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ON CONNECTION BETWEEN COEFFICIENT FUNCTIONS FOR DEEP-INELASTIC AND ANNIHILATION PROCESSES

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In paper [1] the question on the status of the Crewther relation [2] in QCD has been investigated. In fact, using the update results of the multiloop calculations the relation between the coefficient functions for the deep-inelastic and annihilation processes has been considered. The authors of ref. [1] have pointed out various interesting properties of this relation. First of all, it has been shown that the corrections of type $C_F\bar{a}_s$, $C_F^2\bar{a}_s^2$ and $C_F^3\bar{a}_s^3$ are cancelled in the product of coefficient function from the Bjorken sum rule for polarized deep-inelastic lepton-hadron scattering and the Adler function for the two-point correlator of electromagnetic currents. It has also been pointed out that the surviving corrections in the second and third orders of perturbation theory are grouped yielding the two-loop β-function. The result obtained in ref. [1] has the form

$$C_{Bj}(\bar{a}_s)C_R(\bar{a}_s) = 1+\frac{\beta^{(2)}(\bar{a}_s)}{\bar{a}_s}[K_1C_F\bar{a}_s+(K_2N_F+K_AC_A+K_FC_F)C_F\bar{a}_s^2]+O(\bar{a}_s^4), \quad (1)$$

where $\bar{a}_s = \bar{\alpha}_s(\mu^2 = Q^2)/4\pi$, N_F is the number of flavors, C_A and C_F are the Casimir operators (in QCD $C_A = 3$, $C_F = 4/3$), $\beta^{(2)}(\bar{a}_s) = \beta_1\bar{a}_s^2 + \beta_2\bar{a}_s^3 + O(\bar{a}_s^4)$ is the QCD β-function in the two-loop approximation. It is important to point out that this function does not contain the terms of type $C_F^{N-1}\bar{a}_s^N$, $(N \geq 2)$. The numerical multipliers in eq. (1) are determined as

$$K_1 = (-\frac{21}{2} + 12\zeta(3));\quad K_2 = (\frac{326}{6} - \frac{304}{6}\zeta(3));\quad K_A = (-\frac{629}{2} + \frac{884}{3}\zeta(3));$$

$$K_F = (\frac{397}{6} + 136\zeta(3) - 240\zeta(5)).$$

The coefficient function C_{Bj} from the Bjorken sum rule for polarized deep-inelastic lepton-hadron scattering is determined through the following operator product expansion

$$i\int TV_\alpha(x)V_\beta(0)e^{ipx}dx|_{|p^2|\to\infty} \simeq C_{Bj}(\bar{a}_s)\frac{\varepsilon_{\alpha\beta\rho\lambda}p^\rho}{p^2}\frac{1}{12}A^{(3)\lambda}(0) + \ldots$$

Here V_α denotes the electromagnetic current, $A^{(3)\lambda}$ is the third component of the axial isotriplet (interpolating current for π-meson). The expression for this coefficient function is known in the two-loop [3] and three-loop [4] approximations of perturbation theory. In the leading order it has the form $C_{Bj}(\bar{a}_s) = 1 - 3C_F\bar{a}_s + O(\bar{a}_s^2)$. The quantity C_R from eq. (1) is just the coefficient function for the branching ratio

of e^+e^--annihilation into the hadrons. This coefficient function is also known in the two-loop [5] and three-loop [6] approximations. The leading order result has the form

$$C_R(\bar{a}_s) = \frac{D(\bar{a}_s)}{N_c} = 1 + 3C_F\bar{a}_s + O(\bar{a}_s^2),$$

where the Adler function $D(\bar{a}_s)$ is defined as

$$D(\bar{a}_s) = -12\pi^2q^2\frac{d}{dq^2}\Pi(q^2),\quad i\int\langle 0|TA_\alpha^{(3)}(x)A_\beta^{(3)}(0)|0\rangle e^{iqx}dx = (g_{\alpha\beta}q^2 - q_\alpha q_\beta)\Pi(q^2).$$

The aim of the present investigation is to elucidate the reason of cancellation of the $C_F\bar{a}_s$, $C_F^2\bar{a}_s^2$ and $C_F^3\bar{a}_s^3$ corrections in the Crewther relation and to generalize, if possible, this low to the higher orders of perturbation theory. As it will be demonstrated below, the observed cancellation is intimately related to the specific structure of the anomalous triangle and the Adler-Bardeen theorem [7].

Let us consider the following three-point correlation function

$$T_{\mu\alpha\beta}(p,q) = \int\langle 0|TA_\mu^{(3)}(y)V_\alpha(x)V_\beta(0)|0\rangle e^{ipx+iqy}dxdy = \zeta_1(q^2,p^2)\varepsilon_{\mu\alpha\beta\tau}p^\tau +$$

$$+\zeta_2(q^2,p^2)(q_\alpha\varepsilon_{\mu\beta\rho\tau}p^\rho q^\tau - q_\beta\varepsilon_{\mu\alpha\rho\tau}p^\rho q^\tau) + \zeta_3(q^2,p^2)(p_\alpha\varepsilon_{\mu\beta\rho\tau}p^\rho q^\tau + p_\beta\varepsilon_{\mu\alpha\rho\tau}p^\rho q^\tau), \quad (2)$$

where the expansion over the three independent tensor structures is used (the kinematical condition $pq = 0$ is also assumed, for details see ref. [8]).

Following the ideology of ref. [2], we consider the operator product expansion for this correlator in the limit when $|p^2| \to \infty$. Now, using the relation for the various tensor structures [8], it is easy to derive that

$$T_{\mu\alpha\beta}(p,q) \to \frac{1}{12}\frac{1}{p^2}C_{Bj}(\bar{a}_s)\Pi(q^2)(q_\alpha\varepsilon_{\mu\beta\rho\tau}p^\rho q^\tau - q_\beta\varepsilon_{\mu\alpha\rho\tau}p^\rho q^\tau),$$

consequently

$$\zeta_2(q^2,p^2)|_{|p^2|\to\infty} \to \frac{1}{12}\frac{1}{p^2}C_{Bj}(\bar{a}_s)\Pi(q^2). \quad (3)$$

On the other hand, requirement of the gauge invariance leads one to the Ward identity for the Green function under consideration. In our case the vector Ward identity takes the form [8]

$$-\zeta_1(q^2,p^2) = q^2\zeta_2(q^2,p^2) + p^2\zeta_3(q^2,p^2).$$

Differentiating this expression with respect of q^2 and taking into account that the function ζ_1 is just the nonrenormalizable c-number (Adler-Bardeen theorem[7]) we get the following equation for two other invariant functions

$$q^2\frac{d}{dq^2}\zeta_2(q^2,p^2)=-p^2\frac{d}{dq^2}\zeta_3(q^2,p^2)-\zeta_2(q^2,p^2). \quad (4)$$

It should be noticed here, that the statement on the one-loop behavior of the axial anomaly has not strict sense within the perturbation theory. On the language of operator relation the one-loop character is achieved when the normalization of the axial current is strictly fixed in accordance with the relation $(\Lambda^5_\mu)^{Ren}=\gamma_5(\Lambda_\mu)^{Ren}$, where $(\Lambda^5_\mu)^{Ren}$ and $(\Lambda_\mu)^{Ren}$ denote the axial and vector vertex functions respectively. However, this condition does not guarantee the absence of corrections on the language of Green functions (in our case the absence of corrections to ζ_1). As it has been shown in ref. [9], there are anomalous graphs containing light-by-light subdiagrams which cause the renormalization of the axial anomaly on the language of Green functions. However, in our case when the axial current in (2) is the flavor nonsinglet one, diagrams mentioned above renormalyze the quantity ζ_1 in the second order in the fine structure constant, but not in $\bar{a}_s^2$ order. Hence, neglecting the higher electromagnetic corrections, we are able to postulate the one-loop character for ζ_1.

On the other hand, under the condition $|p^2|\to\infty$ and in accordance with the eq. (3) we have

$$q^2\frac{d}{dq^2}\zeta_2(q^2,p^2)\to-\frac{N_c}{(12\pi)^2}\frac{1}{p^2}C_{Bj}(\bar{a}_s)C_R(\bar{a}_s). \quad (5)$$

Let us now expand the expressions for the quantities ζ_2 and ζ_3 in powers of q^2/p^2

$$\zeta_2(q^2,p^2)=\frac{1}{p^2}\sum_{k=0}^{\infty}(\frac{q^2}{p^2})^k\zeta_2^k, \qquad \zeta_3(q^2,p^2)=\frac{1}{p^2}\sum_{n=0}^{\infty}(\frac{q^2}{p^2})^n\zeta_3^n,$$

here ζ_2^k and ζ_3^k are dimensionless coefficients. Substituting these series into the eq. (4) one gets

$$q^2\frac{d}{dq^2}\zeta_2(q^2,p^2)=-\frac{1}{p^2}\sum_{k=0}^{\infty}[(k+1)\zeta_3^{k+1}+\zeta_2^k](\frac{q^2}{p^2})^k. \quad (6)$$

Comparing now eq. (6) with the relation (3) we obtain the following formulae for the product of C_{Bj} and C_R

$$\frac{N_c}{(12\pi)^2}C_{Bj}(\bar{a}_s)C_R(\bar{a}_s)=\zeta_3^1+\zeta_2^0. \quad (7)$$

In the leading order of perturbation theory $\zeta_3^1+\zeta_2^0=N_c/(12\pi)^2$. In so doing, we convinced ourselves that the one-loop or a many-loop behavior for the product $C_{Bj}C_R$ is connected with the renormalizability or nonrenormalizability, respectively, of the

invariant functions ζ_2 and ζ_3. On the other hand, it has been shown in [10], that when the conformal invariance is exactly presented in the theory, the general expression for the three-point correlator function $T_{\mu\alpha\beta}$ has the form totally determined by its one-loop counterpart $\Delta_{\mu\alpha\beta}$

$$T_{\mu\alpha\beta}(p,q)=K(\bar{a}_s)\Delta_{\mu\alpha\beta}(p,q),$$

where $K(\bar{a}_s)$ is the undefined quantity within the approach of ref. [10]. Another way of putting it is that in conformal-invariant theory we have [10]

$$\zeta_1^{exact}=K(\bar{a}_s)\zeta_1^{one\ loop}, \quad \zeta_2^{exact}=K(\bar{a}_s)\zeta_2^{one\ loop}, \quad \zeta_3^{exact}=K(\bar{a}_s)\zeta_3^{one\ loop}. \quad (8)$$

However, it is well known that the renormalization procedure violates the initial conformal invariance of the massless QCD leading to the anomaly in the trace of energy-momentum tensor [2],[11]. The expression for this anomaly [11] in its turn indicates that the factor $\beta(\bar{a}_s)/(\bar{a}_s)$ is the measure of violation of conformal invariance within the framework of perturbation theory. On this basis the relations (8) could be rewritten in QCD as

$$\zeta_1^{exact}=K(\bar{a}_s)\zeta_1^{one\ loop}, \quad \zeta_2^{exact}=[K(\bar{a}_s)+\frac{\beta(\bar{a}_s)}{\bar{a}_s}v_2(p^2,q^2,\bar{a}_s)]\zeta_2^{one\ loop},$$

$$\zeta_3^{exact}=[K(\bar{a}_s)+\frac{\beta(\bar{a}_s)}{\bar{a}_s}v_3(p^2,q^2,\bar{a}_s)]\zeta_3^{one\ loop},$$

where v_2 and v_3 are dimensionless functions satisfying to the Ward identity (4). Arguing now, that in accordance with the Adler-Bardeen theorem $\zeta_1^{exact}=\zeta_1^{one\ loop}$, we obtain $K(\bar{a}_s)=1$. Hence, the invariant functions ζ_2 and ζ_3 are renormalized in the higher orders of perturbation theory by the multiplier containing the factor proportional to $\beta(\bar{a}_s)/\bar{a}_s$ beyond the unity. This fact leads to the following expression for the product of C_{Bj} and C_R

$$C_{Bj}(\bar{a}_s)C_R(\bar{a}_s)=1+\frac{\beta(\bar{a}_s)}{\bar{a}_s}r(\bar{a}_s),$$

$r(\bar{a}_s)$ being polynomial in powers of $\bar{a}_s$, which is not fixed in our approach.

In summary let us stress once again that in this work the reason of cancellation of the $C_F^N\bar{a}_s^N$, $(N\geq1)$ type corrections in the product of coefficient function from the Bjorken sum rule for polarized deep-inelastic lepton-hadron scattering and the Adler function for the two-point correlator of electromagnetic currents has been investigated. It has been shown that the mentioned cancellation appears as a consequence of the Adler-Bardeen theorem for the axial anomaly. It has also been demonstrated that all surviving corrections are grouped producing the factor proportional to the quantity $\beta(\bar{a}_s)/\bar{a}_s$, which in its turn is the measure of violation of conformal invariance in QCD.

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Габададзе Г.Т., Катаев А.Л. E2-95-76

О связи между коэффициентными функциями аннигиляционных и глубоконеупругих процессов

Показано, что однопетлевой характер аксиальной аномалии, проявляющийся при подходящем выборе нормировки аксиального тока, является причиной сокращения поправок типа $C_F^N \overline{\alpha}_s^N$, $(N \geq 1)$ в соотношении Крезера для коэффициентных функций аннигиляционных и глубоконеупругих процессов.

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On Connection between Coefficient Functions for Deep-Inelastic and Annihilation Processes

It has been shown that the one-loop behaviour of the axial anomaly, occurring when the axial current is appropriately normalized, leads to the cancellation of the corrections of type $C_F^N \overline{\alpha}_s^N$, $(N \geq 1)$ in the Crewther relation for the coefficient functions of deep-inelastic and annihilation processes.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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