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HADRONIC COMPONENT OF THE PHOTON SPIN DEPENDENT STRUCTURE FUNCTION g_1^γ FROM QCD

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Since 1977 [1] the deep inelastic electron scattering from a photon target has become a new subject of the intensive theoretical studies in the framework of QCD[1]. Much progress has been achieved in this direction [3]. As was firstly advocated by Witten the non-polarized structure function $F_2(x)$ can be calculated using the perturbative QCD alone. At that time there was considerable optimism that this process was an excellent test for the perturbative QCD and might provide an accurate measurement of α_s. By now the optimism has waned considerably. This happens because Witten's suggestion is true only for the asymptotically large probe-photon momentum transferred squared where a "contact"-type term due to the photon operators in the framework of OPE turns out to be dominant. At smaller values of Q^2 the hadronic component become sizable, the photon admits considerable contribution that comes from the nonperturbative region. Due to this contamination the hope of pure extraction of Λ_{QCD} from this reaction in real experiment fails owing to the large uncertainty in the theoretical prediction for this part. Till recently the only estimations for the latter have been obtained from the Vector Meson Dominance Models [3].

The first QCD based calculation of the hadronic part has been initiated by Balitsky [4]. However, only a few first moments rather than x-dependence of the structure function were found in his paper. Recently a new approach to calculation of the photon structure function in QCD has been developed [5]. It enables one to evaluate the structure function in the region of intermediate x and was successfully applied to the case of spin averaged scattering.

In recent times the polarized photon structure functions have attracted a lot of attention. In ref. [6] OPE and the renomalization group analysis was extended to the polarized sector, while refs. [7] deal with the first moment of the spin dependent structure function $g_1^\gamma(x)$ and its sensitivity to the chiral symmetry realization. In these papers as in the Witten's one the hadronic component was completely disregarded.

In the present investigation we concentrate on the calculation of the polarized structure function $g_1^\gamma(x)$ following the method mentioned above. We shall start with the consideration of the structure function when the target photon virtuality is large and spacelike but $-p^2 \ll Q^2$ and apply the OPE to the discontinuity of the forward photon-photon scattering amplitude, which results in the expansion in the inverse powers of target-photon virtuality p^2. To obtain the correct analytical properties in the photon off-shellness, we adopt a certain model for the structure function and the hadronic spectrum. Comparing the two representations of the same quantity we can fix unambiguously all unknown parameters.

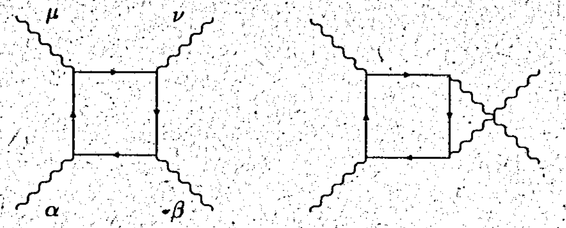
The sensitivity to the QCD radiative corrections is poor until very large Q^2 is attained much larger than is now available. Thus, we restrict ourselves to the lowest order graphs and to the consideration of light quarks only.

To start with we consider the four-point correlation function

$$T_{\mu\nu,\alpha\beta} = 4\pi\alpha_{em} i^3 \int d^4x d^4y d^4z e^{iqx+ip(y-z)} \langle 0|T\{j_\mu(x) j_\nu(0) j_\alpha(y) j_\beta(z)\}|0\rangle, \qquad (1)$$

where $j_\mu = \sum_q Q_q \bar{\psi}_q \gamma_\mu \psi_q$ is the electromagnetic quark current. This amplitude is originated from the T-product of two electromagnetic currents between the photon states and

[1] For the first discussion of the photon structure functions in the context of the gauge theories see the paper by Ahmed and Ross [2] and references given therein.



(a) (b)

Figure 1: Perturbative diagrams for the unit operator in the operator expansion.

application of the Lehmann-Symanzik-Zimmermann reduction formula. The discontinuity across the branch cut on the real axis in the complex plane of $\omega = \frac{1}{x}$, where x is the usual Bjorken variable, gives us the structure function we are interested in. In order to find the polarized spin structure function, we isolate it as a coefficient in front of an appropriate tensor structure, namely

$$\frac{1}{\pi} Im T_{\mu\nu,\alpha\beta}\epsilon_\alpha\epsilon^*_\beta = \frac{i}{(pq)}\epsilon_{\mu\nu\lambda\sigma}q_\lambda s_\sigma g_1^\gamma(x,Q^2,p^2). \quad (2)$$

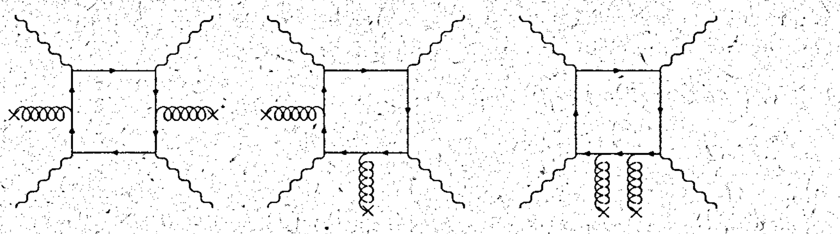
where $s_\sigma = i\epsilon_{\alpha\beta\gamma\sigma}\epsilon_\alpha\epsilon^*_\beta p_\gamma$ and ϵ_α is a photon polarization vector. More precisely, for the purposes of this paper it is enough to pick out the antisymmetric tensor $(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$.

As a first step we have to find the contribution of the unit operator. This result is well known since the "photon-photon fusion" process was calculated even before the advent of QCD [8]. However, to reproduce unambiguously the spectral densities in the dispersion representation for the structure function, which require some intermediate result, such a calculation has to be performed over again. We restrict ourselves to the scaling approximation, i.e. to taking into account the first nonvanishing term in the expansion in powers of p^2/Q^2, therefore limiting to the leading twist-2 contribution. The result is

$$\begin{aligned} & g_1^\gamma(x,Q^2,p^2)_{pert} \\ &= \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle\left\{\left(2-3x+[x^2-(1-x)^2]\int_0^{Q^2/x^2}\frac{dp'^2 p'^2}{(p'^2-p^2)^2}\right)-(1-x)\right\} \\ &= \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle[x^2-(1-x)^2]\left(ln\left(\frac{Q^2}{-p^2x^2}\right)-2\right). \end{aligned} \quad (3)$$

where $\langle Q_q^4\rangle = \frac{1}{N_f}\sum_q Q_q^4$ is an average of the fourth powers of the quark charges. This result is twice that represented by diagrams in fig.1 due to the clockwise and counterclockwise directions of the internal quark lines, each term in the curve brackets corresponds to the graph (a) and (b), respectively. The first line of this equation will be used in the following to fix the parameters of the hadronic spectrum.

From all power corrections up to dimension eight we calculate only one due to the gluon condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle$. This can be elucidated by the facts that the lowest dimension quark condensate $\langle\bar\psi\psi\rangle$ cannot appear due to chiral invariance as it is accompanied by the light quark mass which we set equal to zero in all calculations. The contribution of the three-gluon condensate $\langle g^3fG^3\rangle$ is usually small. Dimension six four-quark condensate



(a) (b) (c)

Figure 2: Gluon condensate contribution to the imaginary part of the forward $\gamma\gamma$-scattering amplitude.

can be omitted because its contribution is proportional to the delta function — $\delta(1-x)$ and turns out to be beyond the scope of the method. Of course, this singular contribution can be smeared over the whole region of the momentum fraction from zero to unity by introducing the concept of nonlocal quark condensate. But as will be discussed at the end of the paper it can be neglected too.

To simplify the calculation of the leading power correction, it is convenient to use fixed-point gauge for the background gluon field $(x-x_0)_\mu B^a_\mu(x) = 0$. We chose the fixed point in the vertex of the hard photon emission $x_0 = 0$. The quark propagator in this gauge up to the order $O(G^2)$ looks like [9]

$$\begin{aligned} S(x,y) = \int\frac{d^4k}{(2\pi)^4}e^{-ik(x-y)}\Bigg\{&\frac{\not k}{k^2} + g\widetilde{G}^a_{\alpha\beta}t^a\frac{k_\alpha}{k^4}\gamma_\beta\gamma_5 + \frac{1}{2}gG^a_{\alpha\beta}t^a y_\alpha\left(\frac{\gamma_\beta}{k^2} - 2\frac{k_\beta \not k}{k^4}\right) \\ &- \frac{\langle g^2G^2\rangle}{3^2 2^5}\left(\frac{y^2\not k}{k^4} + 4\frac{\not k(ky)^2}{k^6} - 2\frac{\not y(ky)}{k^4}\right)\Bigg\}. \end{aligned} \quad (4)$$

A non-zero contribution comes in the leading twist from the diagrams depicted on fig.2 and each term in the curve brackets corresponds to the diagrams (a),(b) and (c), respectively:

$$\begin{aligned} & g_1^\gamma(x,Q^2,p^2)_{\langle\frac{\alpha_s}{\pi}G^2\rangle} \\ &= \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle\frac{1}{12N_c}\frac{\pi^2}{p^4}\langle\frac{\alpha_s}{\pi}G^2\rangle\left\{\left(\frac{8}{3}\frac{1}{x^2}-\frac{4}{3}\frac{1}{x}\right)+\left(\frac{8}{3}\frac{1}{x^2}\right)+\left(\frac{4}{3}\frac{1}{x}\right)\right\} \\ &= \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle\frac{4}{9N_c}\frac{\pi^2}{p^4}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{x^2}. \end{aligned} \quad (5)$$

Collecting all contributions we obtain the following structure function for the off-shell polarized target photon:

$$\begin{aligned} & g_1^\gamma(x,Q^2,p^2) \\ &= \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle\left\{[x^2-(1-x)^2]\left(ln\left(\frac{Q^2}{-p^2x^2}\right)-2\right)+\frac{4}{9N_c}\frac{\pi^2}{p^4}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{x^2}\right\}. \end{aligned} \quad (6)$$

On the other hand we can use the analitical properties in p^2 [10] and represent the structure function via dispersion relation with respect to p^2 in terms of the physical states.

$$g_1^\gamma(x,Q^2,p^2) = G_0(x) + \int_0^\infty dp'^2\frac{G_1(x,p'^2)}{(p'^2-p^2)} + \int_0^\infty dp_1'^2\int_0^\infty dp_2'^2\frac{G_2(x,p_1'^2,p_2'^2)}{(p_1'^2-p^2)(p_2'^2-p^2)} \quad (7)$$

For functions G_i we accept the technique standard for the QCD sum rules, "resonance plus continuum" model:

$$G_1(x,p'^2) = G_1^{(1)}(x)\delta(p'^2 - m_\rho^2) + G_1^{(2)}(x)\theta(p'^2 - p_0{}^2),$$
$$G_2(x,p_1'^2,p_2'^2) = G_2^{(1)}(x)\delta(p_1'^2 - m_\rho^2)\delta(p_2'^2 - m_\rho^2) + G_2^{(2)}(x)\theta(p_1'^2 - p_0{}^2)\theta(p_2'^2 - p_0{}^2), \quad (8)$$

where $p_0^2 = 1.5GeV^2$ is a threshold value for the vector meson channel and $m_\rho^2 = 0.6GeV^2$ is a ρ-meson mass squared.

Requiring that at $-p^2 \to \infty$ eq.(7) must coincide with the bare quark loop, we obtain:

$$G_0(x) = -\frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle[x^2 - (1-x)^2],$$
$$G_1^{(2)}(x) = 0,$$
$$G_2^{(2)}(x) = \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle[x^2 - (1-x)^2]p_1'^2\delta(p_1'^2 - p_2'^2)\theta(Q^2/x^2 - p_1'^2) \quad (9)$$

Substituting them back into the dispersion relation (7) and expanding in the inverse powers of p^2 we can compare the resulting expression with QCD calculated $g_1^\gamma(x,Q^2,p^2)$ (6) and fix the remaining unknown functions $G_1^{(1)}(x)$ and $G_2^{(1)}(x)$, namely:

$$G_1^{(1)}(x) = 0,$$
$$G_2^{(1)}(x) = \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle p_0{}^4\left[\frac{1}{2}[x^2 - (1-x)^2] + \frac{4}{9N_c}\frac{\pi^2}{p_0{}^4}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{x^2}\right]. \quad (10)$$

Finally, we collect all functions and make integration in eq.(7) keeping the leading twist-2 contribution. We come to the polarized virtual structure function which possesses the correct analytical properties in the photon squared mass and accounts for the hadronic part:

$$g_1^\gamma(x,Q^2,p^2) = \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle$$
$$\left\{-[x^2 - (1-x)^2] + [x^2 - (1-x)^2]\left[ln\left(\frac{Q^2}{x^2(p_0{}^2 - p^2)}\right) + \frac{p^2}{(p_0{}^2 - p^2)}\right]\right.$$
$$\left.+\frac{1}{2}\frac{p_0{}^4}{(p^2 - m_\rho^2)^2}\left[[x^2 - (1-x)^2] + \frac{8}{9N_c}\frac{\pi^2}{p_0{}^4}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{x^2}\right]\right\}. \quad (11)$$

Now we make some comments about the introduction of the nonlocal quark condensates . Such an attempt was made in ref. [11] for the spin averaged structure function $F_2(x)$. But there are two shortcomings in this paper. First, the authors claim that the contribution due to the quark condensate improves considerably the description of experimental data, though, there is a numerical error in their answer: the coefficient in front of the nonlocal vector quark condensate is three times smaller. The second fact is connected with improper treatment of nonlocal objects. The diagram with a nonlocal scalar condensate used in their paper does not exist. This can be seen from the following facts. As was shown long ago in Ioffe's paper [12] the virtuality of the active quark lines in the box on which the hard γ-quantum is scattered is of an order of $k^2 \sim xp^2$ or for the transverse component of the quark momentum $k_\perp^2 \sim x(1-x)p^2$. This statement is valid only for the

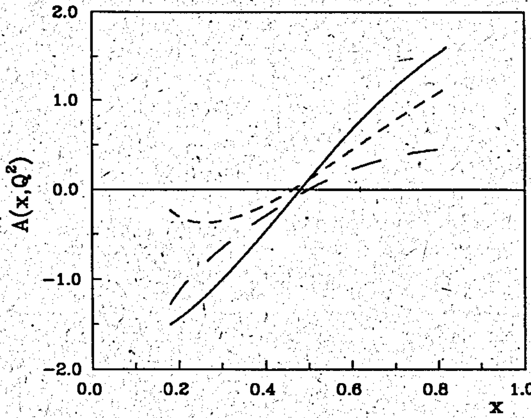


Figure 3: Real photon asymmetry at $Q^2 = 10GeV^2$. The solid curve corresponds to the full spin dependent structure function given by eq. (11), while long- and short-dashed lines correspond to the continuum and hadronic contributions to the latter.

imaginary part of $T_{\mu\nu,\alpha\beta}$ when the propagators corresponding to the horizontal lines are on the mass-shell [13]. The use of OPE in the target-photon virtuality can be justified if $-p^2 \gg R_{conf}^{-2}$, where R_{conf} is the confinement radius. So, the virtuality of the quark line is large in the region of intermediate x where this method can be used, and there is no reason to substitute the perturbative propagator by some function that simulates large distance propagation. In the limit $x \to 0$ the OPE series diverges as well as for $x \to 1$ [13].

Of course, one can convince oneself that from the viewpoint of the operator product expansion the nonlocal quark condensate corresponds to summing up an infinite subset of higher-dimension local condensates. But it is only a certain subset that is summed up while all the other power corrections are ignored at the same time, in spite of the fact that there is no reason to neglect them. Therefore, this cannot resolve the problem of OPE convergence for the problem in question. Moreover, the diagram of this type becomes disconnected in the local limit. Due to the fact that the contribution of nonlocal quark condensate has no numerical enhancement, we disregard the latter in our analysis.

Equation (11) is our main result.

In fig.3 we represent the result of calculation of the real target-photon asymmetry in the central region of the Bjorken variable

$$A^\gamma(x,Q^2) = \frac{2xg_1^\gamma(x,Q^2,0)}{F_2^\gamma(x,Q^2)} \quad (12)$$

for the $Q^2 = 10GeV^2$, where [5]

$$F_2^\gamma(x,Q^2) = \frac{\alpha_{em}}{\pi}N_cN_f\langle Q_q^4\rangle\left\{-1 + 6x(1-x) + [(1-x)^2 + x^2]ln\left(\frac{Q^2}{x^2p_0{}^2}\right)\right.$$
$$\left.+\frac{1}{2}\frac{p_0{}^4}{m_\rho^4}\left[[(1-x)^2 + x^2] - \frac{8}{9N_c}\frac{\pi^2}{p_0{}^4}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{x^2}\right]\right\}. \quad (13)$$