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PHOTOPRODUCTION OF NEUTRAL PION PAIRS IN THE COULOMB FIELD OF THE NUCLEUS

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The nucleus Coulomb interaction method is an effective approach to study in high-energy experiments the low-energy electromagnetic properties of pions in the process $\gamma\gamma \to \pi\pi$. There are two possible ways to realize the Coulomb interaction method in experiment: the radiative scattering $\pi A \to \pi\gamma A$ of a pion on a nucleus and the photoproduction of pion pairs in the Coulomb field of a nucleus $\gamma A \to \pi\pi A$. The experiments under discussion are based on the fact that for sufficiently small momentum transfers the interaction of high-energy particle with nuclei is extremely peripheral and thus dominated by scattering on the virtual photons of the Coulomb field of the nucleus. The reaction $\pi A \to \pi\gamma A$ allows to investigate the $\gamma\gamma \to \pi\pi$ process in the region $m_{\pi\pi}^2 = (p_{\pi 1} + p_{\pi 2})^2 < 0$ and to measure the charged pion polarizability [1] at the point $m_{\pi\pi}^2 = 0$, while the reaction $\gamma A \to \pi\pi A$ occurs in the region $m_{\pi\pi}^2 \geq 4m_\pi^2$ close to threshold. In the case of charged pions the two reactions give complementary information about the process $\gamma\gamma \to \pi^+\pi^-$ in both the physical and the nonphysical region. In the case of neutral pions the photoproduction of a neutral pion pair in the Coulomb field of a nucleus provides a new source of the information on the process $\gamma\gamma \to \pi^0\pi^0$, which was measured in the electron-positron experiment by the Crystal Ball Collaboration [2] and is under active discussion in the context of the physical programme at DAΦNE [3].

The present theoretical interest in the elementary process $\gamma\gamma \to \pi^0\pi^0$ is caused both by experimental data [2] mentioned above and recent progress in Chiral Perturbation Theory (ChPT) [4] up to and including $O(p^6)$. The process $\gamma\gamma \to \pi^0\pi^0$ is very sensitive to the higher-order contributions in ChPT since the first nonvanishing amplitude arises from meson loops at $O(p^4)$ without counterterms. However, the one-loop amplitude at $O(p^4)$ calculated in Ref. [5] does not describe the data even near threshold [6]. This is not surprising, as the analysis of the $\gamma\gamma \to \pi^0\pi^0$ amplitude based on dispersion relations [7] demonstrates the importance of unitarity corrections corresponding to higher orders, next to $O(p^4)$. In fact, the two-loop calculation at $O(p^6)$ carried out in Ref. [8] gives a considerable improvement of the description within ChPT. For completeness we mention that there is also another consideration of $\gamma\gamma \to \pi^0\pi^0$

up to one-loop order corresponding to $O(p^5)$ in the treatment of Generalized ChPT [9].

The contribution from the $O(p^6)$ counterterms was estimated in Ref.[8] [1] from the low-energy meson phenomenology with resonance exchange saturation. In Refs. [10, 11] the counterterms were fixed as effective meson lagrangians with higher-order derivative terms obtained from the bosonization of Nambu-Jona-Lasinio model (NJL). The sensitivity of $\gamma\gamma \to \pi^0\pi^0$ to higher-order corrections including Born contributions from the effective meson lagrangian at $O(p^6)$ makes this process a valuable source of the experimental information essential for the test of bosonized chiral lagrangians at $O(p^6)$.

In this paper we investigate the possibility of studying the process $\gamma\gamma \to \pi^0\pi^0$ near threshold in the photoproduction of neutral pion pairs in the Coulomb field of a nucleus. For the first time the reaction $\gamma A \to \pi\pi A$ was considered in this context in Ref. [12] with a one-loop amplitude of $\gamma\gamma \to \pi^0\pi^0$ at $O(p^4)$. The total cross sections $\gamma A \to \pi\pi A$ were estimated for energies of the incident photon at 20 GeV and 40 GeV for momentum transfer cutoffs 5 MeV and 10 MeV, respectively. However, due to the large incident energy and small cutoff, the nuclear absorption and the offshellness of the virtual photon from the nuclear Coulomb field were not taken into account.

In this note, we extend and improve the previous calculation [12] in a consistent way: we presents results of a calculation of the nuclear Coulomb photoproduction of neutral pion pairs, where we include in the elementary amplitude $\gamma\gamma^* \to \pi^0\pi^0$ both off-shell corrections for the Coulomb virtual photon γ^* and contributions from ChPT up to $O(p^6)$, and where, in addition, nuclear structure and absorption are taken into account in the form factor for the nucleus.

The photoproduction of a $\pi^0\pi^0$ pair in the Coulomb field of a nucleus is schematically described by the diagram in Fig. 1. The virtual photon $\gamma^*(q_2)$ for the interaction of the incident real photon with the stationary Coulomb field of the nucleus has zero energy and transfers only momentum: $q_2 = (0, \mathbf{q}_2)$. Then

[1] The same reference presents a diagrammatical expansion of the amplitude $\gamma\gamma \to \pi^0\pi^0$ up to $O(p^6)$.

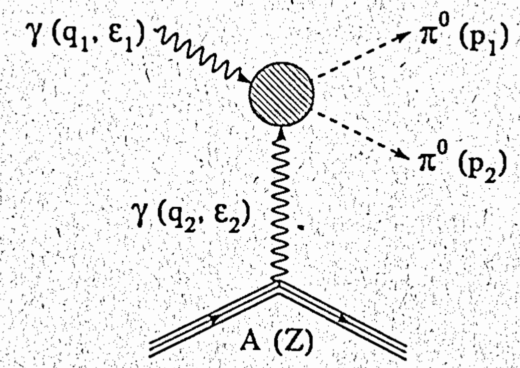

Fig. 1. Photoproduction of pion pairs in the Coulomb field of a nucleus.

the amplitude of the reaction $\gamma(q_1)A \to \pi^0(p_1)\pi^0(p_2)A$ has the form

$$T_C = 2M_A \frac{eZ_A}{|\mathbf{q}_2|^2} F_A(q_{2t}, q_{2l}) \epsilon^\mu T_{\mu 0}^{(\gamma\gamma^* \to \pi^0\pi^0)}, \quad (1)$$

where M_A and Z are, respectively, the mass and charge of the nucleus. In Eq. (1), the nuclear form factor $F_A(q_{2t}, q_{2l})$, which includes nuclear absorption, depends on transverse and longitudinal components of the momentum transfer $\mathbf{q}_2$ measured relative to the momentum $\mathbf{q}_1$ of the incident photon (F_A is normalized to $F_A(0,0) = 1$). $T_{\mu 0}^{(\gamma\gamma^* \to \pi^0\pi^0)}$ is the tensor component of the amplitude of the process $\gamma(q_1)\gamma^*(q_2) \to \pi^0(p_1)\pi^0(p_2)$.

From Lorentz and gauge invariances the general parameterization for the amplitude $T_{\mu 0}^{(\gamma\gamma^* \to \pi^0\pi^0)}$ at $O(p^6)$ has the form

$$\begin{aligned} T_{\mu\nu}^{(\gamma\gamma^* \to \pi^0\pi^0)} &= A(s,t,u;q_2^2)\Big(\frac{\tilde{s}}{2} g_{\mu\nu} - q_{2\mu} q_{1\nu}\Big) \\ &+ B(s,t,u;q_2^2)\big[2\tilde{s}\Delta_\mu\Delta_\nu - \nu^2 g_{\mu\nu} - 2\nu(\Delta_\mu q_{1\nu} - q_{2\mu}\Delta_\nu)\big] \\ &+ D(s,t,u;q_2^2)\Big[\nu q_{2\mu} q_{2\nu} + \tilde{s}\Delta_\mu q_{2\nu} - q_2^2(\nu g_{\mu\nu} + 2\Delta_\mu q_{1\nu})\Big], \quad (2) \end{aligned}$$

where $\Delta_\mu = (p_1 - p_2)_\mu$, $s = (q_1 + q_2)^2 = (p_1 + p_2)^2$, $\tilde{s} = s - q_2^2$, $t = (p_1 - q_1)^2 = (q_2 - p_2)^2$, $u = (p_2 - q_1)^2 = (q_2 - p_1)^2$ and $\nu = t - u$.

The differential cross section for the photoproduction of a neutral pion pair in the Coulomb field of a nucleus is defined as

$$d\sigma_C^{(\gamma A\to\pi\pi A)} = \frac{\delta^{(3)}(\mathbf{p}_1+\mathbf{p}_2-\mathbf{q}_1-\mathbf{q}_2)\delta(E_1+E_2-\varepsilon)}{4\varepsilon M_A(2\pi)^5 8E_1E_2M_A}\frac{1}{2}|T_C|^2 d^3\mathbf{p}_1\, d^3\mathbf{p}_2\, d^3\mathbf{q}_2$$

where E_i and $\mathbf{p}_i$ $(i=1,2)$ and ε are the energies and momenta of the pions and the energy of incident real photon, respectively. For large energies of the incident real photon and for small momentum transfers $|\mathbf{q}_2|$ to the recoil nucleus and neglecting the offshellness of the Coulomb photon and nuclear corrections, the method of equivalent photons [13] allows us to relate the differential cross section for photoproduction of pion pairs on nuclei to the total cross section for the process $\gamma\gamma\to\pi\pi$:

$$\frac{d\sigma_C^{(\gamma A\to\pi\pi A)}}{ds} = \frac{\alpha}{\pi}Z^2\log\left(\frac{\sqrt{s}}{2m_\pi}\right)\frac{1}{s}\sigma^{(\gamma\gamma\to\pi\pi)}(s)\,, \qquad (3)$$

where $s\equiv m^2_{\pi\pi}$. In this limit Eq. (3) enables us to extract model-independent information on the process $\gamma\gamma\to\pi\pi$ from the experimental data on the nuclear Coulomb photoproduction of pion pairs. For a more general kinematics, the nuclear form factor and the offshellness of the Coulomb photon have to be taken into account, however.

The nuclear form factor $F_A(q_{2t},q_{2l})$ in Eq. (1) can be estimated in the same approximation as in Ref. [14] since the amplitude of Coulomb photoproduction on a single proton at small q_{2t} is – upon evaluating Eq. (2) – proportional to q_{2t}:

$$\epsilon^\mu T_{\mu 0}^{(\gamma\gamma^*\to\pi^0\pi^0)} \approx (\epsilon\cdot q_{2t})\Big[A-2\Big(\tilde{s}(E_1-E_2)-\nu\varepsilon+2\nu(E_1-E_2)\Big)B-2\varepsilon D\Big].$$

In the approach of Ref. [14] the nucleus is treated as a completely absorbing sphere, with the form factor

$$\Gamma(q_t,q_l) = q_lRJ_0(q_tR)K_1(q_lR)+\frac{(q_lR)^2}{q_tR}J_1(q_tR)K_0(q_lR)+\Delta F_A(q_t,q_l)\,, \quad (4)$$

where R is the radius of the nucleus, and J_n and K_n $(n=0,1)$ are Bessel functions. In our estimates R is chosen to be $R=1.12A^{1/3}$ fm, where A is atomic weight of a nucleus. The first two terms in Eq. (4) arise from the integration over the three-dimensional space outside a cylinder of radius R of the nucleus. They reflect the assumption, that the nucleus is completely "black" for the outgoing pions for impact parameters $b\leq R$, resulting in a cut $\theta(b-R)$, in the profile function. This drastic assumption is mediated by the correction term $\Delta F_A(q_t,q_l)$, which arises from the integration over the three-dimensional cylinder behind the nucleus and corresponds to the interaction of photons with the nuclear Coulomb field after passing through the nucleus.

In general, the integrations above can be done only numerically. However, when both (q_lR) and (q_tR) are small compared to unity, the correction $\Delta F_A(q_t,q_l)$ can conveniently be expanded to obtain

$$\begin{aligned}\Delta F_A(q_t,q_l) &= \frac{1}{4}\Big\{1-q_lRJ_0(q_tR)K_1(q_lR)-\frac{(q_lR)^2}{q_tR}J_1(q_tR)K_0(q_lR)\\ &+[(q_lR)^2+(q_tR)^2]\Big(-\frac{1}{6}+\frac{i}{8}q_lR+\frac{1}{120}(q_tR)^2\\ &+\frac{4-i15\pi}{480}(q_lR)^2+i\frac{5}{144}(q_lR)^3-\frac{i}{96}q_lR(q_tR)^2\Big)+\ldots\Big\}.\end{aligned}$$

The calculation of the amplitude of $\gamma\gamma^*\to\pi^0\pi^0$ at $O(p^6)$ of the momentum expansion of ChPT involves tree-level, one-loop and two-loop diagrams of a chiral effective lagrangian. The effective meson lagrangian at $O(p^2)$ can be written in the nonlinear parameterization of chiral $SU(2)\times SU(2)$ symmetry as

$$\mathcal{L}^{(2)}_{eff} = \frac{F_0^2}{4}\mathrm{tr}\,(D_\mu U\overline{D}^\mu U^\dagger)+\frac{F_0^2}{4}\mathrm{tr}\,(\chi U^\dagger+U\chi^\dagger)\,, \qquad (5)$$

where

$$U(x)=\exp\left(\frac{i}{F_0}\varphi(x)\right),\quad \varphi=\begin{pmatrix}\pi^0 & \sqrt{2}\pi^+\\ \sqrt{2}\pi^- & -\pi^0\end{pmatrix} \qquad (6)$$

represents the pseudoscalar degrees of freedom, F_0 is the bare π decay constant, and $\chi=\mathrm{diag}\,(\chi_u^2,\chi_d^2)$ is the mass matrix. The covariant derivatives D_μ and $\overline{D}_\mu$ contain the vector and axial-vector degrees of freedom, and are defined as

$$D_\mu U=\partial_\mu U+(A^L_\mu U-UA^R_\mu)\,,\quad \overline{D}_\mu U^\dagger=\partial_\mu U^\dagger+(A^R_\mu U^\dagger-U^\dagger A^L_\mu)\,,$$

where $A^{R/L}_\mu=V_\mu\pm A_\mu$ are the right/left combinations of vector and axial–vector fields. The interaction with the electromagnetic field $\mathcal{A}_\mu$ can be included by replacing $V_\mu\to V_\mu+ie\mathcal{A}_\mu Q$, where Q is the matrix of quark electric charges.

The effective meson Lagrangian of $O(p^4)$ is presented in the general form with structure coefficients L_i and H_i introduced by Gasser and Leutwyler in Ref. [4],

$$\begin{aligned}
\mathcal{L}_{eff}^{(4)} &= \left(L_1-\frac{1}{2}L_2\right)(\mathrm{tr}L_\mu L^\mu)^2 + L_2\mathrm{tr}\left(\frac{1}{2}[L_\mu,L_\nu]^2+3(L_\mu L^\mu)^2\right)\\
&+L_3\mathrm{tr}((L_\mu L^\mu)^2) - L_4\mathrm{tr}(L_\mu L^\mu)\,\mathrm{tr}(\chi U^\dagger + U\chi^\dagger)\\
&-L_5\mathrm{tr}[L_\mu L^\mu(\chi U^\dagger + U\chi^\dagger)] + L_6\left(\mathrm{tr}(\chi U^\dagger + U\chi^\dagger)\right)^2\\
&+L_7\left(\mathrm{tr}(\chi U^\dagger - U\chi^\dagger)\right)^2 + L_8\mathrm{tr}(\chi U^\dagger\chi U^\dagger + U\chi^\dagger U\chi^\dagger)\\
&-L_9\mathrm{tr}\left(F^R_{\mu\nu}R^\mu R^\nu + F^L_{\mu\nu}L^\mu L^\nu\right) - L_{10}\mathrm{tr}\left(UF^R_{\mu\nu}U^\dagger F^{L\mu\nu}\right)\\
&-H_1\mathrm{tr}\left((F^R_{\mu\nu})^2+(F^L_{\mu\nu})^2\right) + H_2\mathrm{tr}(\chi\chi^\dagger)\,. \qquad (7)
\end{aligned}$$

In Eq. (7), $L_\mu = D_\mu U\cdot U^\dagger$, $R_\mu = U^\dagger D_\mu U$, and $F^{R/L}_{\mu\nu} = F^V_{\mu\nu} \pm F^A_{\mu\nu}$ are the right/left combinations of the field strength tensors

$$F^V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + [V_\mu, V_\nu] + [A_\mu, A_\nu]\,,\quad F^A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [V_\mu, A_\nu] + [A_\mu, V_\nu]\,.$$

It is convenient to present the part of the effective lagrangian at $O(p^6)$ for $\gamma\gamma\to\pi^0\pi^0$ with structure coefficients d_i,

$$\begin{aligned}
\mathcal{L}_6 &= \frac{8}{F_0^2}\Big[d_1\mathcal{F}_{\mu\alpha}\mathcal{F}^{\mu\beta}\mathrm{tr}\left(\partial^\alpha U_0\partial_\beta U_0^\dagger Q^2\right) + d_2\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\mathrm{tr}\left(\partial_\alpha U_0\partial^\alpha U_0^\dagger Q^2\right)\\
&+d_3\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\mathrm{tr}\left(\chi(U_0+U_0^\dagger)Q^2\right) + d_4\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\mathrm{tr}(Q^2)\mathrm{tr}\left(\chi(U_0+U_0^\dagger)\right)\\
&+d_5\mathcal{F}_{\mu\alpha}\mathcal{F}^{\mu\beta}\mathrm{tr}\left(Q^2\right)\mathrm{tr}\left(\partial^\alpha U_0\partial_\beta U_0^\dagger\right) + d_6\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\mathrm{tr}\left(Q^2\right)\mathrm{tr}\left(\partial_\alpha U_0\partial^\alpha U_0^\dagger\right)\\
&+d_7\mathcal{F}_{\mu\alpha}\mathcal{F}^{\mu\beta}\mathrm{tr}\left(\partial^\alpha U_0 U_0^\dagger Q\right)\mathrm{tr}\left(\partial_\beta U_0U_0^\dagger Q\right)\\
&+d_8\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\mathrm{tr}\left(\partial_\alpha U_0 U_0^\dagger Q\right)\mathrm{tr}\left(\partial^\alpha U_0U_0^\dagger Q\right)\Big]\,, \qquad (8)
\end{aligned}$$

where $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$ is the standard electromagnetic field strength tensor, and $U_0 = \exp(i\varphi_0/F_0)$, $\varphi_0 = \mathrm{diag}(\pi^0,-\pi^0)$. The lagrangian of Eq. (8) can be obtained from the most general representation of the full lagrangian of Ref.[17].

The structure coefficients of the $O(p^4)$ and $O(p^6)$ chiral lagrangians of Eqs. (7) and (8) can be fixed either from low-energy meson phenomenology, as in Refs. [4, 8], or from the bosonization of NJL-type effective quark models (see Refs. [15, 16, 10, 11] and references therein).

In general, the prediction for the Born amplitude of $\gamma\gamma^*\to\pi^0\pi^0$ at $O(p^6)$ involves eight structure coefficients d_i of the lagrangian of Eq. (8). In the NJL model only the structure constants d_1, d_2, d_3 get nonzero values and contribute to the amplitude:

$$\begin{aligned}
A^{B(p^6)} &= \frac{64e^2}{9F_0^4}\left[\frac{5}{16}d_1(s+q_2^2) + \frac{5}{2}d_2(s-2m_\pi^2) + d_3(4\chi_u^2+\chi_d^2)\right],\\
B^{B(p^6)} &= -\frac{10e^2}{9F_0^4}d_1\,.
\end{aligned}$$

The one-loop amplitude of $\gamma\gamma^*\to\pi^0\pi^0$ at $O(p^4)$, which has no UV divergences, is given as

$$\begin{aligned}
A_\pi^{1l(p^4)} &= -\frac{2e^2}{3F_0^2}\frac{1}{16\pi^2}(6s - 8m_\pi^2 + \chi_u^2 + \chi_d^2)\times\\
&\quad\left[-\frac{2q_2^2}{\tilde{s}^2}\left(J_\pi^1(s) - J_\pi^1(q_2^2)\right) + H_\pi(s,q_2^2)\right],\\
B_\pi^{1l(p^4)} &= D_\pi^{1l(p^4)} = 0\,,
\end{aligned}$$

with

$$H_\pi(s,q_2^2) = \frac{1}{\tilde{s}}\left[1 + \frac{2m_\pi^2}{\tilde{s}}\left(J_\pi^{-1}(s) - J_\pi^{-1}(q_2^2)\right)\right],$$

and

$$J_\pi^n(a) = \int_0^1 d^4x\, x^n \log\left[\frac{m_\pi^2 - a(1-x)x - i\epsilon}{m_\pi^2}\right].$$

In our approach UV divergences, resulting from meson loops at $O(p^6)$, are separated using the superpropagator regularization method [18], which is particularly well-suited for the treatment of loops in nonlinear chiral theories. The result is equivalent to the dimensional regularization technique used in Ref. [8], the difference being that the scale parameter μ is no longer arbitrary but fixed by the inherent scale $\tilde{\mu}$ of the chiral theory, namely, $\tilde{\mu} = 4\pi F_0$. In order to compare the two methods, the constants from the UV divergences in the dimensional regularization have to be replaced by a finite term using the substitution

$$(C-1/\varepsilon) \longrightarrow C_{SP} = 2C + 1 + \frac{1}{2}\left[\frac{d}{dz}\left(\log\Gamma^{-2}(2z+2)\right)\right]_{z=0} + \beta\pi = -1 + 4C + \beta\pi\,,$$

where $C = 0.577$ is Euler's constant, $\varepsilon = (4-D)/2$, and β is an arbitrary constant resulting from the representation of the superpropagator as an integral of the Sommerfeld-Watson type. The splitting of the decay constants F_π and F_K is used at $O(p^4)$ to fix $C_{SP} \approx 3.0$ for $F_0 = 90$ MeV.

The one-loop diagrams at $O(p^6)$ involve the structure coefficients L_i of the lagrangian of Eq. (7) and give the following contributions

$$
\begin{aligned}
A_\pi^{1l(p^6)} = & -\frac{e^2}{3F_0^4}\frac{1}{16\pi^2}\Big\{\Big[2L_9 q_2^2(6s-8m_\pi^2+\chi_u^2+\chi_d^2) \\
& +12L_2\Big(3s^2-8m_\pi^2(s-m_\pi^2)\Big)+24(2L_1-L_2+L_3)(s-2m_\pi^2)^2 \\
& +16L_4\Big(3(s-2m_\pi^2)(\chi_u^2+\chi_d^2)-(3s-4m_\pi^2)\mathrm{tr}\chi\Big) \\
& -16L_5 m_\pi^2(\chi_u^2+\chi_d^2)\Big]2\Big[-\frac{2q_2^2}{\tilde{s}^2}\Big(J_\pi^1(s)-J_\pi^1(q_2^2)\Big)+H_\pi(s,q_2^2)\Big] \\
& +4(L_9+L_{10})(6s-8m_\pi^2+\chi_u^2+\chi_d^2)G_\pi(s) \\
& +12L_2\Big[\frac{2}{3}J_\pi^1(s)\Big(s-2m_\pi^2-\frac{1}{4\tilde{s}}(3q_2^4-48q_2^2m_\pi^2+32m_\pi^4) \\
& \qquad -\frac{1}{2\tilde{s}^2}\Big(80q_2^2m_\pi^4-21q_2^4m_\pi^2+2q_2^6+6\nu^2(q_2^2+12m_\pi^2)\Big) \\
& \qquad -\frac{6}{\tilde{s}^3}\nu^2q_2^2(q_2^2+19m_\pi^2)-\frac{3}{\tilde{s}^4}\nu^2q_2^4(q_2^2+26m_\pi^2)\Big) \\
& \quad +\frac{2}{3}J_\pi^1(q_2^2)\Big(4m_\pi^4-q_2^4+\frac{q_2^2}{\tilde{s}}(q_2^2-10m_\pi^2) \\
& \qquad +\frac{1}{\tilde{s}^2}\Big(40q_2^2m_\pi^4-14q_2^4m_\pi^2+q_2^6+3\nu^2(q_2^2+4m_\pi^2)\Big) \\
& \qquad +\frac{6}{\tilde{s}^3}\nu^2q_2^2(q_2^2+14m_\pi^2)+\frac{3}{\tilde{s}^4}\nu^2q_2^4(q_2^2+26m_\pi^2)\Big) \\
& \quad -2H_\pi(s,q_2^2)\Big(\tilde{s}m_\pi^2-4m_\pi^4+q_2^2m_\pi^2-\nu^2-\frac{2}{\tilde{s}}\nu^2(m_\pi^2+2q_2^2) \\
& \qquad -\frac{3}{\tilde{s}^2}\nu^2q_2^2(m_\pi^2+q_2^2)\Big) \\
& \quad -\frac{1}{18}(s-4m_\pi^2)+\frac{1}{3\tilde{s}}(q_2^4-14q_2^2m_\pi^2-5\nu^2)-\frac{13}{2\tilde{s}^2}\nu^2q_2^2 \\
& \quad -\frac{5}{\tilde{s}^3}\nu^2q_2^4+\frac{1}{3}\widetilde{C}_\pi(\tilde{s}-4m_\pi^2)\Big]\Big\}, \\
B_\pi^{1l(p^6)} = & -\frac{4e^2}{F_0^4}\frac{1}{16\pi^2}L_2\Big\{\frac{1}{3}J_\pi^1(s)\Big[1-\frac{2}{\tilde{s}}(5m_\pi^2-q_2^2)-\frac{q_2^2}{\tilde{s}^2}(10m_\pi^2-q_2^2)\Big] \\
& +\frac{1}{3\tilde{s}}J_\pi^1(q_2^2)\Big[2(4m_\pi^2-q_2^2)+\frac{q_2^2}{\tilde{s}}(10m_\pi^2-q_2^2)\Big]
\end{aligned}
$$

$$
\begin{aligned}
& +m_\pi^2H_\pi(s,q_2^2)-\frac{q_2^2}{6\tilde{s}}-\frac{7}{36}+\frac{1}{6}\widetilde{C}_\pi\Big\}, \\
D_\pi^{1l(p^6)} = & \frac{8e^2}{3F_0^4}\frac{1}{16\pi^2}L_2\frac{\nu}{\tilde{s}^3}\Big\{q_2^2J_\pi^1(s)\Big[\tilde{s}^2+2\tilde{s}(13m_\pi^2+q_2^2)+(26m_\pi^2+q_2^2)\Big] \\
& +J_\pi^1(q_2^2)\Big[\tilde{s}^2(4m_\pi^2-q_2^2)-2\tilde{s}q_2^2(8m_\pi^2+q_2^2)-q_2^4(26m_\pi^2+q_2^2)\Big] \\
& -3q_2^2H_\pi(s,q_2^2)\tilde{s}^2(s+m_\pi^2)+\frac{1}{4}q_2^2\tilde{s}(9s+q_2^2)\Big\},
\end{aligned}
$$

where $G_\pi(s) = \widetilde{C}_\pi + 2J_\pi^1(s)$ and $\widetilde{C}_\pi = C_{SP} + \log\frac{m_\pi^2}{16\pi^2F_0^2}$.

Only two-loop diagrams which are factorizable can be calculated analytically. Their contribution can be presented in the form

$$
\begin{aligned}
A_\pi^{2l(p^6)} = & -\frac{2e^2}{3F_0^4}\frac{1}{(16\pi^2)^2}\Big\{J_\pi^1(q_2^2)\Big(\frac{q_2^2}{\tilde{s}^2}J_\pi^1(q_2^2)+\frac{1}{2}H_\pi(s,q_2^2)\Big)\frac{1}{\tilde{s}^3}\Big[-\frac{32}{3}m_\pi^4\tilde{s}^3 \\
& \quad +2s^4(4m_\pi^2-q_2^2)-\frac{2}{3}s^3q_2^2(32m_\pi^2-9q_2^2)+2s^2q_2^4(8m_\pi^2-3q_2^2) \\
& \quad -2sq_2^4(32m_\pi^4-q_2^4)-\frac{8}{3}m_\pi^2q_2^8+\frac{1}{3}\tilde{s}^3(4m_\pi^2-q_2^2)(\chi_u^2+\chi_d^2)\Big] \\
& -\frac{1}{3\tilde{s}^5}J_\pi^1(q_2^2)\Big[6s^5(4m_\pi^2-q_2^2)-s^4(20m_\pi^4+15m_\pi^2q_2^2-29q_4^4) \\
& \quad -16m_\pi^2\tilde{s}^4-\frac{2}{3}s^3q_2^2(16m_\pi^4+166m_\pi^2q_2^2+9q_2^4) \\
& \quad +2s^2q_2^4(60m_\pi^4+91m_\pi^2q_2^2+15q_2^4) \\
& \quad -2sq_2^6(176m_\pi^4+46m_\pi^2q_2^2+q_2^4)+\frac{1}{3}m_\pi^2q_2^8(276m_\pi^2+35q_2^2) \\
& \quad +\frac{1}{6}\Big(3(\tilde{s}^4+s^4)(4m_\pi^2-q_2^2)-3s^3q_2^2(36m_\pi^2-7q_2^2) \\
& \qquad -24s^2q_2^4(6m_\pi^2-q_2^2)+6sq_2^6(20m_\pi^2-3q_2^2) \\
& \qquad +q_2^8(36m_\pi^2-5q_2^2)\Big)(\chi_u^2+\chi_d^2)\Big] \\
& +J_\pi^1(s)\Big(\frac{q_2^2}{\tilde{s}^2}J_\pi^1(s)-\frac{1}{2}H_\pi(s,q_2^2)\Big)\frac{1}{\tilde{s}^3}\Big[\frac{64}{3}m_\pi^4\tilde{s}^3+12s^5 \\
& \quad -4s^4(8m_\pi^2+9q_2^2)+\frac{4}{3}s^3q_2^2(32m_\pi^2+9q_2^2)-12s^2q_2^4(8m_\pi^2+q_2^2) \\
& \quad +32sm_\pi^4q_2^6+\frac{16}{3}\tilde{s}^3(3s-4m_\pi^2)(\chi_u^2+\chi_d^2)+\frac{7}{3}\tilde{s}^3(\chi_u^2+\chi_d^2)^2\Big] \\
& +\frac{1}{3\tilde{s}^5}J_\pi^1(s)\Big[6s^5m_\pi^2-2s^4(20m_\pi^4+88m_\pi^2q_2^2+19q_2^4)+32m_\pi^4\tilde{s}^4 \\
& \quad +2s^3q_2^2\Big(136m_\pi^4-\frac{502}{3}m_\pi^2q_2^2-5q_2^4\Big)
\end{aligned}
$$

$$
\begin{aligned}
&-6s^2 q_2^4\Big(96m_\pi^4 + \frac{124}{3}m_\pi^2 q_2^2 + q_2^4\Big)\\
&+2sq_2^6(248m_\pi^4 + 43m_\pi^2 q_2^2 + q_2^4) - \frac{8}{3}m_\pi^2 q_2^8(57m_\pi^2+q_2^2)\\
&+\frac{1}{3}\Big(-72s^5 + 36s^4(3m_\pi^2+8q_2^2) + 24\tilde{s}^4(3s-4m_\pi^2)\\
&\quad -s^3q_2^2(420m_\pi^2+433q_2^2) + 3s^2q_2^4(204m_\pi^2+97q_2^2)\\
&\quad -3sq_2^6(132m_\pi^2+25q_2^2) + q_2^8(96m_\pi^2+q_2^2)\Big)(\chi_u^2+\chi_d^2)\Big]\\
+&\frac{q_2^2}{\tilde{s}^5}J_\pi^1(q_2^2)J_\pi^1(s)\Big[-12s^5 + 2s^4(12m_\pi^2+19q_2^2) - \frac{32}{3}m_\pi^4\tilde{s}^3\\
&-\frac{14}{3}s^3q_2^2(16m_\pi^2+3q_2^2) + 2s^2q_2^4(40m_\pi^2+9q_2^2)\\
&-2sq_2^6(16m_\pi^2+q_2^2) + \frac{8}{3}m_\pi^2 q_2^8 + \frac{1}{3}\Big(20m_\pi^2\tilde{s}^3 - 48s^4 + 145s^3q_2^2\\
&-147s^2q_2^4 + 51sq_2^6 - q_2^8\Big)(\chi_u^2+\chi_d^2) - \frac{7}{3}\tilde{s}^3(\chi_u^2+\chi_d^2)^2\Big]\\
-&\frac{1}{2\tilde{s}^3}H_\pi(s,q_2^2)\Big[-\frac{2}{3}s^4(36m_\pi^2+q_2^2) + \frac{2}{9}s^3q_2^2(328m_\pi^2+9q_2^2)\\
&+\frac{112}{3}m_\pi^4\tilde{s}^3 - 2s^2q_2^4(38m_\pi^2+q_2^2) + \frac{2}{3}sq_2^6(40m_\pi^2+q_2^2)\\
&-\frac{8}{9}m_\pi^2q_2^8 + \frac{1}{9}\tilde{s}^3(12m_\pi^2-q_2^2)(\chi_u^2+\chi_d^2)\Big]\\
-&\frac{1}{9\tilde{s}^5}\Big(s^4q_2^2(109m_\pi^2+3q_2^2) - 48m_\pi^4\tilde{s}^4 + 2m_\pi^2 s^3q_2^4 - 4m_\pi^2 sq_2^8 - 3m_\pi^2q_2^{10}\Big)\\
+&\widetilde{C}_\pi\Big(\frac{q_2^2}{\tilde{s}^2}(J_\pi^1(s) - J_\pi^1(q_2^2)) - \frac{1}{2}H_\pi(s,q_2^2)\Big)\frac{1}{\tilde{s}^3}\Big[6s^5 + s^4(8m_\pi^2-17q_2^2)\\
&-\frac{1}{3}s^3q_2^2(76m_\pi^2-45q_2^2) - \frac{80}{3}m_\pi^4\tilde{s}^3 + s^2q_2^4(28m_\pi^2-q_2^2)\\
&-sq_2^6(12m_\pi^2+q_2^2) + \frac{4}{3}m_\pi^2q_2^8 - \frac{1}{6}(72m_\pi^2\tilde{s}^3 - 48s^4 + 143s^3q_2^2\\
&-141s^2q_2^4 + 45sq_2^6 + q_2^8)(\chi_u^2+\chi_d^2) + \frac{7}{6}\tilde{s}^3(\chi_u^2+\chi_d^2)^2\Big]\\
+&\frac{1}{\tilde{s}^5}\widetilde{C}_\pi\Big[s^5m_\pi^4 - \frac{23}{6}s^4m_\pi^2q_2^2 + 4m_\pi^4\tilde{s}^4 + \frac{1}{3}s^3q_2^4(3m_\pi^2+16q_2^2)\\
&-s^2q_2^6(20m_\pi^2+q_2^2) + \frac{1}{3}sq_2^8m_\pi^2 + \frac{1}{6}m_\pi^2q_2^{10}\\
&+\frac{2}{3}m_\pi^2\tilde{s}^4(\chi_u^2+\chi_d^2)\Big]\Big\},
\end{aligned}
$$

$$B_\pi^{2l(p^6)} = D_\pi^{2l(p^6)} = 0\,.$$

It should be stressed that additional two-loop diagrams, such as box diagrams and acnode graphs, which cannot be evaluated analytically, can be neglected: the numerical estimates in Ref. [8] indicate the smallness of their contributions in the photoproduction process under consideration.

Monte Carlo techniques were used to compute the total cross sections for the photoproduction of $\pi^0\pi^0$-pairs in the Coulomb field of the carbon ($Z = 6$) and silicon nuclei ($Z = 14$). It is important to note that for a momentum transfer cutoff $q_{max} \equiv |\mathbf{q}_2|_{max} \ll \varepsilon$, the effective mass of $\pi\pi$ system varies in the range $4m_\pi^2 \le m_{\pi\pi}^2 \le 2\varepsilon q_{max}$.

The dependence of the total cross section of the reaction $\gamma A \to \pi^0\pi^0 A$ on the momentum transfer cutoff q_{max} for different energies ε of the incident real photon is shown in Figs. 2 and 3. (In the calculation, the additional cutoff $m_{\pi\pi} < 700$ MeV was used, corresponding to the range of validity of the chiral theory.) In Fig. 2 we demonstrate explicitly the influence of the nuclear form factor: our numerical estimates indicate that the nuclear effects are mainly saturated by the contribution of the two first terms of Eq. (4), while the correction ΔF_A does not exceed 10%. For our numerical estimates we used for the parameters L_i and d_i the values from Eq. (14) and Table 1 of Ref. [11] obtained from the NJL model. These values include the resonance-exchange contributions effectively taken into account by integrating out the vector, axial-vector and scalar degrees of freedom. Moreover, for numerical comparison with the phenomenological approach to $\gamma\gamma \to \pi^0\pi^0$, we used the structure constants L_i and d_i corresponding to Tables 1 and 2 of Ref. [8]. The results of our calculations with the parameters of Ref. [8] are also shown in Fig.2.

The main background for the Coulomb photoproduction of $\pi^0\pi^0$ pairs is the photoproduction of pion pairs following π, ρ and ω exchange from the nucleus. The π and ρ meson exchange can be suppressed by choosing an isoscalar target. An estimate of the ω meson exchange was obtained in Ref. [12]. It was shown that even without nuclear absorption, the ω exchange contribution is strongly suppressed in the energy and momentum transfer regions under discussion in comparison with Coulomb photoproduction. In addition, absorption on the

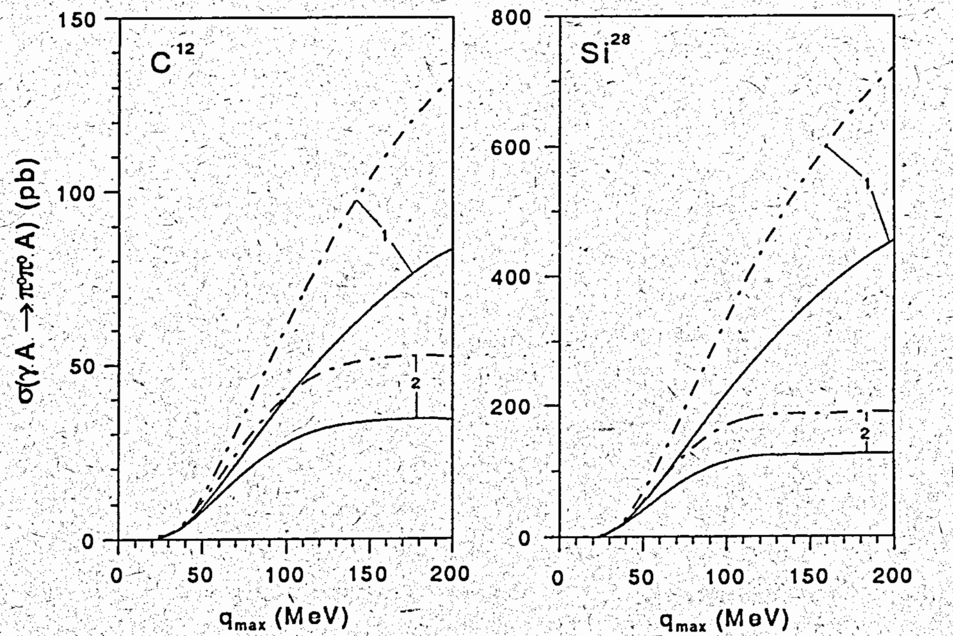

Fig. 2. Dependence of the $\gamma A \to \pi^0\pi^0 A$ cross section on the momentum transfer cutoff q_{max} for the energy $\varepsilon = 4$ GeV of the incident photon. Compared are: L_i, d_i from the bosonization of the NJL model (Ref. [11], full line) and from phenomenology (Ref. [8], dashed line), without and with a nuclear form factor (indicated by (1) and (2)).

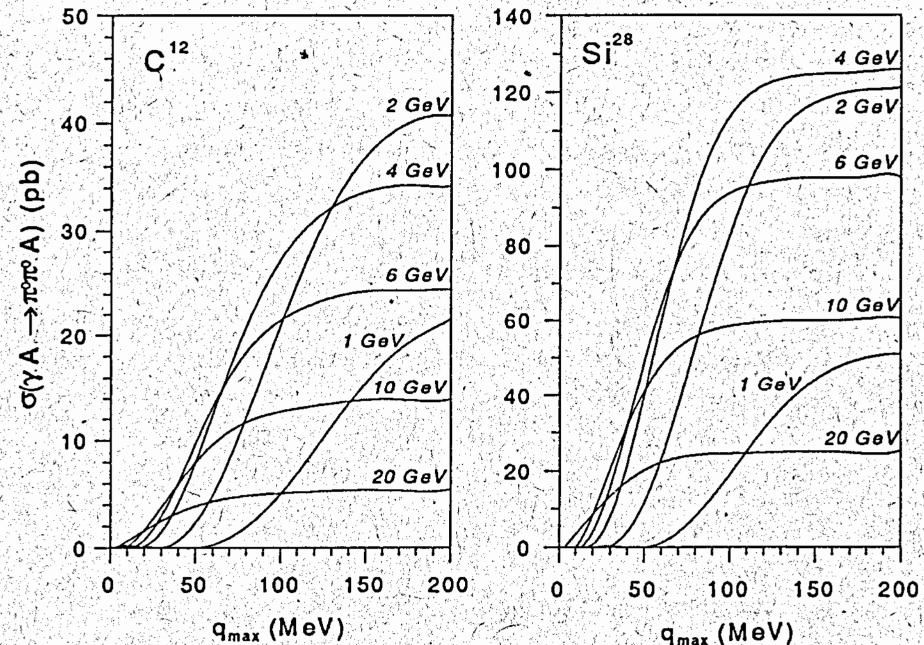

Fig. 3. Dependence of the $\gamma A \to \pi^0\pi^0 A$ cross section on the momentum transfer cutoff q_{max} for different photon energies, with parameters L_i and d_i from the NJL model (Ref. [11]).

nucleus should suppress the ω exchange mechanism even much stronger than for coherent Coulomb production.

Summarizing, we find that for energies of the incident photon $\varepsilon \leq 4$ GeV and for momentum transfers $|\mathbf{q}_2| \leq 200$ MeV, $\pi^0\pi^0$ pairs with $2m_\pi \leq m_{\pi\pi} \leq$ 700MeV in the reaction $\gamma A \to \pi^0\pi^0 A$ are produced with a cross section of typically $\sigma \approx 40$ pb and $\sigma \approx 120$ pb for carbon and silicon nuclei, respectively. Our results demonstrate that this reaction can be experimentally investigated with presently available photon beams as a new source of the low-energy data on the process $\gamma\gamma \to \pi^0\pi^0$.

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