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SU(2) GAUGE THEORY IN THE MAXIMALLY ABELIAN GAUGE WITHOUT MONOPOLES

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1 Introduction

In the last ten years there have been extensive studies of abelian monopole dynamics inspired by conjectured dual-superconductor confinement mechanism [1, 2]. The maximaly abelian (MA) gauge projection [3] was found having abelian dominance and monopole condensation [4] in $SU(2)$ gauge theory. It suggests the existence of an effective $U(1)$ theory of confinement.

But questions about role of Gribov's copies [5] and lattice artifacts are not clear yet. Calculation of monopole condensate like [6] and direct investigation of the model without monopoles are also interesting. For these reasons a simulation of $SU(2)$ gauge theory under the MA gauge constraints is desirable.

This is the purpose of this work to present an algorithm for simulation of $SU(2)$ gauge theory under the MA projection and first results of studying topology of the field manifold.

In the next section we briefly discuss the Faddeev-Popov operator for the MA projection of $SU(2)$ lattice gauge theory and a partial solution of the gauge constraints. We also give there a short description of a hybrid Monte Carlo algorithm of the simulation [7]. The numerical results and concluding remarks are presented in the last two sections.

2 $SU(2)$ under the MA gauge

The MA projection for a lattice gauge field configuration $\{\tilde{U}\}$ in $SU(2)$ theory is defined by [3]

$$\max_{\{g\}} R(\{\tilde{U}\},\{g\}) = \max_{\{g\}} \sum_{l=(i,\ j)} Tr[\sigma_3 U_l \sigma_3 U_l^+]$$

where $U_l = g_i \tilde{U}_l g_j^+$ are gauge transformed link variables and g_i, g_j are arbitrary $SU(2)$ matrixes of the gauge transformation at sites i and $j = i+\mu$. Written as a stationarity condition it reads

$$\left.\frac{\delta R}{\delta \tilde{g}_i}\right|_{\tilde{g}=1} = \sum_\mu Tr[\sigma_3(U_{i+\mu}\sigma_3 U_{i+\mu}^+ + U_{i-\mu}^+\sigma_3 U_{i-\mu})] = Tr(\sigma_3 X_i) = 0 \qquad (1)$$

or equivalently

$$X_i^\perp = 0$$

where $\tilde{g}_i$ is an $SU(2)$ matrix with $\tilde{g}_{3,i}$ taken to be zero. $X = \sigma_1 X_1 + \sigma_2 X_2 + \sigma_3 X_3$ is a traceless 2×2 antihermitian matrix and $X^\perp = \sigma_1 X_1 + \sigma_2 X_2$.

Partition function for the theory with gauge conditions (1) could be written in the form

$$Z = \int \delta(X^\perp)\ det(F)\ [d\mu(U)] \exp(-S_W\{U\}) \qquad (2)$$

where $d\mu(U)$ is the invariant integration measure, F is the Faddeev-Popov operator for the MA gauge and S_W is the Wilson action.

Follow Gribov's idea [8] stationarity eqs.(1) should be supplemented with the stability condition that the Faddeev-Popov operator be positively defined $F > 0$.

On the lattice $F = f_{ij} = -\frac{\delta^2 R}{\delta g_i \delta g_j}$ is a square symmetric matrix with nonzero diagonal elements ($i = j$) and elements for sites i and j connected by a link. On surface (1) f_{ij} reads

$$f_{ij} = \begin{cases} \begin{vmatrix} X_{3,i} & 0 \\ 0 & X_{3,i} \end{vmatrix} & \text{for } i = j \\ \begin{vmatrix} Tr(\sigma_2 U_{ij}\sigma_2 U_{ij}^+) & Tr(\sigma_2 U_{ij}\sigma_1 U_{ij}^+) \\ Tr(\sigma_1 U_{ij}\sigma_2 U_{ij}^+) & Tr(\sigma_1 U_{ij}\sigma_1 U_{ij}^+) \end{vmatrix} & \text{for } i \neq j \end{cases} \qquad (3)$$

where X_i is defined by (1) and for U_{ij} we suppose $U_{ij} = U_{ji}^+$. So F is a sparse real $2N \times 2N$-matrix where N is number of sites.

2.1 Partial solution of the MA gauge constraints

Having in mind a hybrid scheme [9] to guide a MC simulation of theory with partition function (2) we now define a set of independent, with respect to constraints (1), variables $\{q\}$.

Let a field configuration $C_{MA} = \{U\}$ be a solution of (1). We consider three link variables $S = \{U_{si}, U_{ij}, U_{jk}\}$ of the configuration C_{MA} such that links $L_S = \{(s,i),\ (i,j),\ (j,k)\}$ form a continual path ($3l$-path) on the lattice.

Equations (1), which include link variables of set S, read

$$\begin{aligned} (U_{si}^q\sigma_3 U_{si}^{q+})^\perp &= (U_{si}\sigma_3 U_{si}^+)^\perp \\ (U_{si}^{q+}\sigma_3 U_{si}^q + U_{ij}^q\sigma_3 U_{ij}^{q+})^\perp &= (U_{si}^+\sigma_3 U_{si} + U_{ij}\sigma_3 U_{ij}^+)^\perp \\ (U_{ij}^{q+}\sigma_3 U_{ij}^q + U_{jk}^q\sigma_3 U_{jk}^{q+})^\perp &= (U_{ij}^+\sigma_3 U_{ij} + U_{jk}\sigma_3 U_{jk}^+)^\perp \\ (U_{jk}^{q+}\sigma_3 U_{jk}^q)^\perp &= (U_{jk}^+\sigma_3 U_{jk})^\perp \end{aligned} \qquad (4)$$

where the terms independent on S are omitted. Substitution $U_{si}^q = U_{si} * \exp(-i\sigma_3 q)$ and $U_{jk}^v = \exp(i\sigma_3 v) * U_{jk}$ solves the first and last of equations (4). Two others are equivalent to 4 algebraic equations with 4 independent variables, say $u_{1,ij}, u_{2,ij}, u_{3,ij}$ and v, and allow to find a single solution U_{ij}^q and $v(q)$. So, for any $3l$-set S we can locally define a new set of variables

$$\vec{Q}_S = \{q,\ Q_1^\perp,\ Q_2^\perp,\ Q_3^\perp,\ Q_4^\perp\}_S = \{q,\ \vec{Q}^\perp\}_S \qquad (5)$$

where $Q_i^\perp$ is the right-hand side of the i-th equation (4). If the link variables apart from S are kept fixed, then the gauge-fixing conditions require $\vec{Q}^\perp$ to be constant, whereas q is a free parameter.

Choosing a set of $3l$-paths $\{S\}$ such that $S_i \cap S_j = \emptyset$ for $i \neq j$ we can locally define a set of independent variables $\{q\}$. It allows to make transition $C_{MA} \to \tilde{C}_{MA}$ where the both configurations lay on the surface defined by the gauge constraints (1).

2.2 Hybrid Monte Carlo method for SU(2) gauge theory under the MA gauge

By following the well known procedure we can eliminate the determinant in (2) and write the partition function in the form

$$Z = \int \delta(X^{\perp})\,[d\mu(U)d(\Phi)] \exp(-S_W\{U\} - \Phi^* F^{-1}\Phi) \qquad (6)$$

where Φ is an additional complex scalar field. It looks very similar to a partition function of gauge theory with pseudefermions except the δ-functions and form of the interaction matrix for the scalar field.

For numerical simulation of the system a Markov process should be constructed with the fixed-point distribution defined by (6), and the only requirements for the transition probability of the process are detailed balance and ergodicity.

The algorithm suggested in this paper includes two steps. First, we choose a set $\{S\}$ of N_{site} nonintersecting $3l$-paths on the lattice. The choice is made at random from a uniform distribution. Second, we generate a new field configuration by hybrid Monte Carlo method [9], so the detailed balance and the gauge-fixing conditions are satisfied. We also repeat the transition for different $\{S\}_i$ to gain ergodicity.

To specify the second step of the algorithm, let us take a configuration $C_{MA}\{U\}$ satisfying the constrains (1) and select a set $\{S\}$ of nonintersecting $3l$-paths on the lattice. Restricted to the link variables associated with $\{S\}$, the desired probability distribution reads

$$\left[\delta(\vec{Q}^{\perp} - \vec{Q}_0^{\perp})\left|\Delta^{-1}(\vec{Q},U)\right| d\vec{Q}d\Phi\right] \exp(-S_W\{U(q)\} - \Phi^* F^{-1}\Phi)$$

where $\vec{Q}_0 \equiv \vec{Q}\Big|_{C_{MA}}$ and Jacobian Δ is given by

$$\Delta(\vec{Q},U_S) = Tr(\sigma_3 U_{ij}\sigma_3 U_{ij}^{+}) Tr[\sigma_3 (U_{ij}\sigma_3 U_{ij}^{+})(U_{ik}\sigma_3 U_{ik}^{+})]$$

After integration over $\vec{Q}^{\perp}$ the probability distribution takes the form

$$[dqd\Phi]\exp(-S_W\{U(q)\} - \Phi^* F^{-1}\Phi - \sum_i \ln(\left|\Delta(\vec{Q},U)\right|)) = [dqd\Phi]\exp(-S_{eff}) \qquad (7)$$

where the sum in the exponent is taken over all $3l$-paths of set $\{S\}$.

We apply the hybrid Monte Carlo algorithm [9] with evolution in pseudotime defined by S_{eff} to generate a new MA configuration $\tilde{C}_{MA}$.

It is easy to see that Markov process converging to (7) in variables $\vec{Q}$ yields a correct simulation of system (6). Moreover, if step-size in the integration of the equations of motion is taken small, a quasicontinuous evolution in Langevin time keeps the configuration within the Gribov horizon [10].

3 Numerical simulation

The algorithm has been implemented for simulation of $SU(2)$ gauge theory on the 4^4 lattice. We started with a configuration very close to zero field because matrix F for the completely ordered configuration has zero eigenvalues. The usual relaxation algorithm has been employed for initial MA gauge fixing.

For the described in the previous section hybrid Monte Carlo algorithm, we chose duration $\delta\tau = 0.0005$ for Langevin time steps and 400 steps for every run. The choice was not optimized but yielded acceptance rate ~ 0.7. The generated configurations were separated by 12 Langevin runs with different (random) sets of 256 $3l$-paths. It approximately reproduces a real number of degrees of freedom of the system.

The first 500 configurations were skipped for thermolisation at β values 4.0, 3.0, 2.5, 2.3 and $\simeq 2000$ configurations at $\beta = 2.25$. The next 100 configurations were used to calculate average plaquette. Every configuration was searched for abelian monopoles [11] but no monopoles were found.

The results are summarized in Table 1. Here we also presented data on a one-loop calculations of the plaquettes [12] for 8^4 lattice and values of the plaquette calculated by the heatbath algorithm. From the collected data we could observe that the calculations under the MA gauge are in an excellent agreement with the one-loop perturbative results at all β, whereas the heatbath data essentially differ at $\beta \leq 2.5$.

Table 1

β	N_{mon}	$< Pl_{MA} >$	$Pl_{one-loop}$	$< Pl_{HB} >$
2.25	0	0.642(2)	0.637	0.593
2.3	0	0.647(3)	0.645	0.609
2.5	0	0.672(2)	0.676	0.655
3.0	0	0.729(3)	0.733	0.724
4.0	0	0.799(2)	0.803	0.800

We additionally tested the stability of the generated configurations with respect to a small random gauge transformation with a further gauge fixing by the relaxation algorithm. In all cases the configurations appear to be stable. The situation changes crucially if matrix F is replaced by $\tilde{F}$ with $\tilde{f}_{ii} = f_{ii}$ and $\tilde{f}_{ij} = 0$ for $i \neq j$. At $\beta = 2.25$ after ~ 200 sweeps starting from an almost ordered MA configuration we receive a nonstable one sometimes having abelian monopoles. So we come to the conclusion that it is the Faddev-Popov determinant that keeps us within the Gribov horizon.

4 Conclusion

We have presented an algorithm for the numerical simulation of $SU(2)$ gauge theory under the MA gauge. The first numerical results give evidence that Gribov region for the MA gauge projection is split into at least two separated sectors. The configurations laying within the locus of the first zeros of $det(F)$ have no abelian monopoles. Without monopoles the smallest Wilson loop - plaquette becomes very close to its one-loop perturbative value. It supports the idea that an effective theory could be built with nonperturbative interaction of the residual abelian field with monopoles and the other interactions are perturbative.

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