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MULTI-INSTANTON EFFECTS IN QCD SUM RULES FOR THE PION*

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The QCD sum rule approach [1] allows the investigation of hadron properties in a systematic manner. It provides, in particular, the possibility to describe static characteristics of particles, such as masses, decay constants, form factors, etc., in an energy region where perturbative methods are not applicable [2, 3, 4, 5, 6]. Within this approach, the effects of large distances are effectively parametrized in terms of local matrix elements of quark-gluon operators averaged over the physical vacuum (vacuum condensates), quantities which are independent of hadron properties. On the other hand, short-distance physics is contained in the Wilson coefficients of the Operator Product Expansion (OPE) entering the calculation of correlators.

However, the nonperturbative matrix elements contain contributions that are not taken into account in the OPE. These are, so-called, "direct" small-size instantons (see, e.g., [7]) which give essential nonlocal contributions to current correlators in the channels where they are allowed by quantum numbers. These contributions are not sufficiently accounted for in the *local* condensates, since the latter correspond to vacuum fluctuations with infinite correlation length. They should rather be taken into account within the Wilson coefficients along with the factors calculated by perturbative methods.

The instanton liquid model of the QCD vacuum, originally suggested in [8, 9], has later been further generalized by an analytic approach, based on the Feynman variational principle [10, 11, 12]. (For lattice calculations using this vacuum model, see, e.g., [13].)

As it was shown in [12], the instanton-induced vacuum fluctuations are responsible for the spontaneous breaking of chiral invariance. This chiral-symmetry-breaking mechanism is based on the idea of mixing and delocalization of fermion zero modes in the field of the instanton (I) and anti-instanton (A) pairs. The QCD vacuum is modeled as an $I-A$ diluted liquid, characterized by a small ratio $\rho_c/R \simeq 1/3$, where $\rho_c \simeq 1/600$ MeV$\simeq 1/3$ Fm is the average instanton size in the vacuum, and R is the average distance between pseudoparticles.

A summary of some successful applications of this approach includes: The calculation of current correlators in the background of I and A external fields which provides a useful procedure for extracting the static features of the pseudoscalar meson octet [8, 14]. More recently [15], a possible mechanism for the bound-state formation in the vector-meson channel has been proposed. In a series of works [16], several main properties of hadron spectroscopy have been quantitatively determined. Evidence was provided there that large spin-flip high-energy amplitudes [17] are the result of the spin-dependent interaction between quarks, induced by the small-size vacuum fluctuations.

The role of direct instantons in stabilizing the QCD sum rules for the nucleon [2, 3, 4] was first discussed in [18] and later also in [19]. These analyses show that the inclusion of the instanton contributions amount to a significant enlargement of the stability region of the Borel parameter.

The instanton contribution to different vacuum matrix elements is defined basically by the quark zero modes in external I, A fields. Due to the specific chiral and flavor properties of these fields, instanton effects depend strongly on the channel under consideration. In the channel with the quantum number 0^-, the instanton contribution is dominant [7]. The single instanton contribution to the QCD sum rule for the pion, within the effective approach given in [9, 20], has been first calculated in [9]. There was shown that a self-consistent description of the pion as a pseudo-Goldstone mode is possible only if the contribution of direct instantons is taken into account.

It is the purpose of this paper to investigate the multi-instanton contributions to QCD sum rules for the pion in the framework proposed in [12]. The main conclusion of this investigation is that the large-distance behavior of the pion correlator in the singular gauge is essentially the same as in the effective single instanton approach [9]. The behavior of the correlator in the regular gauge is also explored but found to give a negligible contribution at large distances.

The QCD sum rules for the pion are evaluated from the correlator function

$$\Pi(q) = i\int d^4x\, e^{iqx}\langle 0|T(j(x)j^+(0)|0\rangle\,, \tag{1}$$

which is considered at $Q^2 = -q^2 \simeq 1$ GeV. We will analyze the sum rules for a charged pion, so that

$$j(x) = q_u q_{\bar d}\,[\bar v_R i\gamma_5 u_L + \bar v_L i\gamma_5 u_R]\,(x)\,. \tag{2}$$

Here q_i denotes quark annihilation operators, and $u_{L(R)} = (\frac{1\pm\gamma_5}{2})$ are left-(right-) handed spinors.

The single instanton contribution has been computed in [9], assuming that the quark Green function in the background of the instanton field

$$S_I(x,y) = S_0(x,y) + S_\pm(x,y) \tag{3}$$

can be approximated by the expression

$$S_\pm(x,y) = \langle q^a_\alpha(x)\bar q^b_\beta(y)\rangle = \int d^4z\,\frac{\left[\Psi^0_z(x)\bar\Psi^y_z(0)\right]^{ab}_{\alpha\beta}}{m^*} \tag{4}$$

which retains only the zero modes, given in singular gauge by

$$\Psi_\pm(x) = \Phi(x-z)\frac{1\pm\gamma_5}{2}(\not x - \not z)U \tag{5}$$

with

$$\Gamma_5(q) = -\int \frac{d^4k}{(2\pi)^4} \frac{\sqrt{M(k)M(k+q)}\,(k^2+(k+q)^2-q^2)}{[M^2(k)+k^2][M^2(k+q)+(k+q)^2]}$$
$$\underset{Q^2\simeq 1\ \mathrm{GeV}^2}{\simeq} -\frac{\sqrt{M(0)M(Q)}\,\pi^2}{(2\pi)^4\, 2Q^2\rho^4} \tag{14}$$

where $M_{1(2)}$ is a short-hand notation for $M(k \mp \frac{q}{2})$. Assuming fixed values of the instanton radii, $\rho = \rho_c$, it follows $M(p) \sim p^2\varphi^2(p)$, $\varphi(p)$, being associated with the zero mode representation in momentum space. It has the following asymptotics

$$\varphi_s(p) = \begin{cases} -\frac{2\pi\rho}{|p|}, & \rho p \ll 1 \\ -\frac{12\pi}{p^4\rho^2}, & \rho p \gg 1 \end{cases} \tag{15}$$

$$\varphi_r(p) = \begin{cases} \frac{4\pi\rho}{|p|}, & \rho p \ll 1 \\ -\frac{4\pi\rho}{p}\,e^{-p\rho}, & \rho p \gg 1 \end{cases} \tag{16}$$

in singular and regular gauges, respectively. Since $M(p)$ is a rapidly increasing function with p for $\rho p \gg 1$ [11, 24], we cut off the k-integration in (13) and (14) at values $\sim 1/\rho^2$.

Then by using Eq. (4) in conjunction with Eqs. (12)-(14), we obtain

$$\Pi(q) = \frac{N_c}{Q^4}\,\frac{M(Q)}{M(0)}\,\frac{1}{2^6\rho^6\pi^2}, \tag{17}$$

and utilizing the explicit expressions for $\varphi(p)$, given in [11, 23], viz.

$$\varphi_s(p) = \pi\rho^2 \frac{d}{dz}\left[I_0(z)K_0(z) - I_1(z)K_1(z)\right]_{z=\frac{\rho p}{2}},$$
$$\varphi_r(p) = \frac{4\pi}{p}\,e^{-p\rho}, \tag{18}$$

we find the corresponding results for the correlators

$$\Pi(q)^{sing} = \frac{N_c}{Q^2\rho^4 2^6\pi^2}\left[I_1(z)K_0(z) - I_0(z)K_1(z) + \frac{I_1(z)K_1(z)}{z}\right]^2_{z=\frac{Q\rho}{2}}$$
$$\Pi(q)^{reg} = \frac{N_c}{Q^4\rho^6 2^6\pi^2}\,e^{-2Q\rho}. \tag{19}$$

Using now the integral representations

$$K_\nu(z) = \frac{(\frac{z}{2})^\nu\,\Gamma(1/2)}{\Gamma(\nu+1/2)}\int_1^\infty e^{-zt}\,(t^2-1)^{\nu-1/2}dt, \tag{20}$$

$$I_\nu(z) = \frac{(\frac{z}{2})^\nu}{\Gamma(\nu+1/2)\Gamma(1/2)}\int_0^\pi e^{\pm z\cos\theta}\,\sin^{2\nu}(\theta)\,d\theta, \tag{21}$$

and the following Borel transforms, in accordance with Eq. (8),

$$B[e^{-a\sqrt{s}}] = \frac{a}{\sqrt{4\pi\tau^3}}\,e^{-\frac{a^2}{4\tau^2}}$$
$$B\left[\frac{1}{s^2}e^{-a\sqrt{s}}\right] = \left(\tau^2+\frac{a^2}{2}\right)\left[1-\mathrm{erf}\left(\frac{a}{2\tau}\right)\right] - \frac{\tau a}{\sqrt{\pi}}\,e^{-\frac{a^2}{4\tau^2}}, \tag{22}$$

it follows

$$\Pi^{sing}(\tau) = \frac{N_c}{8(2\pi)^4\sqrt{\pi}\,z\tau^4}\,I, \tag{23}$$

where

$$I = \int_0^\pi d\theta_1\int_0^\pi d\theta_2\int_1^\infty dt_1\int_1^\infty dt_2\, C\,t\,e^{-\frac{z^2t^2}{16}}; \tag{24}$$

with $t = \cos\theta_1 + \cos\theta_2 + t_1 + t_2$ and

$$C = \left(\frac{1}{8} - \frac{1}{4}\sin^2\theta_1 + 2\sin^2\theta_1\sin^2\theta_2\right)\sqrt{t_1^2-1}\sqrt{t_2^2-1}$$
$$- \frac{1}{4}\sin^2\theta_1\cos^2\theta_2\frac{\sqrt{t_1^2-1}}{\sqrt{t_2^2-1}} + \frac{1}{8}\sin^2\theta_1\sin^2\theta_2\frac{1}{\sqrt{t_1^2-1}\sqrt{t_1^2-1}}. \tag{25}$$

The values of the integral I are tabulated below.

z	1	1.5	2	2.5	3
I	279	26.4	4.4	1	0.29

The analogous result to Eq. (23) in regular gauge is

$$\Pi^{reg}(\tau) = \frac{N_c}{16(2\pi)^2 z^6\tau^4}\,f(z) \tag{26}$$

with

$$f(z) = (1+2z^2)\,[1-\mathrm{erf}(z)] - \frac{2z}{\sqrt{\pi}}\,e^{-z^2}, \tag{27}$$

where

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-y^2}dy. \tag{28}$$

Comparing these results with the effective single-instanton contribution, given by Eq. (11), at $\tau = \rho$, we deduce

$$\Pi^{sing}_{mult.}(\tau) \approx \frac{7}{11}\,\Pi^{sing}_{eff.}(\tau). \tag{29}$$

[19] H. Forkel and M. K. Banerjee, Phys. Rev. Lett. **71**, 484 (1993).

[20] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B163**, 45 (1980).

[21] A. E. Dorokhov, S. V. Esaibegyan, and N. I. Kochelev, submitted to Yad. Fiz..

[22] E. V. Shuryak, Rev. Mod. Phys. **61**, 1 (1993).

[23] M. Hutter, München preprint # LMU-95-01 (1995).

[24] M. Hutter, München preprint # LMU-95-03 (1995).

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Многоинстантонные эффекты в КХД правилах сумм для пиона

В модели — вакуум КХД-инстантонная жидкость — исследованы многоинстантонные вклады в КХД правила сумм для пиона. Показано, что в сингулярной калибровке сумма планарных диаграмм в лидирующем порядке по $1/N_c$-разложению приводит к вкладу, близкому (на больших расстояниях) к результату, найденному в эффективном одноинстантонном приближении. Анализ проведен также в регулярной калибровке, которая приводит к исчезающе малому вкладу, что указывает на предпочтительность использования сингулярной калибровки в описании поведения корреляционной функции на больших расстояниях.

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Multi-Instanton Effects in QCD Sum Rules for the Pion

Multi-instanton contributions to QCD sum rules for the pion are investigated within a framework which models the QCD vacuum as an instanton liquid. It is shown that in singular gauge the sum of planar diagrams in leading order of the $1/N_c$ expansion provides similar results as the effective single-instanton contribution. These effects are also analyzed in regular gauge. Our findings confirm that at large distances the correlator functions are more adequately described in the singular gauge rather than in the regular one.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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