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PHASE STRUCTURE OF THE LINEAR σ-MODEL IN R^{1+1}

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1 Introduction

Investigation of the phase structure of quantum field systems relates to several significant points of quantum field theory (QFT). These are the dynamical symmetry reconstruction and dynamical generation of fermion masses, composite fields and some others. Most of these problems can be reformulated as general problem of description of actual ground state and calculating the physical spectrum of given theory. This means that starting with a given classical Lagrangian we have to determine, what particles are described by this Lagrangian after quantization, and how they interact for different regimes over the coupling constants, the temperature and other possible parameters. However, the ultraviolet (UV) divergences occurring in most QFT models hamper simple realization of this program and make trouble for some powerful methods, e.g., the variational one. Investigation of simple superrenormalizable models (like the two-dimensional linear σ-model considered in present paper) can help to develop an appropriate language and methods for studying the phase structure problem in QFT.

There is also particular reason why investigation of the phase structure of the Yukawa-type theories in R^{1+1} is quite interesting. It gives a straightforward opportunity to clarify relationship between lattice (regularized) and continuous (renormalized) formulations of QFT. Lattice calculations indicate a rich phase structure of the Yukawa-type theories [1]. In particular, it has been shown in paper [2] that in the two-dimensional theories with the Yukawa interaction fermions get nonzero dynamically generated mass for arbitrary weak Yukawa coupling. This interesting result, obtained within the lattice formulation of QFT, cannot be extended directly to the case of renormalized continuous theory. Some analytical methods are need to test this prediction of lattice QFT.

A popular approach to the problem of phase structure is provided by variational estimations of the effective potential [3, 4, 5]. Meanwhile, applicability of the variational method to QFT is restricted by some problems (see Feynman's paper in [6]). The Hamiltonians of most QFT models are ill-defined operators in the Hilbert space due to the higher order UV divergences. As a result, variational estimations with the help of trial wave functionals are not defined either [7]. This issue is in the φ^4_d-theory for $d > 2$. The same thing is true for the two-dimensional linear σ-model as soon as the diagrams in Fig. 1 are divergent. Another problem arises from impossibility to control an accuracy of approximation within the variational calculations [8].

A way to overcome these difficulties of the variational approach was proposed in [9]-[11] where the phase structure of various scalar field models at zero and finite temperature was considered. The detailed description of the method we use can be found in Ref. [11]. Here, we will formulate main ideas of the method.

It is based on a combination of the canonical transformation method and renormalization group. The idea of this combination originates from the basic properties of the local QFT: existence of nonequivalent representations of canonical (anti)commutation relations (CR) and UV divergences (see, e.g., [12] and references therein). From a physical viewpoint, existence of nonequivalent representations means that field system may have a set of vacua. Internal dynamics of the system chooses among them a vacuum which is an actual ground state of the system for given values of parameters of the model. The nonequivalent representations give a tool for classification and description of different phases. Below we will identify a phase structure of given QFT model with a set of nonequivalent CR representations realized in this model for different values of dynamical and external parameters.

We can formulate the problem as follows:

what representation of CR is suitable for different values of coupling constants and what physical picture corresponds to this representation?

A way to recover the dynamical reasons for a formation of phases is provided by the fact that UV divergencies give a main contribution to the physical parameters like masses, effective coupling constants and so on. Renormalization removes these divergencies and means actually taking into account the leading radiative corrections which have a major influence on a formation of the phases.

The scattering theory as well as the canonical formalism of QFT indicate the following *correct* form of the total Hamiltonian:

$$\mathrm{H} = \mathrm{H}_0 + \mathrm{H}_\mathrm{I} + \mathrm{H}_\mathrm{ct} + VE\,. \tag{1.1}$$

The standard free part H_0 describes a ground state of field system. The interaction Hamiltonian H_I does not contain terms linear or quadratic in fields and describes small corrections to H_0 if the coupling constants are small. The counter-term operator H_ct removes all UV divergencies. The form of H_ct is determined by H_0, H_I and renormalization scheme. The renormalization scheme should be fixed. The constant term E is the vacuum energy density, which coincides with the free energy density for zero temperature.

The method we use is based on two ideas. First, the total Hamiltonian of field system should be written in the *correct* form in some particular representation which seems to be suitable for specific values of the parameters (e.g., in the weak coupling regime at zero temperature). Second, the *canonical transformations* of the field variables and the requirement that the Hamiltonian expressed in new variables has the *correct* form lead to equations defining unitary nonequivalent CR representations at any values of parameters. Each representation is characterized by the effective coupling constants and vacuum energy density. The system is considered to be in a definite phase if the effective coupling constants and free energy density in the representation describing this phase are the smallest ones. The effective coupling constants are used to control an accuracy of approximation.

The results of present paper can be summarized as follows. We find the boson and fermion masses, effective coupling constants, order parameter and free energy density as functions of (G, Y) for different CR representations. Here, G and Y are boson self-interaction and Yukawa coupling constants, correspondingly. The phase diagram in the (Y, G)-plane is constructed. There are two phases with broken chiral symmetry: φ^4-type phase conditioned by the self-interaction of boson fields and the Yukawa-type phase coursed by the Yukawa coupling. There is a critical point (Y_c = .13..., G_c = .84...) where the difference between these phases disappears. The φ^4-type phase with violated symmetry is realized in the strong coupling regime $G \gg Y$. This agrees with the phase structure of the pure $(\vec{\varphi}^2)^2_2$ theory [4, 9] and is not surprising. Instability of the symmetric phase at $Y \ll 1$ and $G \ll 1$ seems to be actually unexpected and interesting. Arbitrary small Yukawa coupling leads to the symmetry breaking. As a result, the fermions get dynamically generated mass in the weak coupling regime, that agrees with the results of lattice calculations [2]. This agreement is only qualitative, since statement of the problem of phase structure of a field system and investigation techniques within the regularized (lattice) and renormalized (as in our case) formulations of quantum field theory are basically different (for details see [13]).

The symmetric phases are realized for the case ($Y = 0$, $G < 1.317...$), for some intermediate coupling regime and for $Y \gg G$. The points ($Y = 0$, $G = 1.317...$) and ($Y = .08...$, $G = 1.13..$) turn out to be the triple points in which the symmetric phase and two phases with broken symmetry (the φ^4- and Yukawa-type ones) coexist.

The effective coupling constants are small in the cases: $G \ll 1$ and $Y \ll 1$, $G \gg Y$, $Y \gg G$. For other values of G and Y they are large enough, that indicates that our method gives quite accurate description of the phases outside the critical regions, while the phase boundaries are defined rather approximately. More accurate description of the critical regions can be achieved by incorporating into below calculations not only all divergent diagrams but also some classes of the UV finite diagrams [14].

There is another important aspect of these results. According to Goldstone theorem, massless bosons should appear in the phases with broken chiral symmetry. Naively, pseudoscalar triplet should play the role of the Goldstone bosons as is realized for spontaneous breaking of chiral symmetry at classical level. Meanwhile, as our calculations show, dynamical violation of the symmetry can be realized with nonzero mass of pseudoscalar fields, although the ratio of masses of the pseudoscalar and scalar particles tends to zero in the strong coupling limit (see also the paper [4] where the same result was obtained for the O(N) invariant φ^4_2 theory within the Gaussian approximation of the effective potential). This situation is treated in [4] as approximate realization of the Goldstone theorem. Meanwhile, the reason may be deeper. As is pointed out in the original paper of Goldstone, Salam and Weinberg [15], the massless bosons accompanying symmetry breaking are the composite particles rather than the elementary ones. In other words, for complete solution of the problem of physical spectrum we have to incorporate into consideration the CR representations corresponding to composite fields, that is the problem for further investigations. The underlining theoretical aim of present paper is to demonstrate by explicit calculations how the renormalization of UV divergences affects the nonequivalent representations of the canonical relations.

2 The Symmetry of Classical Lagrangian

In this section we recall the well-known properties of the classical Lagrangian of the linear σ-model. The Lagrangian density has the form

$$\begin{aligned} L &= \bar{\psi}_j(x) i\hat{\partial}\psi_j(x) + \frac{1}{2}\varphi_j(x)(\Box - m^2)\vec{\varphi}_j(x) + \frac{1}{2}\sigma(x)(\Box - m^2)\sigma(x) \\ &+ y\bar{\psi}(x)(\sigma(x) + i\gamma_5\tau_j\varphi_j(x))\psi(x) - \frac{g}{4}(\varphi_j^2(x) + \sigma^2(x))^2, \end{aligned} \tag{2.2}$$

where $x = (x_0, x_1) \in R^{1+1}$. In the two-dimensional space-time the Dirac matrices can be represented as

$$\gamma_0 = \tau_3, \quad \gamma_1 = i\tau_2, \quad \gamma_5 = \tau_1,$$

with τ_j being the Pauli matrices.

Lagrangian (2.2) describes interaction of the pseudoscalar fields φ_i $(i = 1, 2, 3)$, scalar field σ and isodoublet of the fermion fields ψ_j $(j = 1, 2)$. This Lagrangian is invariant under the chiral $SU_L(2) \times SU_R(2)$ transformations:

$$\begin{aligned} &\psi_L \to \psi_L' = \psi_L + i\alpha_j\tau_j\psi_L, \quad \psi_R \to \psi_R' = \psi_R + i\beta_j\tau_j\psi_R, \\ &\varphi_k \to \varphi_k' = \varphi_k - \frac{i}{2}\varepsilon_{ijk}\varphi_j(\beta_i - \alpha_i) + \sigma(\beta_k - \alpha_k), \\ &\sigma \to \sigma' = \sigma + \varphi_j(\alpha_j - \beta_j). \end{aligned}$$

If the mass parameter m^2 of boson fields is positive then the chiral symmetry is exact at the classical level. Intuitively, it seems natural to expect that this symmetry will be not violated by quantum corrections if the dimensionless (perturbation) coupling constants

$$G = \frac{g}{2\pi m^2}, \quad Y = \frac{y^2}{2\pi m^2} \tag{2.3}$$

are small enough.

Does this intuitive suggestion actually true and what particles are described by the field system under consideration in the strong coupling regime, where one or both coupling constants (2.3) are large? In order to clarify these points we will use a kind of the canonical transformation method shortly described in the introduction (see also [11]). First of all we have to construct the Fock representation of the canonical (anti-)commutation relations which plays the role of starting or normalization point for further consideration.

3 Quantization

Canonically quantized Hamiltonian density corresponding to Lagrangian (2.2) has the form

$$\begin{aligned} H &= H_0 + H_I + H_{ct} && (3.4) \\ H_0 &= \frac{1}{2} : [\pi^2 + (\nabla\sigma)^2 + m^2\sigma^2] : + \frac{1}{2} : [\pi_j^2 + (\nabla\varphi_j)^2 + m^2\varphi_j^2] : + : \bar{\psi} i\gamma_1\partial_1\psi :, \\ H_I &= \frac{g}{4} : (\sigma^2 + \varphi_j^2)^2 : - y : \bar{\psi}(\sigma + i\gamma_5\tau_j\varphi_j)\psi : && (3.5) \\ H_{ct} &= \frac{1}{2} : [\delta m_\phi^2\varphi_j^2 + \delta m_\sigma^2\sigma^2] : + \delta E. && (3.6) \end{aligned}$$

The fields φ_j, π_j, σ, π, ψ_i and ψ_i^+ satisfy the canonical relations

$$\begin{aligned} &[\varphi_j(x_0, x_1), \pi_k(x_0, x_1')]_- = i\delta_{kj}\delta(x_1 - x_1') \quad (k, j = 1, 2, 3), \\ &[\sigma(x_0, x_1), \pi(x_0, x_1')]_- = i\delta(x_1 - x_1'), \\ &[i\psi_k^+(x_0, x_1), \psi_j(x_0, x_1')]_+ = i\delta_{kj}\delta(x_1 - x_1') \quad (k, j = 1, 2). \end{aligned} \tag{3.7}$$

The rest of commutators and anti-commutators are equal to zero. According to division of the total Hamiltonian (3.4) into the free H_0 and interaction H_I Hamiltonians, the fields are defined on the Fock space corresponding to massless fermions and scalar and pseudoscalar bosons with renormalized mass m. Vacuum vector $|0\rangle$ of this Fock space obeys usual conditions

$$\varphi_j^{(-)}(x)|0\rangle = 0, \quad \sigma^{(-)}(x)|0\rangle = 0, \quad \psi_j^{(-)}(x)|0\rangle = 0, \quad (\forall x, j), \tag{3.8}$$

where $\varphi_j^{(-)}$, $\sigma^{(-)}$ and $\psi_i^{(-)}$ are negative-frequency parts of fields.

The model under consideration is superrenormalizable. Besides the lowest order bubble diagrams, there are few divergent diagrams shown in Fig. 1. Ultraviolet divergences corresponding to bubble diagrams are removed in representation (3.4-3.6) by normal ordering of the Hamiltonian with respect to the vacuum $|0\rangle$. The divergences related to the diagrams shown in Fig. 1a require mass renormalization of fields φ_i and σ, while vacuum diagrams given in Fig. 1b contribute to vacuum energy renormalization.

From physical viewpoint, the following renormalization scheme is preferable:

- *mass renormalization*: the counterterms δm_φ and δm_σ are given by the diagrams in Fig. 1a with external momenta squared being equal to renormalized mass squared, that corresponds to the on-shell renormalization scheme;
- *vacuum energy renormalization*: divergent vacuum diagrams in Fig. 1b are subtracted completely.

These prescriptions and the form of H_0 and H_I determine the counterterm part H_{ct} given by (3.6) with

$$\begin{aligned}\delta m_\varphi^2 &= \mathrm{y}^2\tilde{\Pi}^{\mathrm{P}}_{\mathrm{reg}}(m^2|0), \\ \delta m_\sigma^2 &= \mathrm{y}^2\tilde{\Pi}^{\mathrm{S}}_{\mathrm{reg}}(m^2|0), \\ \delta E &= \frac{\mathrm{y}^2}{16\pi}\mathrm{reg}\int_0^\infty \frac{du}{u+m^2}\left[\tilde{\Pi}^{\mathrm{S}}_{\mathrm{reg}}(-u|0)+3\tilde{\Pi}^{\mathrm{P}}_{\mathrm{reg}}(-u|0)\right].\end{aligned} \tag{3.9}$$

The pseudoscalar $\tilde{\Pi}^{\mathrm{P}}_{\mathrm{reg}}(m^2|0)$ and scalar $\tilde{\Pi}^{\mathrm{S}}_{\mathrm{reg}}(m^2|0)$ polarization functions look as

$$\begin{aligned}\tilde{\Pi}^{\mathrm{S}}_{\mathrm{reg}}(m^2|0) &= 2i\mathrm{reg}\int\frac{\mathrm{d}^2q}{(2\pi)^2}\mathrm{Tr}\left\{\tilde{S}(q-p|0)\tilde{S}(q|0)\right\}, \\ \tilde{\Pi}^{\mathrm{P}}_{\mathrm{reg}}(m^2|0) &= 2i\mathrm{reg}\int\frac{\mathrm{d}^2q}{(2\pi)^2}\mathrm{Tr}\left\{i\gamma_5\tilde{S}(q-p|0)i\gamma_5\tilde{S}(q|0)\right\},\end{aligned} \tag{3.10}$$

where

$$\tilde{S}(q|m_F) = \frac{1}{m_F-q-i\varepsilon}$$

For the case of massless fermions ($m_F=0$) the counterterms for the pseudoscalar and scalar fields coincide:

$$\tilde{\Pi}^{\mathrm{P}}_{\mathrm{reg}}(m^2|0) = \tilde{\Pi}^{\mathrm{S}}_{\mathrm{reg}}(m^2|0) = \frac{1}{\pi}\ln\frac{\Lambda^2}{m^2},$$

where Λ is a cutoff parameter.

Now the S-matrix of the model

$$S = \lim_{\Lambda\to\infty}\mathrm{reg}\,\mathrm{T}\exp\left\{-i\int \mathrm{d}^2x\,[H_{\mathrm{I}}+H_{\mathrm{ct}}]\right\}$$

is defined. It means that we have definite rules for calculating the S-matrix elements, which are UV finite in each order of perturbation expansion over the coupling constants G and Y. The calculation prescriptions are appropriate unless G and Y are small enough. The next step consists in looking for other possible representations of the canonical relations (3.7).

4 Canonical Transformation

Let us transform the canonical variables as follows

$$\begin{aligned}&\{i\psi_j^+,\psi_j\}\to\left\{i\Psi_j^+\exp\left(-i\frac{\alpha}{2}\gamma_5\right),\exp\left(i\frac{\alpha}{2}\gamma_5\right)\Psi_j\right\},\quad (j=1,2) \\ &\{\pi_i,\varphi_i\}\to\{\Pi_i,\Phi_i\}\quad (i=1,2,3), \\ &\{\pi,\sigma\}\to\{\Pi,\Sigma+\sigma_0\},\end{aligned} \tag{4.11}$$

where Σ is a scalar field with a new mass $M_\Sigma^2 = s\cdot m^2$, Φ_i is a triplet of the pseudoscalar fields with a new mass $M_\Phi^2 = t\cdot m^2$ and Ψ_j are the fermion fields with a mass $M_F^2 = f\cdot m^2$. The constant σ_0 has the sense of vacuum condensate of scalar field.

The fields are defined on the Fock space unitary nonequivalent to the original one. Vacuum vector in the new Fock space $|0\rangle\rangle$ satisfies conditions

$$\Phi_j^{(-)}(x)|0\rangle\rangle = 0,\quad \Psi_j^{(-)}(x)|0\rangle\rangle = 0,\quad \Sigma^{(-)}|0\rangle\rangle = 0\quad (\forall x,j). \tag{4.12}$$

The Hamiltonian density expressed in terms of new canonical variables takes the form

$$H = H_0'+H_{\mathrm{I}}'+H_{\mathrm{ct}}'+H_1+E \tag{4.13}$$

$$\begin{aligned}H_0' &= \frac{1}{2}:[\Pi_\Sigma^2+(\nabla\Sigma)^2+M_\Sigma^2\Sigma^2]:+\frac{1}{2}:[\Pi_i{}^2+(\nabla\Phi_i)^2+M_\Phi^2\Phi_i^2]: \\ &+:\bar{\Psi}(i\gamma_1\partial_1+M_F)\Psi:,\end{aligned} \tag{4.14}$$

$$\begin{aligned}H_{\mathrm{I}}' &= \frac{g}{4}:(\Sigma^2+\Phi_i^2)^2:+g\sigma_0\Sigma\Phi_i^2+g\sigma_0\Sigma^3 \\ &-\mathrm{y}:\bar{\Psi}\left[\Sigma(\cos(\alpha)+i\gamma_5\sin(\alpha))-\tau_j\Phi_j(\sin(\alpha)-i\gamma_5\cos(\alpha))\right]\Psi:.\end{aligned} \tag{4.15}$$

The total Hamiltonian is written in normal form with respect to the new vacuum $|0\rangle\rangle$. The counterterm operator H_{ct}' is determined by the new free Hamiltonian H_0', new interaction Hamiltonian H_{I}' and renormalization scheme which is equivalent to the initial one. This means that mass renormalization satisfies the on-shell condition with the new masses M_Φ, M_Σ and M_F, and vacuum energy renormalization removes new vacuum diagrams completely:

$$H_{\mathrm{ct}}' = \frac{1}{2}:[\delta M_\Phi^2\Phi_i^2+\delta M_\Sigma^2\Sigma^2]:+\delta E', \tag{4.16}$$

$$\delta M_\Phi^2 = \mathrm{y}^2\tilde{\Pi}^{\mathrm{PS}}_{\mathrm{reg}}(M_\Phi^2|M_F),\quad \delta M_\Sigma^2 = \mathrm{y}^2\tilde{\Pi}^{\mathrm{SP}}_{\mathrm{reg}}(M_\Sigma^2|M_F),$$

$$\begin{aligned}\tilde{\Pi}^{\mathrm{PS}}_{\mathrm{reg}}(M_\Phi^2|M_F) &= 2i\mathrm{reg}\int\frac{\mathrm{d}^2q}{(2\pi)^2}\mathrm{Tr}\left\{i\gamma_5e^{i\alpha\gamma_5}\tilde{S}(q-p|M_F)i\gamma_5e^{i\alpha\gamma_5}\tilde{S}(q|M_F)\right\} \\ &= \frac{1}{\pi}\left[\ln\frac{\Lambda^2}{m^2}-\ln f-F\left(\frac{f}{t}\right)\right],\end{aligned} \tag{4.17}$$

$$\begin{aligned}\tilde{\Pi}^{SP}_{\mathrm{reg}}(M_\Sigma^2,M_F) &= 2i\,\mathrm{reg}\int\frac{\mathrm{d}^2q}{(2\pi)^2}\mathrm{Tr}\left\{e^{i\alpha\gamma_5}\tilde{S}(q-p|M_F)e^{i\alpha\gamma_5}\tilde{S}(q|M_F)\right\} \\ &= \frac{1}{\pi}\left[\ln\frac{\Lambda^2}{m^2}-\ln f-\left(1-4\frac{f}{s}\right)F\left(\frac{f}{s}\right)\right],\end{aligned} \tag{4.18}$$

$$\delta E' = \frac{\mathrm{y}^2}{16\pi}\mathrm{reg}\int_0^\infty du\left[\frac{\tilde{\Pi}^{\mathrm{SP}}_{\mathrm{reg}}(-u|M_F)}{u+M_\Sigma^2}+3\frac{\tilde{\Pi}^{\mathrm{PS}}_{\mathrm{reg}}(-u|M_F)}{u+M_\Phi^2}\right]. \tag{4.19}$$

Here, the function $F(z)$ looks as

$$F(z) = \int_0^1 \frac{dx}{x(1-x)-z} = \begin{cases} \frac{1}{\sqrt{1-4z}} \ln\left(\frac{1+\sqrt{1-4z}}{1-\sqrt{1-4z}}\right), & \text{if } z \le \frac{1}{4} \\ \frac{2}{\sqrt{4z-1}} \mathrm{arctg}\sqrt{4z-1}, & \text{if } z \ge \frac{1}{4}. \end{cases} \tag{4.20}$$

The vacuum energy density $E(G,Y)$ in Eq.(4.13) is a c-number function and has the form

$$\begin{aligned} E &= E_0 + E_{\mathrm{I}} + E_{\mathrm{ct}}, \\ E_0 &= \frac{m^2\sigma_0^2}{2} + L_{\mathrm{B}}(s,t) + L_{\mathrm{F}}(f), \\ E_{\mathrm{I}} &= \frac{g}{4}\left[\sigma_0^4 - 6\sigma_0^2 D(s) - 6\sigma_0^2 D(t) + 3D^2(s) + 15D^2(t) + 6D(s)D(t)\right] \\ &\quad - \mathrm{y}\cos\alpha\sigma_0 \langle\langle 0|\bar\Psi\Psi|0\rangle\rangle, \\ E_{\mathrm{ct}} &= \delta E - \delta E' + \frac{1}{2}\delta m_\sigma^2 \sigma_0^2 - \frac{3}{2}\delta m_\varphi^2 D(t) - \frac{1}{2}\delta m_\sigma^2 D(s), \end{aligned}$$

where

$$\begin{aligned} L(s,t) &= \frac{1}{2}\langle\langle 0|\left[\Pi_\Sigma^2 + (\nabla\Sigma)^2 + M_\Sigma^2\Sigma^2 + \Pi_j^2 + (\nabla\Phi_j)^2 + M_\Phi\Phi_j^2\right]|0\rangle\rangle \\ &\quad - \frac{1}{2}\langle 0|\left[\pi^2 + (\nabla\sigma)^2 + m^2\sigma^2 + \pi_j^2 + (\nabla\varphi_j)^2 + m^2\varphi_j^2\right]|0\rangle \\ &= \frac{m^2}{8\pi}(3t - 3\ln t + s - \ln s - 4), \\ L_{\mathrm{F}}(f) &= \langle\langle 0|\bar\Psi(i\gamma_1\partial_1 + M_F)\Psi|0\rangle\rangle - \langle 0|\bar\psi i\gamma_1\partial_1\psi|0\rangle, \end{aligned}$$

and

$$D(z) = \int \frac{d^2q}{2\pi^2 i}\left[\frac{1}{m^2-q^2} - \frac{1}{zm^2-q^2}\right] = \frac{1}{4\pi}\ln z. \tag{4.21}$$

The terms H_0', H_{I}', H_{ct}' and E gives the total Hamiltonian in new representation written in the *correct* form. Other terms appearing with the canonical transformation (4.11) are absorbed into the term H_1 which can be written as

$$\begin{aligned} H_1 &= \left[m^2 - M_\Phi^2 + g\sigma_0^2 - gD(s) - 5gD(t) + \delta m_\varphi^2 - \delta M_\Phi^2\right] : \Phi_j^2 : \\ &\quad + \left[m^2 - M_\Sigma^2 + 3g\sigma_0^2 - 3gD(s) - 3gD(t) + \delta m_\sigma^2 - \delta M_\Sigma^2\right] : \Sigma^2 : \\ &\quad + \left[m^2\sigma_0 + g\sigma_0^3 - 3g\sigma_0 D(s) - 3g\sigma_0 D(t) + \delta m_\sigma^2\sigma_0 - \mathrm{y}\cos\alpha \mathrm{Tr}\tilde S(0|M_F)\right]\Sigma \\ &\quad - \left[M_F - \mathrm{y}\sigma_0\cos\alpha\right] : \bar\Psi\Psi : \\ &\quad - \mathrm{y}\sigma_0\sin\alpha : \bar\Psi i\gamma_5\Psi : . \end{aligned} \tag{4.22}$$

In order to provide the *correct* form of the total Hamiltonian we should put $H_1 = 0$. This constraining condition for canonical transformation (4.11) gives a set of equations defining dependence of parameters M_Σ, M_Φ, M_F, σ_0 and α on the coupling constants G and Y:

$$\begin{aligned} &\mathrm{y}\sigma_0\sin\alpha = 0 \\ &M_F - \mathrm{y}\sigma_0\cos\alpha = 0 \\ &m^2 - M_\Phi^2 + g\sigma_0^2 - gD(s) - 5gD(t) + \delta m_\varphi^2 - \delta M_\Phi^2 = 0 \\ &m^2 - M_\Sigma^2 + 3g\sigma_0^2 - 3gD(s) - 3gD(t) + \delta m_\sigma^2 - \delta M_\Sigma^2 = 0 \\ &m^2\sigma_0 + g\sigma_0^3 - 3g\sigma_0 D(s) - 3g\sigma_0 D(t) + \delta m_\sigma^2\sigma_0 - \mathrm{y}\cos\alpha\mathrm{Tr}\tilde S(0|M_F) = 0. \end{aligned} \tag{4.23}$$

Using the dimensionless notation

$$b^2 = 2\pi\sigma_0^2, \quad t = \frac{M_\Phi^2}{m^2}, \quad s = \frac{M_\Sigma^2}{m^2}, \quad f = \frac{M_F^2}{m^2}$$

and taking into account (3.9), (3.10), (4.16), (4.17) and (4.21) one can rewrite (4.23) as

$$\begin{aligned} &b\sin\alpha = 0 \\ &f = Yb^2\cos^2\alpha \\ &3Gb^2 + 1 - s + 2Y\left[\ln f + \left(1 - 4\frac{f}{s}\right)F\left(\frac{f}{s}\right)\right] - \frac{3G}{2}\ln(st) = 0 \\ &Gb^2 + 1 - t + 2Y\left[\ln f + F\left(\frac{f}{t}\right)\right] - \frac{G}{2}\ln s - \frac{5G}{2}\ln t = 0 \\ &b\left[Gb^2 + 1 + 2Y\ln f - \frac{3G}{2}\ln(st)\right] = 0. \end{aligned} \tag{4.24}$$

Taking into account these equations we represent the energy density (4.21) in the form

$$\begin{aligned} E &= \frac{m^2}{8\pi}\left\{2b^2 + 4f\ln f + 3t - 3\ln t + s - \ln s - 4 + G\left[b^4 - 3b^2\ln(st) + \frac{3}{2}\ln s\ln t \right.\right. \\ &\quad \left.\left. + \frac{3}{4}\ln^2 s + \frac{15}{4}\ln^2 t\right] - Y\left(\ln^2 s + 3\ln^2 t\right) + YJ^S\left(\frac{s}{f}\right) - YJ^P\left(\frac{t}{f}\right)\right\}, \end{aligned} \tag{4.25}$$

$$J^S(z) = 4\int_0^1 \frac{dx(1-x^2)}{x((1-x)^2+zx)}\left[\frac{x}{x-1}\ln x - \ln(1-x)\right] > 0,$$

$$J^P(z) = 12\int_0^1 \frac{dx(1-x^2)}{x((1-x)^2+zx)}\left[\ln(1-x) - \frac{x}{1+x}\ln x\right] > 0. \tag{4.26}$$

It should be noted that J^S and J^P enter into (4.25) with different signs – scalar and pseudoscalar fields contribute into the vacuum energy density with opposite signs.

Different solutions of (4.24) define nonequivalent representations of the canonical relations which are identified with possible phases of the system under considera- tion. Each phase is characterized by the energy density (4.25) and effective coupling constants

$$G_P(G,Y) = \frac{G}{t(G,Y)}, \quad G_S(G,Y) = \frac{G}{s(G,Y)},$$
$$Y_P(G,Y) = \frac{Y}{t(G,Y)}, \quad Y_S(G,Y) = \frac{Y}{s(G,Y)},$$

and is described by the Hamiltonian given by Eqs. (4.14), (4.15) and (4.16).

5 Phase Structure

It is convenient to formulate the following definitions. Let us suppose that Eqs.(4.24) have N different solutions, which can be denoted as

$$S_j(Y,G) = \{t_j(G,Y), s_j(G,Y), f_j(G,Y), b_j(G,Y), \alpha_j(G,Y)\} \ (j = 1, ..., N).$$

The effective coupling constants (4.27) and energy density (4.25) corresponding to the j-th solution are defined by

$$Y_P^{(j)}(G,Y) = \frac{Y}{t_j(G.Y)}, \quad G_P^{(j)}(G,Y) = \frac{G}{t_j(G,Y)},$$
$$Y_S^{(j)}(G,Y) = \frac{Y}{s_j(G.Y)}, \quad G_S^{(j)}(G,Y) = \frac{G}{s_j(G,Y)},$$
$$E_j(G,Y) = E\left(t_j(G,Y), s_j(G,Y), f_j(G,Y), b_j(G,Y), \alpha_j(G,Y), G, Y\right).$$

We shall say that in the region $\Gamma_k \subset R^2_+ = \{(Y,G) : Y \geq 0, G \geq 0\}$ the field system (2.2) exists in the phase described by the solution $S_k(Y,G)$ if for $(Y,G) \in \Gamma_k$

$$\min_j E_j(Y,G) = E_k(Y,G)\,, \tag{5.27}$$

$$\min_j Y^{(j)}_{P(S)}(Y,G) = Y^{(k)}_{P(S)}(Y,G)\,, \quad \min_j G^{(j)}_{P(S)}(Y,G) = G^{(k)}_{P(S)}(Y,G)\,. \tag{5.28}$$

The regions Γ_k cover all the space R^2_+, i.e., $\cup\Gamma_k = R^2_+$. It is quite possible that some solutions are not realized as actual phases of the system, since they do not minimize the effective coupling constants and energy density for any Y and G.

Usually, criterion (5.27) based on comparison of the vacuum energy densities is used in the phase transition theory. At the same time, it is natural to suppose that large coupling constants in H mean that representation determined by H_0 does not describe real states and can not be considered as a suitable representation for the total Hamiltonian H. Nevertheless, our calculations show that both criteria give similar results [9, 10, 11, 13].

Solving the equations (4.24) and comparing the energy densities and effective coupling constants we arrive at the phase diagram represented in Fig. 2. The phases with broken chiral symmetry (denoted as III and V) are realized for sufficiently small coupling constants G and Y and for the strong coupling regime $G \gg Y$, that is in agreement with the lattice results [2]. The original symmetric representation (I) occurs for $G \in (0, T_1)$ at the G-axis and for some intermediate coupling regime, while for the strong Yukawa coupling we get the second symmetric phase (II). The points T_1 and T_2 are the triple points, in which the phases I, III and V are in equilibrium.

It should be stressed that the method we use gives an approximate description of the phase picture. Moreover, the effective coupling constants are sufficiently small out of the critical regions, that indicates that we have qualitatively correct picture. At the same time, the phase boundaries in Fig. 2 are defined very approximately, since the effective coupling constants of all solutions of (4.24) are of order of unity in the phase transition regions, and neither solution gives appropriate representation of the canonical relations.

Now let us consider different phases in details.

5.1 Pure Yukawa Interaction

To begin with, consider the case $G = 0$, i.e., the model with pure Yukawa interaction. One gets from (4.24):

$$\begin{aligned} & b\sin\alpha = 0 \\ & f = Yb^2\cos^2\alpha \\ & 1 - s + 2Y\left[\ln f + (1 - 4\frac{f}{s})F\left(\frac{f}{s}\right)\right] = 0 \\ & 1 - t + 2Y\left[\ln f + F\left(\frac{f}{t}\right)\right] = 0 \\ & b(1 + 2Y\ln f) = 0\,. \end{aligned} \tag{5.29}$$

Free energy density (4.25) takes the form

$$\begin{aligned} E = & \frac{m^2}{8\pi}\left[3t - 3\ln t + s - \ln s - 4\right. \\ & \left. -Y\left(\ln^2 s + 3\ln^2 t\right) + YJ^S\left(\frac{s}{f}\right) - YJ^P\left(\frac{t}{f}\right)\right]. \end{aligned}$$

Equations (5.29) have two solutions for b:

$$b = 0 \ \text{(symmetric) and } \ b \neq 0 \ \text{(broken symmetry)}.$$

According to the solutions of the third and fourth equations (5.29) there are three possible phases – two symmetric phases and one with dynamically broken chiral symmetry.

Symmetric solutions correspond to $b = 0,\ f = 0,\ s = t$.

I. $s_1 = t_1 \equiv 1,\ E_1 \equiv 0$. This solution leads to the original representation (3.4-3.6).

II. $s_2(0,Y) = t_2(0,Y) \not\equiv 1,\ E_2(0,Y) \not\equiv 0$. This is the second, nontrivial, symmetric representation. In this phase, the mass of boson fields and free energy density are functions of the coupling constant Y. The mass is defined by the equation:

$$\frac{t_2 - 1}{\ln t_2} = 2Y. \tag{5.30}$$

Energy density (4.25) can be written in the form

$$E_2 = \frac{m^2}{8\pi}\{4t_2 - 4\ln t_2 - 4 - 2(t_2 - 1)\ln t_2\}. \tag{5.31}$$

In this representations neither energy density nor boson mass depend on the parameter α. There is a set of nonequivalent representations with degenerate energies and masses, and with different interaction Hamiltonians. For $\sin\alpha \neq 0$ parity violation takes place due to terms in the interaction Hamiltonian (4.15) responsible for interaction of the scalar (pseudoscalar) field with the pseudoscalar (scalar) fermion current. For definiteness we will consider below the case $\sin\alpha = 0$, i.e., the symmetric representation.

Equations (5.30) and (5.31) show that in the strong coupling regime ($Y \gg 1$) the mass, effective coupling constant and free energy density behave as

$$\begin{aligned} &t_2(0,Y) \to 2Y\ln Y, \\ &Y_P^{(2)}(0,Y) = Y_S^{(2)}(0,Y) \to \frac{1}{2\ln Y} \\ &E_2(0,Y) \to -\frac{m^2}{2\pi} Y \ln^2 Y. \end{aligned} \tag{5.32}$$

On the other hand, in the weak coupling limit ($Y \ll 1$) we get

$$t_2(0,Y) \to \exp\left\{-\frac{1}{2Y}\right\},$$

$$Y_P^{(2)}(0,Y) = Y_S^{(2)}(0,Y) \to Y\exp\left\{\frac{1}{2Y}\right\} \tag{5.33}$$

$$E_2(0,Y) \to \frac{m^2}{8\pi}\frac{1}{Y}$$

For intermediate values of Y the boson mass and energy density $E_2(0,Y)$ are shown in Figs. 2, 3. Comparing the energy densities and effective coupling constants for the phases I and II (see Figs. 2, 3 and asymptotic relations (5.32,5.33)) we see that a kind of phase transition from phase I to the nontrivial symmetric phase II takes place at $Y = 0.5$. The symmetry of the system is not changed.

Equations (5.29) have single solution corresponding to broken chiral symmetry.

III. $b_3 = \frac{1}{\sqrt{Y}}\exp\left\{-\frac{1}{4Y}\right\},\ f_3 = \exp\left\{-\frac{1}{2Y}\right\},\ \sin\alpha = 0,\ t_3 \neq s_3 \neq 1$. The boson masses are defined by the equations

$$\begin{aligned} &s_3 - 2Y\left(1 - 4\frac{f}{s_3}\right)F\left(\frac{f}{s_3}\right) = 0, \\ &t_3 - 2YF\left(\frac{f}{t_3}\right) = 0. \end{aligned} \tag{5.34}$$

Free energy density looks in this case as

$$\begin{aligned} E_3 = \frac{m^2}{8\pi}\Big[&3t_3 - 3\ln t_3 + s_3 - \ln s_3 - 4 \\ &-Y\left(\ln^2 s + 3\ln^2 t\right) + YJ^S\left(\frac{s}{f}\right) - YJ^P\left(\frac{t}{f}\right)\Big]. \end{aligned} \tag{5.35}$$

Equations (5.34) has not real solutions for $0.15.. < Y < 2.58..$, while for other values of Y there is unique real solution.

In the weak coupling regime $Y \to 0$ one can get

$$\begin{aligned} &t_3(0,Y) \to 1 + 2\exp\left\{-\frac{1}{2Y}\right\}, \\ &s_3(0,Y) \to 1 - 2\exp\left\{-\frac{1}{2Y}\right\}, \\ &Y_P^{(3)}(0,Y) \to Y\left(1 - 2\exp\left\{-\frac{1}{2Y}\right\}\right), \\ &Y_S^{(3)}(0,Y) \to Y\left(1 + 2\exp\left\{-\frac{1}{2Y}\right\}\right); \\ &E_3(0,Y) \to -\frac{m^2}{\pi}\exp\left\{-\frac{1}{2Y}\right\}. \end{aligned} \tag{5.36}$$

The dependence of the masses (f_3, s_3, t_3), order parameter b_3 and energy density E_3 on the coupling constant Y is nonanalytical at $Y = 0$.

Free energy density (5.36) is negative for small Y. This is due to negative contribution of the polarization effect of the pseudoscalar fields into the energy density of broken symmetry phase (the term J^P in Eq.(5.35)). Comparing energy densities of all the possible phases I, II and III for $Y \to 0$ we conclude that phase transition accompanied by chiral symmetry breaking occurs at the origin $Y = 0$. This phase transition is of the second order, because the order parameter

$$b_3 = \frac{1}{\sqrt{Y}} \exp\left\{-\frac{1}{4Y}\right\}$$

is continuous in the transition point $Y = 0$.

Numerical solution of Eqs. (5.34) and calculation of the energy density show (see Figs. 3,4) that the energy density in the phase III becomes positive in the point $Y = .11...$, so that we get the transition from the phase III to the symmetric phase I. Further increasing of the coupling constant Y leads to the above-mentioned transition between the symmetric phases I and II.

Now let us consider how the self-interaction of boson fields affects the ground state of the system under consideration.

5.2 Effect of the boson self-interaction

When the self-interaction is switched on, i.e. $G > 0$, two solutions corresponding to broken chiral symmetry appear in addition to the above-considered ones. So that, in general case, we get five different solutions of equations (4.24).

I. $b = 0$, $f_1 = 0$, $t_1 = s_1 = 1$, $E \equiv 0$. This phase is nothing more than the original representation (3.4-3.6).

II. $b = 0$, $f_2 = 0$, $s_2 = t_2 \neq 1$. This is the second symmetric phase. The boson masses $s_2 = t_2$ are defined by the equation

$$\frac{t_2 - 1}{\ln t_2} = 2Y - 3G. \tag{5.37}$$

Energy density (4.21), with account of (5.37), can be written as

$$E_2 = \frac{m^2}{8\pi}\left[4t_2 - 4\ln t_2 - 4 - 2(t_2 - 1)\ln t_2\right]. \tag{5.38}$$

One can see that (5.37) has unique real solution if $2Y - 3G \geq 0$. Otherwise there are no real solutions. Moreover, $t_2 \geq 1$ and $E_2 \leq 0$ for $2Y - 3G \geq 1$ that indicates

the curve $2Y - 3G = 1$ in the (Y, G) plane (see Fig. 2) to be a boundary between the phases I and II. This is according to both criteria based on comparison of the free energy densities and effective coupling constants. In the strong coupling regime $Y \gg G$ we get the asymptotic formulas which are precisely coincide with relations (5.32). Equations (4.24) have not other symmetric solutions.

Solutions III, IV and V with nonzero boson condensate:
$b_j^2(G, Y) = f_j/Y, f_j(G, Y),\ t_j(G, Y) \neq s_j(G, Y)$ $(j = 3, 4, 5)$.
The free energy density for this case is given by (4.25). For description of these solutions it is convenient to rewrite equations (4.24) for the boson and fermion masses in terms of variables f, $r = f/t$ and $q = f/s$:

$$\begin{aligned} f &= 2Y\frac{q(4q-1)F(q)}{(2\frac{G}{Y}q - 1)} \\ rq &= \exp\left\{-\frac{2}{3G}\left(\frac{G}{Y}f + 1 + (2Y - 3G)\ln f\right)\right\} \\ \frac{2}{3} &+ \frac{2G}{3Y}f + \frac{f}{r} - 2YF(r) + 2G\ln q + \frac{2}{3}(2Y - 3G)\ln f = 0. \end{aligned} \tag{5.39}$$

The last equation (5.39) can be considered as an equation on variable q if the first and second equations are taken into account.

Numerical analysis shows that system (5.39) has one or three real solutions for different values of the coupling constants G and Y. Namely, outside the region D restricted by the dashed lines in Fig. 2 we have only one real solution which is a continuation of the solution III of the pure Yukawa model (see previous subsection) on the (Y, G) plane. Its presence is conditioned by the Yukawa coupling. Below we will refer to corresponding phase as the Yukawa-type phase III. Inside the region D two additional solutions IV and V occur, that is caused by the self-interaction of boson fields. They are a continuation of the symmetry breaking solutions of O(4) invariant $(\vec{\varphi}^2)^2_2$ theory [4, 9] on the (Y, G) plane and correspond to the φ^4 type phases.

On the lower dashed line in Fig. 2 solutions IV and V terminate, while on the upper dashed line solutions III and IV disappear. All solutions are equal to each other at the point C in Fig. 2 ($Y_c = 0.13...$, $G_c = 0.84...$). The point C is analogous to the critical point known in the classical thermodynamical systems like gas-liquid [16]. Different phases do not exist and the system is always homogeneous outside the region D in Fig. 2. One can say that at the critical point C the difference between phases disappears. As soon as the critical point exists, a continuous transition between the phases III and V is possible, in which the separation into phases does not occur at any point. To do this, the change of coupling constants must take place along some curve in the (Y, G) plane nowhere cutting the lower dashed line in Fig. 2. This curve may pass through the critical point C.

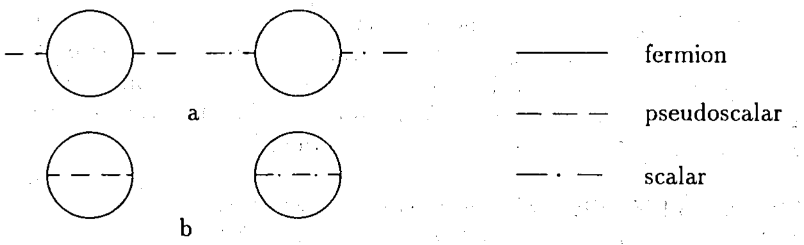

Figure 1: Divergent diagrams.

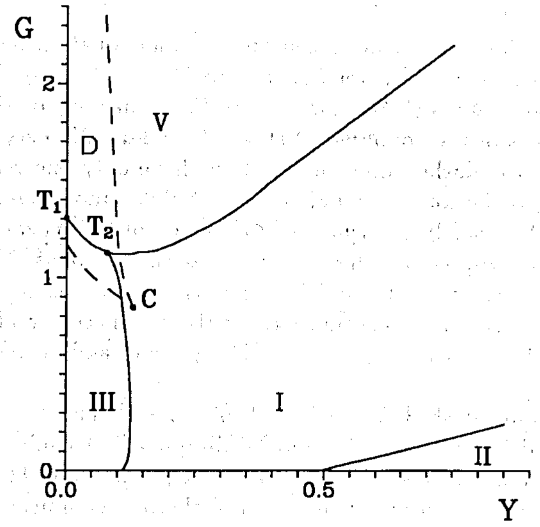

Figure 2: Phase diagram in the plane (Y, G). The dashed lines restrict the region D where Eqs. (5.39) have three solutions.

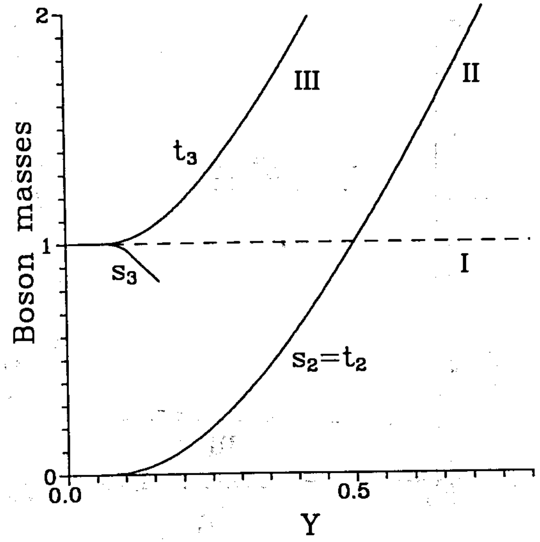

Figure 3: Boson masses in different phases for $G = 0$.

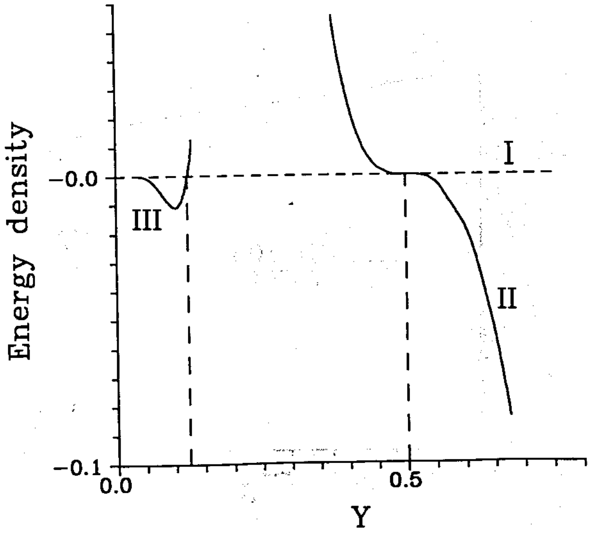

Figure 4: Energy density in different phases for $G = 0$.

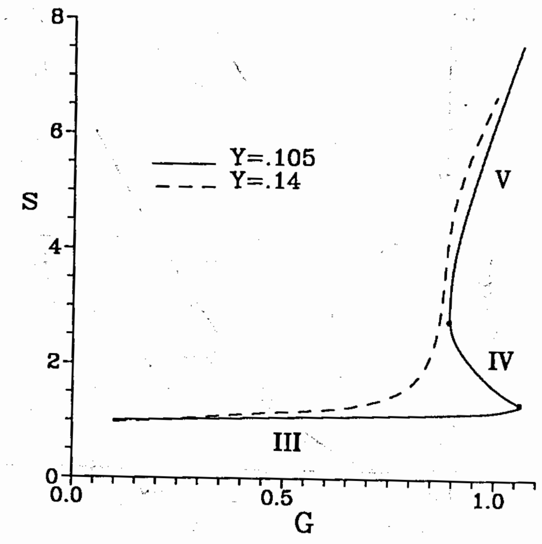

Figure 5: The mass of scalar field for the phases III, IV and V with broken symmetry.

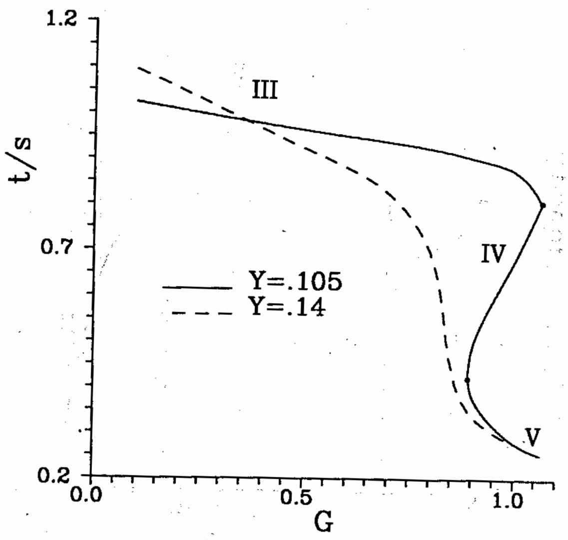

Figure 6: The ratio of masses of the pseudoscalar field t and scalar field s for the phases III, IV and V.

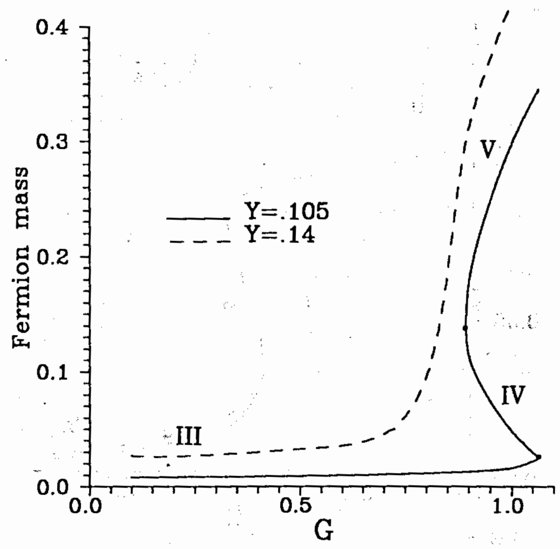

Figure 7: the fermion mass for the phases III, IV and V.

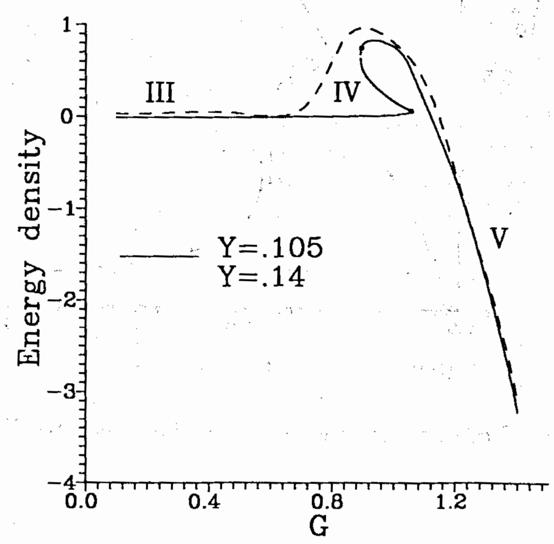

Figure 8: Energy density for the phases III, IV and V.

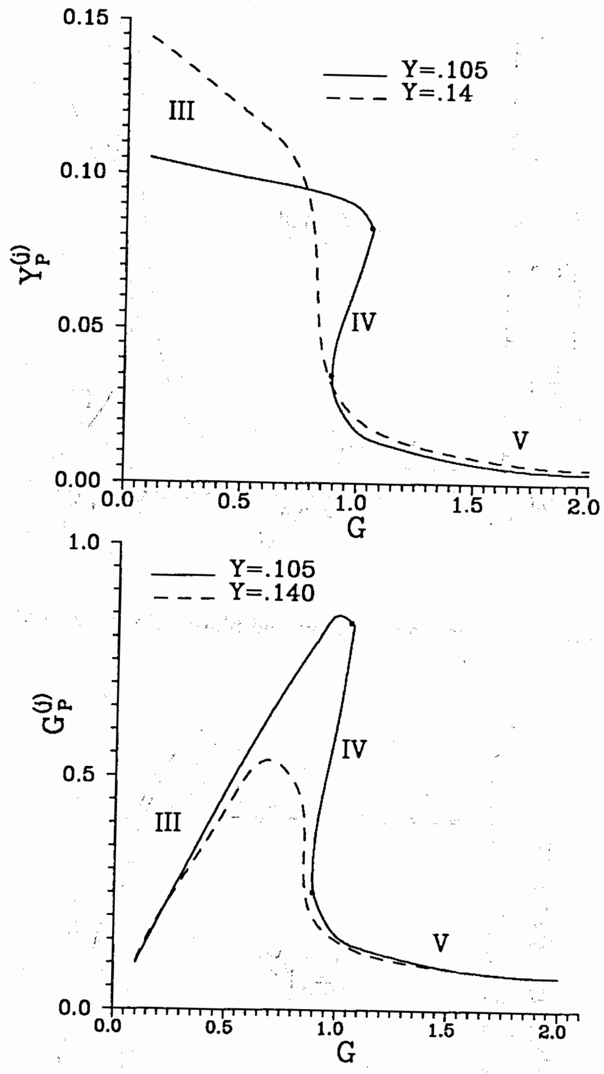

Figure 9: Effective coupling constant for the phases III, IV and V.

Boson and fermion masses as functions of G for a fixed value of Y are shown in Figs. 5-7 for two different paths in the (Y, G) plane. The solid line represent the case $Y < Y_c$: the path cuts the region D and we see the separation into phases III, IV and V. The dashed line corresponds to $Y > Y_c$: the path does not cut the region D, the separation does not occur and a continuous transition from the Yukawa-type phase III to the φ^4-type phase V takes place. The difference between these two phases is purely quantitative, they are characterized by the same symmetry. Strictly speaking, one can speak of two phases only in the case when they exist at the same time touching each other, i.e., for points situated inside the region D.

To find the phase boundaries in the (Y, G) plane we have to compare the free energy densities and effective coupling constants of all the phases of the system. The free energy densities and some of the effective coupling constants for the phases with broken symmetry are shown in Figs. 8,9. In the strong coupling regime $G \gg Y$ $(G \gg 1)$ one can find the relations

$$
\begin{aligned}
&t_5 \to G \ln\ln G \gg 1, \quad s_5 \to 6G\ln G \gg 1, \quad f_5 \to 3Y \ln G \gg 1,\\
&Y_P \to \frac{Y}{G\ln\ln G} \ll 1, \quad Y_S \to \frac{Y}{6G\ln G} \ll 1,\\
&G_P \to \frac{1}{\ln\ln G} \ll 1, \quad G_S \to \frac{1}{6\ln G} \ll 1,\\
&E_5 \to -\frac{m^2}{8\pi}\frac{3}{G}\ln^2 G < 0.
\end{aligned}
\tag{5.40}
$$

This asymptotic formulas show that in the strong coupling regime $G \gg 1$ the phase V with broken symmetry is realized. Numerical solution of equations (5.39) and comparison of the energy densities leads to the phase picture represented in Fig. 2. The Yukawa phase III with broken chiral symmetry is realized for small enough coupling constants G and Y. The φ^4-type phase V exists above the upper solid line in Fig. 2. The original symmetric representation I is realized for the points $(0, T_1)$ at the G-axis and for some intermediate coupling regime, while for the strong Yukawa coupling we get the second symmetric phase II.

The points T_1 $(Y = 0,\ G = 1.317...)$ and T_2 $(Y = .08...,\ G = 1.13...)$ are the triple points where the phases I, III and V are in equilibrium, their energies are equal to zero at these points.

Besides that, the segment $(0, T_1)$ of the G-axis and, in particular, the origin $(0,0)$ corresponds to the second order phase transition between the phases I and III, since the order parameter vanishes continuously at $Y = 0$ (see also previous section).

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Абрамова С.В., Ефимов Г.В., Неделько С.Н. E2-95-72

Фазовая структура линейной σ-модели в R^{1+1}

Фазовая структура двумерной линейной σ-модели исследуется с помощью методов канонических преобразований и ренормгруппы. Построена фазовая диаграмма в плоскости (Y, G), где Y и G — константы связи Юкавы и самодействия бозонов. Получены гамильтонианы, описывающие систему в каждой из фаз. Показано, что вклад псевдоскалярных полей уменьшает плотность свободной энергии фазы с нарушенной киральной симметрией. Это ведет к довольно сложной фазовой структуре линейной σ-модели, наиболее репрезентативными чертами которой являются динамическое нарушение киральной симметрии при сколь угодно малой константе связи Юкавы, а также наличие в системе критической точки и тройных точек.

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Препринт Объединенного института ядерных исследований. Дубна, 1995

Abramova S.V., Efimov G.V., Nedelko S.N. E2-95-72

Phase Structure of the Linear σ-Model in R^{1+1}

Phase structure of the two-dimensional linear σ-model is investigated within the method based on the canonical transformations and renormalization group formalism. The phase diagram in the (Y, G)-plane is constructed, where Y and G are the Yukawa and boson self-interaction coupling constants. The Hamiltonians describing the system in each phase are obtained. It is shown that the contribution of the pseudoscalar fields leads to describing of the vacuum energy density in the phase with broken chiral symmetry. This results is rather complicated phase structure of the linear σ-model. The most representative features of the phase picture are the dynamical breaking of the chiral symmetry for arbitrary small Yukawa coupling and presence of the critical nad triple points at the phase diagram.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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