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THE DIPOLE ASYMPTOTIC BEHAVIOUR OF THE PROTON FORM FACTOR AS A CONTRIBUTION OF VECTOR MESONS AND CENTRAL REGION

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In paper [1] a new relativistic relative coordinate introduced in [2] has been used to describe the particle spatial distribution. An expansion over the unitary irreducible representations of the Lorentz group [3] is used instead of the Fourier transform in passing to a new configurational representation. For the proton form factor $F(t)$ this transformation due to its spherical symmetry has the form [1,2] (in (1) we put $\hbar = c = 1$)

$$F(t) = 4\pi \int \frac{\sin rMy}{rM \sinh y} F(r) r^2 dr . \tag{1}$$

Here M is the proton mass and the hyperbolic angle $y = \mathrm{Ar}\cosh\left(\frac{2M^2 - t}{2M^2}\right)$ is the rapidity corresponding to the momentum transfer $t = (p-k)^2$.

The new coordinate has the important property that its modulus r is the relativistic invariant, as it parametrizes eigenvalues of the invariant operator, the Casimir operator of the Lorentz group $\hat{C} = -\frac{1}{4} M^{\mu\nu} M_{\mu\nu} = \vec{N}^2 - \vec{M}^2 = \frac{\hbar^2}{M^2 c^2} + r^2$. Therefore the spatial distribution function $F(r)$ like $F(t)$ is also the relativistic invariant and describes the particle distribution in an arbitrary reference frame (and not in the Breit frame only in contrast with the usual Fourier transform). In terms of $F(r)$ one has the following expression of the invariant mean square radius (m.s.r.) [1]

$$\langle r_0^2\rangle = \frac{6\frac{\partial F(t)}{\partial t}\Big|_{t=0}}{F(0)} = \frac{\hbar^2}{M^2c^2} + \frac{\int r^2 F(r)\, d\vec{r}}{\int F(r)\, d\vec{r}} = \frac{\hbar^2}{M^2c^2} + \langle r^2\rangle \quad (2)$$

Consequently, when $\langle r^2\rangle$ is positive * both the new coordinate r and F(r) describe not the total distribution of a particle but only the region at distances larger than its Compton wave length. From (2) it follows that to the central sphere with $\langle r_0^2\rangle = \frac{1}{M^2} = \frac{\hbar^2}{M^2c^2}$ there corresponds the spatial distribution $F(r) = \delta(r)/4\pi r^2$.

According to (1) this provides the following contribution to the form factor from the central region

$$F(t)\Big|_{r_0=\frac{h}{Mc}} = \frac{\sin rMy}{rM\sinh y}\Big|_{r=0} = \frac{y}{\sinh y} = 2M^2\,\frac{\mathrm{Ar}\cosh\left(\frac{2M^2-t}{2M^2}\right)}{\sqrt{t(t-4M^2)}} \quad (3)$$

So, in the case of $\langle r^2\rangle$ positive the standard form factor F(t) may be represented in the form

$$F(t) = \frac{y}{\sinh y}\cdot\Phi(y), \quad (4)$$

where the "external" form factor $\Phi(y)$ corresponds to the proton distribution outside the sphere with $r_0 = \hbar/Mc$. It should be noted that the factor $y/\sinh y$ thus separated and the corresponding region with $r_0 = \hbar/Mc$ have no nonrelativistic analogs since $\frac{\hbar}{Mc} \xrightarrow[c\to\infty]{} 0$ and $y/\sinh y \xrightarrow[c\to\infty]{} 1$.

Now let us analyze in terms of the new coordinate which region of a proton is described by the vector

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* $\langle r^2\rangle$ is positive if F(r) is of constant sign.

meson contribution. The image of a meson propagator $\frac{1}{\mu^2 - t}$ in a new r-space depends essentially on the relation between the mass of a particle itself M and that of a meson μ /2/:

$$F(r) = \begin{cases} \dfrac{1}{4\pi r}\,\dfrac{\cosh(rMa_1)}{\sinh(rM\pi)} & \mu^2 < 4M^2, \quad a_1 = \arccos\left(\dfrac{\mu^2-2M^2}{2M^2}\right) \quad (5a) \\[2ex] \dfrac{1}{4\pi r}\,\dfrac{\cos(rMa_2)}{\sinh(rM\pi)} & \mu^2 > 4M^2, \quad a_2 = \mathrm{Ar}\cosh\left(\dfrac{\mu^2-2M^2}{2M^2}\right) \quad (5b) \end{cases}$$

According to formulae (2) and (5) we have

$$\langle r^2\rangle = \langle r_0^2\rangle - \frac{h^2}{M^2c^2} = \frac{\int r^2F(r)\,d\vec{r}}{\int F(r)\,d\vec{r}} = \frac{6M^2-\mu^2}{\mu^2M^2}. \quad (6)$$

Since the masses of all presently discovered ρ, ω, ϕ and ρ'' (1550) vector mesons satisfy the inequality $\mu^2_{V=\rho,\omega,\phi,\rho''} < 4M_P^2$ the function $F_P(t)$, which describes their contribution to the proton structure, has the form (5a), i.e., it is of constant sign, and $\langle r^2\rangle_P$ (6) is positive. Consequently, these vector mesons give the proton structure at distances larger than its Compton wave length, and contribute to the external form factor $\Phi(y)$ *.

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* For pion, to the same vector mesons there correspond the oscillating functions $F_\pi(r)$ (5b) and negative $\langle r^2\rangle_\pi$ because $\mu^2_V > 4M_\pi^2$. This makes the value of the pion m.s.r. smaller than its Compton wave length, that is in agreement with experimental data.

Therefore, to represent the total proton structure in the momentum space, allowing for the contribution of vector mesons with $\mu_V^2 < 4M_p^2$ by the VDM, one should add the contribution from the central part. As a result, the proton electromagnetic form factor takes the form

$$F_p(t) = \frac{y}{\sinh y} \cdot \sum_{V=\rho,\omega,\phi,\rho''} \frac{a_V}{\mu_V^2 - t} . \qquad (7)$$

From (3) it is easy to see that formula (7) gives the correct "almost dipole" asymptotic behaviour of the proton form factor at large $-t$

$$F_p(t) \xrightarrow[|t| \gg M_p^2]{} \frac{\ln \frac{|t|}{M^2}}{t^2} . \qquad (8)$$

At small transfer momenta $0 \le -t \le 1\ (\mathrm{GeV}/c)^2$ the factor $\frac{y}{\sinh y} \simeq 1$ that makes the pure VDM be valid, and at large $-t$ it provides an additional decrease as $\frac{\ln |t|/M^2}{|t|}$. An idea of the modification of the VDM through introducing an additional factor was suggested by other authors [4] as well. However, their choice of a propagator as such a factor was rather random considering the propagator to be the simplest and the most popularly used function (see also ref. [5]). We emphasize that the form of the factor $\frac{y}{\sinh y}$ corresponding to the central part contribution is due to the formalism itself, i.e., the expansion (1).

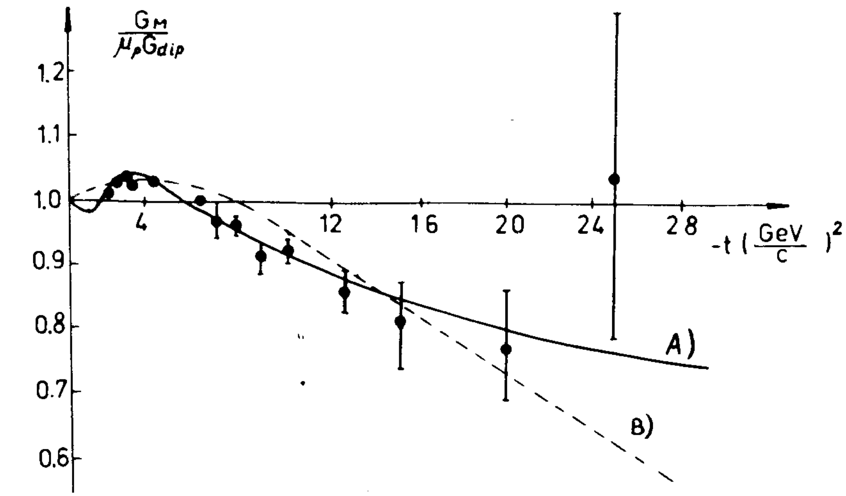
From the view point of quark models the obtained picture may be interpreted as follows. According to estimate by these models [6], the relative motion of three quarks constituting a proton is within the region of order of the proton Compton wave length. Consequently, they produce

the contribution of the central part of the proton (3). The quark-antiquark pairs excited by scattering of electrons on the proton compose the vector mesons, responsible, as it follows from (2) and (6), for the distribution at distances larger than the proton Compton wave length if $\mu_V^2 < 4M^2$.

We have compared the experimental data on the proton magnetic form factor with predictions following from (7). We have also used another VDM parametrization of the form factor which corresponds to a possible contribution from a "kern". (In our approach the role of the "kern" plays the central part with $r_0 = \frac{\hbar}{Mc}$)

$$F_p(t) = \frac{y}{\sinh y} \cdot \left[(1 - \sum_V a_V) + \sum_V \frac{a_V}{1 - t/\mu^2} \right] \qquad (9)$$

The results are presented in the table, where we give the values of χ^2 per one degree of freedom: $\chi_F^2 = \chi^2 / \mathrm{deg.fr.}$, found both by formulae (7) and (9), and compare them with the results of pure VDM.



Comparison between our model A) and the usual VDM B). The data points are taken from ref. [7] and normalized to the dipole fit.

Table

Comparison of χ^2_F *values obtained by fitting of the data in our approach and the usual vector meson pole models*

№	Model	χ^2_F with the central part contribution		χ^2_F without the central part contribution		Free parameters
		ref. [7] 14 points	ref. [8] 84 points	ref. [7] 14 points	ref. [8] 84 points	
1.	4-pole (ρ,ω,ϕ,ρ' (1250))	0.65	0.90	1.72	2.02	3
2.	4-pole (ρ,ω,ϕ,ρ''(1550))	0.67	1.04	1.78	2.36	3
3.	VDM ($\rho,\omega,\phi,\rho',\rho''$)	0.81	1.90	2.05	3.61	3
4.	VDM with kem (ρ,ω,ϕ,ρ')	0.76	0.88	1.90	1.12	4
5.	VDM with kem (ρ,ω,ϕ,ρ'')	0.69	0.92	1.89	1.31	4

Thus, we see that the consideration of the contribution from the central part with $\langle r_0^2\rangle = \frac{\hbar^2}{M^2c^2}$ modifying the VDM at distances of order of the Compton wave length results in a rather good description of the data allowing for the experimentally found vector mesons ρ, ω, ϕ and ρ'' (1550) only (in contrast with [9]). *Besides, our approach contains the prediction (see formula (8)) that at asymptotical* $-t \gg M_P^2$ *the decrease of the form factor, which is presently observed to be more rapid than the dipole* t^{-2}, *should be more slow, as (8).* Hence, the curve defined by our model should intercept the straight line $G_M^P/\mu_P G^P_{dip} = 1$ at large $-t$.

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