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HADRON SCATTERING AT ACCESSIBLE AND ASYMPTOTIC ENERGIES IN THE POMERON THEORY WITH $\alpha(0) > 1$

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I. Introduction

New data on high energy hadronic scattering, which appeared in the last years, intensified the interest in the properties of the Pomeranchuk singularity. The position of this singularity on the j-plane at $t = 0$ $\alpha(0)$ is the most important one. In the case when a Pomeron is a Regge pole, there are known two selfconsistent variants of the theory with asymptotically constant [1] and growing [2] total cross sections. It turns out, however, that one cannot describe high energy data on the total cross section, the elastic scattering cross section slope and particle diffraction production together. It is not yet clear now if this is caused by the incorrect calculation of cut contribution or is due to the $\alpha(0)$ value. In any case, the interpretation of the high energy data becomes much simpler if one adopts $\alpha(0) > 1$, ref. [3]

II. Modern Energies

Let us consider some consequences which follow for the observable quantities in the case of $\alpha(0) > 1$. All the results mentioned in this section are obtained under restriction by the unenhanced graphs only. One believes that enhanced graphs are small at modern energies due to the smallness of multipomeron vertices. But as the energy increases, their contribution becomes important and it will be brought into the play below in connection with the problem of unitarity.

It is worth noting first that the natural explanation is obtained for the so-called "geometrical scaling" (GS) observed in pp-scattering, ref. [4]. This phenomenon resulting from the empirical observation that the partial elastic scattering amplitude $f(b, s)$, where b is an impact parameter, at the high energy depends on one variable $b^2/B(s)$ only, where $B(s)$ is the elastic scattering slope.

The Pomeron contribution to the elastic scattering amplitude in the impact parameter represenation has the following form:

$$\rho_0(b, y) = \frac{g_{\alpha 1} g_{\beta 1} e^{\Delta y}}{R_0^2 + \alpha' y} \, e^{-\frac{b^2}{4(R_0^2 + \alpha' y)}} \qquad (1)$$

Here $g_{\alpha 1}(K_\perp^2) g_{\beta 1}(K_\perp^2) = g_{\alpha 1} g_{\beta 1} \exp(-R_0^2 K_\perp^2)$ is the Pomeron residue in the $\alpha\beta$-scattering amplitude; α' is a slope of the Pomeron trajectory; $y = \ln(s/s_0)$; $\Delta = \alpha(0) - 1$. In the pole approximation $B(s) = 2(R_0^2 + \alpha' y)$, so one can easily see from (1) that in the wide energy region, where

$$Z = \frac{g_{\alpha 1} g_{\beta 1} e^{\Delta y}}{R_0^2 + \alpha' y} \tag{2}$$

is approximately a constant, i.e.,

$$\Delta \simeq \frac{2\alpha'}{B(s)} \simeq 0{,}06 \tag{3}$$

the GS will take place. As the energy increases, considerable deviation from GS will emerge.

On the other hand, in πp and Kp scattering, where the value of B(s) is smaller than in pp GS demands much higher energy. The same conclusion can be made for the pp $\to$ pX reaction, where R_0^2 is about two times smaller than the elastic one.

2. As it follows from (1), the partial amplitude will exceed the unity and violate the unitarity at sufficiently high energy. But as interaction becomes stronger, the rescattering corrections grow. The mutual shadowing of elastic and inelastic channels will decrease the amplitude value, so the unitarity can be restored. This is easy to see from an example of unenhanced graphs, shown in fig. 1.

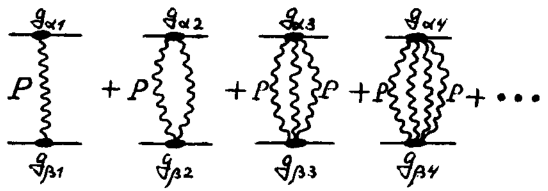


Fig. 1

This sum calculated in the eikonal approach gives in the b-representation

$$\rho(b,y) = 1 - e^{-\rho_0(b,y)} . \tag{4}$$

It follows from (4) that $\rho(b,s)$ asymptotically has the form of a black disk with the radius $2\sqrt{\alpha'\Delta}\cdot y$ /5/

$$\rho(b,y) \simeq \theta(4\alpha'\Delta y^2 - b^2) \tag{5}$$

3. The energy dependence of the total interaction $\sigma_{tot}(y)$ and the total inelastic interaction $\sigma_{in}(y)$ cross sections, which corresponds to (4) is given by the expressions:

$$\sigma_{tot}(y) = 8\pi(R_0^2 + \alpha' y)\varphi(Z) , \tag{6}$$

$$\sigma_{in}(y) = 4\pi(R_0^2 + \alpha' y)\varphi(2Z) , \tag{7}$$

where

$$\varphi(z) = C + \ln z - Ei(-z) . \tag{8}$$

Here Z is defined in (2); $C \simeq 0.577$; Ei (-z) is the integral exponent function. The variation of the diffraction slope B (y) with energy is given by the expression

$$B(y) = 2(\alpha' y + R_0^2)\int_0^z \varphi(x)\frac{dx}{x} \Big/ \varphi(z) \tag{9}$$

It is clear from (6)-(9) that σ_{tot}/B , σ_{el}/σ_{tot} ,etc. remain energy-dependent in the FNAL-ISR energy region, where (3) takes place. These values will acquire energy dependence at higher energies.

From (9) and (8) it follows that as $y \to \infty$, $B(y) \to \alpha'\Delta y^2$. It is interesting to note that due to (9) and (5) GS asymptotically will be restored, what agrees with the common result of ref. /6/

The ratio of the real part of the forward scattering amplitude to the imaginary one for the eikonal graphs in fig. 1 is given by

$$\varepsilon = \frac{\mathrm{Re}\, f(y,0)}{\mathrm{Im}\, f(y,0)} = \frac{\pi}{2}\left\{\frac{\alpha'}{R_0^2 + \alpha' y} + \frac{1}{\varphi(Z)}\left(\Delta - \frac{\alpha'}{R_0^2 + \alpha' y}\right)\left(1 - e^{-Z}\right)\right. . \tag{10}$$

At energies, where GS holds, ε is almost constant and close to $\Delta\pi/2$. In the asymptotic region it becomes $\varepsilon = \pi/y$.

4. It has been claimed in ref. /7/ that the fast growth of $\sigma_{tot}(y)$, which follows from (6), contradicts experimental data from extensive air showers at energies up to $s \simeq 10^9$ GeV^2 . These data correspond to the absorption of protons by air nuclei with the mean atomic number $\bar{A}$ = 14.4. The inelastic cross section $\sigma_{in}^{PA}(y)$ has been calculated in ref. /6/ by using the Glauber approximation. It is clear, however, that at such high energies the interaction radius becomes very large and is comparable with the radius of the air nucleus. So, the most of the nucleon parton

clouds in the nucleus cover each other in the impact parameter plane and the multiple scattering model fails, ref. [8].

5. If the Pomeron gives a contribution to the spin-flip amplitude, it will be small amount of polarization in elastic scattering resulting from vacuum exchange. This polarization should be an approximate constant in the region, where GS takes place. The spin correlation effects in high energy total cross sections also should be almost energy independent in a wide energy region.

6. Let us consider particle production now. In the scheme under discussion one immediately finds explanation for the Koba, Nielsen, Olesen (KNO)- scaling of topological cross sections [9]:

$$\langle n\rangle \frac{\sigma_n}{\sigma_{in}} = \Psi\left(\frac{n}{\langle n\rangle}\right) \tag{11}$$

here $\langle n\rangle$ is the mean multiplicity of particles produced;

σ_n is the n-particle production cross section;

$\Psi\left(\frac{n}{\langle n\rangle}\right)$ is some function independent of energy. In order to prove (11), it is sufficient to show the energy independence of the moments $\langle n^k\rangle/\langle n\rangle^k$ ($k = 1,2,3 \ldots$). The graphs which contribute to σ_n are obtained by cutting diagrams in fig. 1 in accordance with Abramovski, Gribov, Kancheli rules [10,11]. By averaging n^k over this contributions one can find

$$\sigma_{in}\langle n^k\rangle = \int e^{-\rho_0(y,b)}\left(a y\, \rho_c \frac{d}{d\rho_c}\right)^k \left(e^{\rho_c(b,y)} - 1\right) e^{-\rho_0(b,y)}\, d^2 b \tag{12}$$

Here $\rho_c(b,y) = 2\,\mathrm{Im}\,\rho_0(b,y)$ is the Green function of the cut Pomerons; the operator $(a y \rho_c\, d/d\rho_c)^k$ extracts the value of n^k for each graph; $a y$ is a number of particles in one cut Pomeron; the factor $\exp(-\rho_0)$ takes into account the absorption corrections.

After the integration one obtains from (12)

$$\sigma_{in}\langle n^k\rangle = 4\pi(R_0^2+\alpha' y)(a y)^k \sum_{i=1}^{k} i^{k-i-1}(2z)^i \tag{13}$$

where Z is the variable (2). It follows from (13) and (7) that

$$\frac{\langle n^k\rangle}{\langle n\rangle^k} = \left[\varphi(2z)\right]^{k-1} \sum_{i=1}^{k} \frac{i^{k-i-1}}{(2z)^{k-i}} \tag{14}$$

Thus, in the energy region where Z is approximately constant, i.e., GS takes place, the KNO-scaling also occurs. But at

higher energies Z varies and both types of scaling are violated. It is interesting to note that unlike in GS, KNO is not restored asymptotically.

7. The multiplicity distribution σ_n will oscillate [10] at high energy with the period $\Delta n = a y$. But unlike in the case of $\alpha(0) = 1$, the maximum amplitude is attributed to a peak with the number $2z/\varphi(2z)$. It is also seen from (13) and (5) that the mean multiplicity increases with the energy as

$$\langle n\rangle = c + a y\cdot 2z/\varphi(2z)$$

8. It is obvious that due to $\alpha(0) > 1$ the Feinman scaling is also violated. The inclusive cross section in the central region should grow with energy as

$$\frac{d\sigma}{dy} \sim e^{\Delta y}. \tag{15}$$

It has been shown by M.S.Dubovikov and K.A.Ter-Martirosyan [12] that (15) holds after the enhanced graph inclusion. In the triple Regge region the cross section also increases with energy [13]

$$\frac{d\sigma}{dx\,dt} = G_{PPP}(t)\frac{e^{\Delta y}}{(1-x)^{1+\Delta+2\alpha' t}}. \tag{16}$$

III. The s-Channel Unitarity and Asymptotic Regimes

1. The problem of the s-channel unitarity is not exhausted, of course, by the eikonal expansion. Generalization to a wider class of enhanced graphs has been made by Cardy [14]. He has shown that if the vertices $g_{\alpha n}$ and $g_{\beta n}$ of n Pomeron emission by particles α and β have unique analytical expansion to the complex n-plane, the disk (5) acquires the factor $g_{\alpha 0}\, g_{\beta 0}$, so it can be "grey". The inclusion of enhanced graphs makes the problem of unitarity much more complicated. Following Cardy, it is convenient to substitute the sum of graphs in fig. 1 by the graph shown in fig. 2,

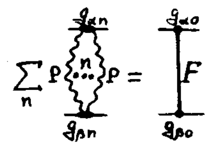
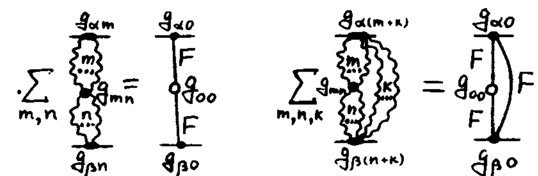


Fig. 2



a) Fig. 3 b)

which has been called "Froissaron" by K.A.Ter-Martirosyan. From Froissarons one can build more complicated graphs, for instance, those shown in Fig. 3. The vertex g_{00} has been introduced by Cardy. It is a result of analytical continuation of the g_{mn}- vertex which couples m Pometons with the n ones.

Let us note that summing up in Fig. 3 begins from m = n = 1. The term $g_{11}\Psi^+\Psi$ in the Pomeron field Lagrangian has a form of the mass term, so g_{11} is included in the Δ value. If one wants to develop a Froissaron calculus, one must redefine the value Δ in the bare Pomeron substituting it by Δ_0

$$\Delta_0 = \Delta - g_{11} \tag{17}$$

to avoid the double counting.

It is easy to see that some new Froissaron graphs violate the unitarity. The singularity ω^{-6} in the ω-plane (ω = j-1) corresponds to the graph in Fig. 3a, for instance, at t = 0. So, its contribution to $\sigma_{tot.}$ grows as y^5. Cardy has noted[14] that there is a considerable compensation between graphs in Fig. 3a and b . He proposed such a procedure of summing up the Froissaron graphs that for any graph, which violates the unitarity, there exists another one, which compensates it. Nevertheless, these compensation are not sufficient to guarantee the unitarity. Indeed, if one takes into account the F Green function behaviour at distances $b \sim 2\sqrt{\alpha'\Delta_0}$, one can find that the compensation is not complete and the sum of graphs in Fig. 3a, b grows as $\sqrt{y}$ at $b \simeq 2\sqrt{\alpha'\Delta_0}\,y - \sqrt{\alpha'/\Delta_0}\,\ln y$

2. Another method for summing up the graphs is proposed here. It is clear that the s-channel iteration of the graph in Fig.3a carried out analogously to Fig 1 , leads to the unitarity result. Let us denote the result of the s-channel interation of some graph $\rho(b,y)$ by the symbol $E[\rho(b,y)]$. The function $E(\rho_0)$ in eikonal approximation has the form of (4). It is clear that the operation $E(\rho)$ can he applied to any graph or sum of graphs, which are irreducible in the s-channel, i.e., they cannot be divided by a vertical line without crossing Pomeron lines. So, after summing up all the graphs the exact Green function T (b, y), if any, can be written as follows

$$T(b,y) = E\left[\sum_k \rho_k(b,y)\right]. \tag{18}$$

It has been shown by M.S.Dubovikov and K.A.Ter-Martirosyan[12] that the condition of the positivity of the sum in (18) can be satisfied if g_{00} is small enough and $T(b,y)$ has the form of the Froissaron

$$T(b,y) = (4\alpha'\Delta_0 y^2 - b^2) \tag{19}$$

We give here their method of graphs ρ_k classification and show that some non-Froissaron-like solution can exist.

The sum $\sum_k \rho_k$ can be divided into three groups

$$\sum_k \rho_k = \rho_0 + \mathcal{D}(T) + C(T). \tag{20}$$

Here $\rho(b,y)$ is given by (1), where Δ is substituted by Δ_0 from (17). The group D(T) contains graphs irreducible both in t- and s-channels. It is clear that one can consider these graphs as skeleton ones built from the exact Green function T. The group C(T) includes graphs shown in Fig. 4,

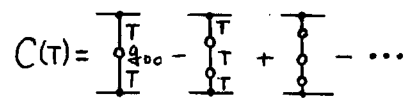


Fig. 4

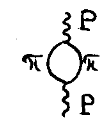


Fig. 5

In (ω, $k_\perp^2$)-representation it is equal[12] to

$$C(\omega,k_\perp^2) = T(\omega,k_\perp^2) - \left[T^{-1}(\omega,k_\perp^2) + g_{00}\right]^{-1} \tag{21}$$

Now let us draw our attention to the fact that an addition of enhanced graphs to the series in Fig. 1 causes the renormalization of the Δ value in accordance with (17). That is,in some high energy region a change of the asymptotic regime occurs. But the possibility is not excluded that $\Delta_0 \leq g_{11}$, so $\Delta_0 \leq 0$. This danger follows from a simple estimation of g_{11} by the graph, shown in Fig. 5 which gives

$$g_{11} \simeq \frac{(\sigma_{tot}^{\pi N})^2}{16\pi^3 \sigma_{tot}^{NN}}\left[\mu^2 - 2\mu^2 \ln(\mu^2 R^2) + R^{-2}\right]. \tag{22}$$

Here μ is the pion mass; $R^2/2$ characterizes the dependence of $\sigma_{tot}^{\pi N}$ from the mass-square of the virtual pion. If one takes $R^2 \simeq 1\ (\mathrm{GeV}/c)^2$, one obtains $g_{11} \simeq 0.08$, i.e., of the same order as Δ in (3).

Then, return to equations (18),(20) and consider the case of $\Delta_0 \leq 0$. The asymptotic behaviour cannot saturate the Froissart-bound in this case. If the total cross section rises as

$$\sigma_{tot}(y) \sim y^{\eta} \tag{23}$$

then $T\ (\omega, k^2=0) \sim \omega^{-\eta-1}$. It is seen from (15) that in order to avoid singularities in the positive ω -half plane the following condition, at least, should be satisfied

$$\eta \leq 1\ . \tag{24}$$

In this case the inclusive spectrum in the pionization region is not flat:

$$\frac{d\sigma}{dy_1} \sim y_1^{\eta}(y-y_1)^{\eta} \tag{25}$$

and the mean multiplicity rises as

$$\langle n \rangle \sim y^{\eta+1}\ . \tag{26}$$

As a possible realization in (y,b) space of such solution with $\eta = 1$ one can consider a ring with constant thickness d and the rising radius $\sim a \cdot y$.

Another value of Δ , for which non-Froissart-like behaviour is known to take place is a critical one $\Delta = \Delta_c$ corresponding to the strong coupling variant of the theory /2/. If $\Delta = \Delta_c$, the sum of graphs (20) has singularity at $\omega = 0$ and should be classified as one of the cases discussed just above. So one can demand that

$$\Delta_c \leq g_{11}\ . \tag{27}$$

This inequality is in accordance with the estimations of Δ_c /2/ and g_{11} from (22).

It is worth noting that different types of asymptotic behaviour can have no effect in the energy region accessible now.

It is possible even that some Froissart-like behaviour can take place at $s \simeq 10^6 - 10^{10}\ \mathrm{GeV}^2$ but a change of the regime may occur at much larger energy values, where enhanced Froissaron graphs causing compensation reach asymptotics. It seems now that only new experiments on the new generation of high energy accelerators and possibly in cosmic rays can give the answer to what kind of asymptotic energy behaviour for cross-sections takes place.

IV. Conclusion

The theory of the Pomeron with $\alpha(0) = 1$ has large amount of attractive features and explains many general properties of hadronic scattering at high energies. But a number of subtle effects, experimentally observed recently, have shown the necessity of considering the case of $\alpha(0) > 1$.

The main consequences for experimentalists discussed above are the following:

1. Due to an "accidental" play of parameters the approximate geometrical scaling takes place at modern energies. It can be violated with increasing energy.

2. The total cross section and the diffraction slope varying slow now, can grow faster and saturate the Froissar limit at energies at $10^6 - 10^{10}$ GeV.

3. Fast increase of the cross section does not contradict experimental data from accelerators or cosmic rays.

4. The ratio of real-to-imaginary parts of the forward scattering amplitude can reach the value of about $\pi/2 \cdot \Delta$ and very long can remain approximately a constant.

5. If a Pomeron gives any contribution to the spin-flip amplitude, it will be a small energy-independent part of elastic scattering polarization in the energy region of the GS validity.

6. The natural explanation is obtained for the KNO scaling at modern energies, where Z =constant . The mean multiplicity acquires the supplementary factor $2Z/\varphi(2)$ and has a small deviation from an ordinary $\ln(s/s_0)$-dependence at accelerator energies.

7. The Feinman scaling is violated in the central and triple Regge regions both. The inclusive cross section rises as $\exp(\Delta y)$.

The problems of unitarity arise at asymptotic energies. The Froissaron calculus by Cardy-Gribov is a convenient tool for studying this problem. A procedure is proposed for the summing up of the Froissaron graphs which quarantees the s-channel unitarity. The existence of the Froissaron-type solution has been proved by M.S.Dubovikov and K.A.Ter-Martirosyan. It is noted here that other solutions can exist, if $\Delta \leq g_{11}$. One of such solutions is a strong coupling variant /2/, which takes place, when $\Delta = \Delta_c$ is a critical value.

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