $\hat{\boldsymbol{\nabla}} \times$

$\hat{\nabla}\hat{\mathbf{A}} = \nabla \times \nabla\mathbf{A} = 0$ and replace in the obtained expressions the vector products $\nabla \times \mathbf{A}$ and $\hat{\nabla} \times \hat{A}$ by the fields $\mathbf{H}$ and $-\hat{G}$:

$$\hat{\nabla}_i H_k - \epsilon_{kmn} \nabla_m E_{ni} = 0 \quad (6)$$

$$\nabla_i G_k + \epsilon_{kmn} \hat{\nabla}_m E_{in} = 0 \quad (7)$$

where ϵ_{kmn} is the antisymmetrical unit tensor.

One can get two additional equations by multiplying the expressions $\mathbf{H} = \nabla \times \mathbf{A}$ and $\hat{G} = -\hat{\nabla} \times \hat{A}$, respectively, by the operators ∇ and $\hat{\nabla}$:

$$\nabla\mathbf{H} = \hat{\nabla}\hat{G}. \quad (8)$$

Besides, one has to add still the extended Lorentz condition

$$\hat{\nabla}\hat{\mathbf{A}} \equiv \hat{\nabla}\hat{A} - \nabla\mathbf{A} = 0. \quad (9)$$

Together with this condition the equations (4) — (8) completely describe a behavior of electromagnetic field and a charge particle in the six-dimensional world. The generalization over the case with several interacting particles will not be very difficult.

By the transition to an one-time case when all world-lines in t-subspace are parallel to a fixed direction $\hat{\tau}$ the operator $\hat{\nabla} \rightarrow \hat{\tau}\partial/\partial t$, the potential $\hat{\mathbf{A}} \rightarrow \varphi\hat{\tau}$, the "time-magnetic" field $\hat{G} \rightarrow c^{-1}\dot{\varphi}\hat{\tau} \times \hat{\tau} = 0$, electric tensor $E_{ik} \rightarrow -\tau_k(\dot{A}_i + \partial\varphi/\partial x^i) = E_i\tau_k$, where E_i is the one-time theory electric field. So, all generalized relations reduce to the well-known Maxwell equations,

The particular cases of multi-dimensional electrodynamics were discussed by many authors (see, for example, papers [22 – 24]), however, in a complete form the generalized Maxwell equations were formulated by E.A.B.Cole [25 – 27].

In the simplest case of a time-independent motionless point charge with a time trajectory $\hat{\tau}$ we get a simple solution of these equations

$$\hat{A}(\mathbf{x}) = q\hat{\tau}/r$$

where $r = |\mathbf{x}|$ [3].

A generalization of the solution over the case of a moving charge has been done in paper [26]. Like the one-time theory, the magnetic field $\mathbf{H}$ is perpendicular to the velocity vector $\mathbf{v}$ and to the radius-vector $\mathbf{r}$, linking the space points, where the charge and the observer are located: $\mathbf{H} \sim \mathbf{v} \times \mathbf{r}$. The field $\hat{G}$ is perpendicular to the time trajectories of the charge and the observer ($\hat{G} \sim \hat{\tau} \times \hat{\tau}'$) and is very small ($\sim \sin\theta$) if the latter's are near one to an other. The electric field components reducing in the limit $\theta \rightarrow 0$ to the ordinary three-dimensional field $\mathbf{E}$ are long-shaped, as it is in Maxwell theory, along the charge velocity $\mathbf{v}$. The other components of $\hat{\mathbf{E}}$ are proportional to $\sin\theta$ and turn into zero if $\theta \rightarrow 0$.

When the charge passes by a observer, the field $\hat{G}$ and all multi-time corrections to the electric field increase up to some maximal value depending on a charge impact parameter and the angle θ, decreases thereupon up to zero and changes their sign (see Fig. 1). The presence of two force pulses with opposite signs causes trouble by their registration because a possessing of some inertia detector practically has no time to respond to them.

[3] One should note that in the multi-time world the field of a charge is visible only within a time interval Δt, the duration of which depends on an angle θ between the charge and the observer time trajectory. Now we shall not discuss this aspect of the multi-time theory.

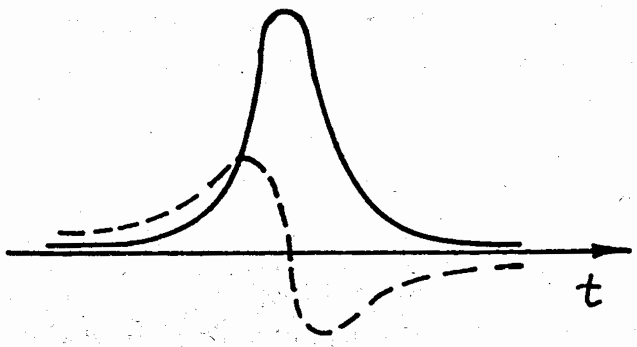

Figure 1: The solid curve shows a time dependence of the magnetic field $\mathbf{H}$ and all electric components reducing into the ordinary Maxwell field $\mathbf{E}$. The dashed curve represents a time behavior of the multi-time corrections.

In the next paragraphs we consider another important example — a motion of plane electromagnetic waves in vacuum and its energy vector [28].

Electromagnetic waves in multi-time space

Introducing a potential $\hat{\mathbf{A}}$ into the equation (4) with $\mathbf{J} = \hat{\rho} = 0$ we get:

$$\hat{\boldsymbol{\nabla}}^2\mathbf{A} - \boldsymbol{\nabla}(\hat{\boldsymbol{\nabla}}\hat{\mathbf{A}}) = 0,$$

or

$$\hat{\boldsymbol{\nabla}}^2\mathbf{A} \equiv \hat{\nabla}^2\mathbf{A} - \boldsymbol{\nabla}^2\mathbf{A} = 0, \tag{10}$$

if we take into account the extended Lorentz condition (9). Analogously, it follows from the equation (5):

$$\hat{\boldsymbol{\nabla}}^2\hat{A} = 0. \tag{11}$$

Because a rectilinear world-line of the plane wave can be defined by the vector $\hat{\mathbf{n}} = (1,0,0,1,0,0)^T$, the equations

(10), (11) and the extended Lorentz condition (9) can be written in the form

$$\partial^2\hat{\mathbf{A}}/\partial t^2 - c^2\partial^2\hat{\mathbf{A}}/\partial x^2 = 0 \tag{12}$$

$$\partial A_4/\partial t + \partial A_1/\partial x = 0. \tag{13}$$

Further, just as in the known one-time theory, the first term in (13) is turned into zero by means of a gauge and we get then from the equation (12):

$$\partial A_1/\partial t = \mathrm{const},$$

i.e. the component A_1 bears no relation to a wave process and can be removed. It means out that the potentials $\mathbf{A}$ and $\hat{A}$ are transverse vectors: $\hat{\mathbf{A}} \cdot \hat{\mathbf{n}} = 0$.

Now the electric field tensor looks as

$$\hat{\mathbf{E}} = c^{-1}\begin{pmatrix} 0 & \dot{A}_5 & \dot{A}_6 \\ -\dot{A}_2 & 0 & 0 \\ -\dot{A}_3 & 0 & 0 \end{pmatrix} \tag{14}$$

(The solution of a wave equation corresponding to a wave moving along the vector $\mathbf{n}$ satisfies the condition $\partial A_\mu/\partial x = -\dot{A}_\mu/c$). Let us introduce the three-dimensional electric fields $\mathbf{E}_k = (E_{1k}, E_{2k}, E_{3k})$ and $\hat{E}_k = (E_{k1}, E_{k2}, E_{k3})$. It is easy to prove that the fields $\mathbf{E}_k$ and $\hat{E}_k$ for $k = 2,3$ are longitudinal vectors, but $\mathbf{E}_1$ and $\hat{E}_1$ are transversal ones: $\mathbf{E}_1 \cdot \mathbf{n} = \hat{E}_1 \cdot \hat{n} = 0$.

From the formal point of view the appearance of the longitudinal electric field is stipulated by the impossibility to remove by means of the gauge transformation more than one component of the three-vector $\hat{A}$.

Using for $\mathbf{E}_i$ and $\hat{E}_i$ the expressions from (14), one can prove that magnetic and "time-magnetic" fields

$$\mathbf{H} = c^{-1}\frac{\partial}{\partial t}\mathbf{n}\times\mathbf{A} = \mathbf{n}\times\mathbf{E}_1$$
$$\hat{G} = c^{-1}\frac{\partial}{\partial t}\hat{n}\times\hat{A} = \hat{n}\times\hat{E}_1$$

are also transverse ones: $\mathbf{H}\cdot\mathbf{n} = \hat{G}\cdot\hat{n} = 0$. The absolute values

$$H = |\mathbf{E}_1| = |\hat{E}_L| \quad , \quad G = |\hat{E}_1| = \mathbf{E}_L,$$

where $\hat{\mathbf{E}} = \hat{\mathbf{E}}_2 + \hat{\mathbf{E}}_3$ is the summary electric vector of a longitudinal wave.

As we see, in the multi-dimensional world the electromagnetic wave becomes apparent in two essence: in x-subspace it looks as a superposition of the transverse wave $(\mathbf{E}_1, \mathbf{H})$ and the longitudinal wave $\mathbf{E}_L$, in t-subspace it is a sum of the transverse wave $(\hat{E}_1, \hat{G})$ and the longitudinal one with the vector $\hat{E}_L$. The structure of the plane wave in x- and t-subspaces is completely symmetrical — that part of the wave which in x-subspace is transversal in t-subspace becomes longitudinal and conversely (see Fig.1). According to their signs, the longitudinal field strengths $\mathbf{E}_k$ and $\hat{E}_k (k = 2, 3)$ can be in parallels or antiparallels to the direction of the space and time vectors $\mathbf{n}$ and $\hat{n}$.

Momentum-energy of a plane wave

The momentum-energy tensor of electromagnetic field

$$T^{\mu\nu} = \frac{1}{4\pi}\left(\mathcal{F}^{\nu}_{\delta}\mathcal{F}^{\delta\mu} + \frac{1}{4}g^{\mu\nu}\mathcal{F}_{\lambda\gamma}\mathcal{F}^{\lambda\gamma}\right)$$

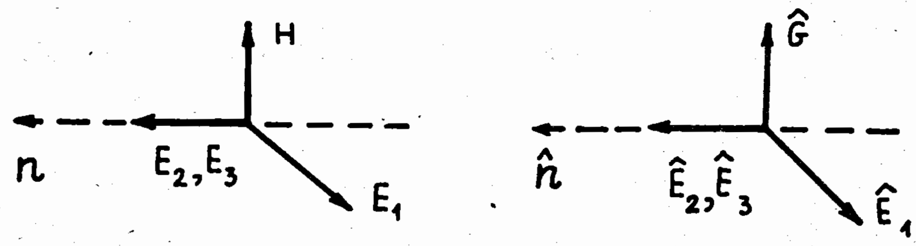

Figure 2: The components of a plane wave in x- (left) and t- (right) subspaces.

derived with the help of canonical rules distinguishes from the respective tensor of the one-time theory only by a number of components. If the electromagnetic field tensor is presented in the form where $\hat{\mathbf{H}}$, $\hat{\mathbf{G}}$, $\hat{\mathbf{E}}$ are 3×3 matrixes then $T^{\mu\nu}$ can also be written in the similar matrix form:

$$T^{\mu\nu} \equiv \begin{pmatrix} \hat{\mathbf{T}}_1 & \hat{\mathbf{T}}_2 \\ \hat{\mathbf{T}}_3 & \hat{\mathbf{T}}_4 \end{pmatrix}^{\mu\nu} =$$

$$\frac{1}{4\pi}\begin{pmatrix} -\hat{\mathbf{H}}\hat{\mathbf{H}} - \hat{\mathbf{F}}\hat{\mathbf{E}} & -\hat{\mathbf{H}}\hat{\mathbf{E}} - \hat{\mathbf{E}}\hat{\mathbf{G}} \\ \hat{\mathbf{E}}^T\hat{\mathbf{H}} + \hat{\mathbf{G}}\hat{\mathbf{E}}^T & \hat{\mathbf{E}}^T\hat{\mathbf{E}} + \hat{\mathbf{G}}\hat{\mathbf{G}} \end{pmatrix}^{\mu\nu} + \frac{1}{8\pi}g^{\mu\nu}\left(\mathbf{H}^2 + \hat{G}^2 - \hat{\mathbf{E}}^2\right),$$

where

$$\hat{\mathbf{E}}^2 = \sum_{i=1}^{3}\mathbf{E}_i^2 = \sum_{i=1}^{3}\hat{E}_i^2.$$

Taking into account the relations (2) and (14) we get the following expressions for the tensors $\hat{\mathbf{T}}_i$:

$$4\pi\hat{\mathbf{T}}_1^{ik} = -\hat{E}_i\hat{E}_k - H_iH_k + \tfrac{1}{2}\delta_{ik}(\mathbf{H}^2 - \hat{G}^2 + \hat{\mathbf{E}}^2)$$
$$4\pi\hat{\mathbf{T}}_4^{ik} = \mathbf{E}_i\mathbf{E}_k + G_iG_k + \tfrac{1}{2}\delta_{ik}(\mathbf{H}^2 - \hat{G}^2 - \hat{\mathbf{E}}^2)$$
$$4\pi\hat{\mathbf{T}}_3^{ik} = 4\pi\hat{\mathbf{T}}_2^{ik} = (\mathbf{E}_k\times\mathbf{H})_i - (\hat{E}_i\times\hat{G})_k$$

(For details see ref. [28]).

Let us define a six-dimensional momentum-energy vector

$$P^{\mu} = \int T^{\mu\nu} ds_{\nu},$$

where $ds_{\nu} = n_{\nu} dV$ and dV is an element of a three-dimensional hypersurface. In a particular case of the plane wave, when the direction of its time-trajectory is taken as the t_1-axis,

$$P^{\mu} = \int T^{\mu k} \tau_k dV = \begin{cases} \int \hat{\mathbf{T}}_3^{1k} dV & \mu = k \leq 3 \\ \int \hat{\mathbf{T}}_4^{1k} dV, & \mu = 3 + k \end{cases}$$

So, the field momentum-energy density

$$\hat{\mathbf{p}} = (W_T + W_L)\hat{\mathbf{n}}, \qquad (15)$$

where

$$W_T = (\dot{A}_2^2 + \dot{A}_3^2/\pi c^2 = (\mathbf{E}_1^2 + \mathbf{H}^2)/8\pi$$

and

$$W_L = -(\dot{A}_5^2 + \dot{A}_6^2)/8\pi = -(\hat{E}_1^2 + \hat{G}^2)/8\pi$$

are the energies of the transverse and longitudinal fields.

As we see the three-dimensional energy and momentum vectors of the plane wave are directed along its world-line $\hat{\mathbf{x}}$ and their transverse components $p_k, k \neq 1, 4$, always equal to zero. The momentum-energy of the transverse field component $(\mathbf{E}_1, \mathbf{H})$ is positive, meanwhile not only the momentum but also the energy of the longitudinal field $\mathbf{E}_L$ are negative.

It would seem strange, that a part of the wave energy is negative, however, an analogous situation takes place in the

Maxwell electrodynamics also where a "scalar photon" energy is negative. Because the multi-component electromagnetic wave is an unitary object, only its summary energy is physically significant and its negative part has been never observed. In the six-dimensional theory the total wave energy (15) must be positive too, i. e. $W_T \geq |W_L|$ and the transverse component prevails always over the longitudinal one: $|\mathbf{E}_1| \geq |\hat{E}_1|$. In this case the plane wave momentum is directed along the x_1-axis and the wave is developing along t_1-axes from the past into the future.

The mathematically acceptable solution with negative energy $W_T + W_L < 0$ corresponding to an incoming wave with a backward space and time directions must be rejected, if we take into account the causality principle.

In the boundary case of equal amplitudes $\mathbf{E}_1^2 = \hat{E}_1^2 = \mathbf{H}^2 = \hat{G}^2$ the complete compensation of the transverse and longitudinal components occurs (similar to the compensation of the scalar and longitudinal components of the plane wave in the one-time theory) the momentum-energy (15) vanishes and the wave disappears.

So, as in the usual Maxwell electrodynamics the energies of all outgoing plane waves in the six-dimensional world are positive and these waves develop along the positive directions of all time axes t_i.

The considered example of plane electromagnetic waves promotes that the analogous situation takes place in the general cases: due to the causality principle all wave solutions describing the motion of particles correspond to the time trajectories with the positive projections $\tau_i \geq 0$ and, respectively, to the energy vectors with the positive components. That forbids spontaneous creations of groups of particles from vacuum and exotic decays in which the mass of secondaries exceeds the decaying particle mass, since every such an event is accompanied always by a time-reverse motion at least along one axis t_i and by a violation of causation.

It should be emphasized that the time reversibility only is an approximate property of theories with the finite number of particles and interconnection, meanwhile due to a non-exhaustive huge number of real interconnection in Nature the time reversibility does not realize, strictly speaking, even in microscopic processes, since it would demand the time turning of all these innumerable interconnection [3].

From the cosmological point of view the condition $t_i \geq 0$ is equivalent to a supposition of the availability of a preferable (relict) reference frame (a "time arrow" in the one-time case) fixed by the events order which has become settled at the first moments of the existence of Universe. In spite of this, an observer moving along a trajectory $\hat{t}$ can describe what is going on by means of the measured along his own trajectory one-dimensional proper time t, since the latter can be always used as a parameter defining the trajectory in the relict frame. The fact of his world time multidimensionality he may ascertain meeting a body with some different time trajectory or discovering in an experiment a longitudinal electromagnetic wave.

In order to describe the events under way, one can also use, of course, the co-ordinate frames turned with respect to the relict one. It is equivalent to a formal remarking of the time co-ordinates similar to a reverse time-reading

used sometimes in our practice. In this respect x- and t-subworlds differ essentially. In the six-dimensional theory space and time, as before, are not equal in their right.

We see that multi-time electrodynamics is a consistent theory with a plausible physical interpretation. Meanwhile, one has to answer now the question about provenance of bodies whose time trajectories are distinguish from the our one.

In conclusion I wish to thank my colleagues B.F.Kostenko, A.B.Pestov and M.Z.Yur'ev for the fruitful discussions.

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Received by Publishing Department
on January 16, 1996.