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RESONANCES IN SUBATOMIC PHYSICS

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We develop[1-4] the following general physical conception of resonances: the periodic motion and refraction of waves in the restricted region of space are responsible for creation of resonances in any resonating system. This conception is considered here for quantum mechanical systems, whose wave nature plays a decisive role in our approach. Within the R-matrix formalism we put at the boundary of this region a condition of radiation of physical particles which can be observed at large (asymptotic) distances and require proper matching of the corresponding "external" wave functions with the "inner" part of the wave function of the considered system. This "inner" part can be constructed by using any reasonable existing model and must be projected at the boundary into physically observed states for matching with the "external" part.

The new quantization condition for asymptotic momenta of decay products of a resonance was obtained in the framework of this conception. It results in the Balmer-like mass formula used in our study; its accuracy is surprisingly high and unusual for this branch of physics. Following the outlined conception we carried out the systematical investigation of the gross structure of spectra and mass distributions of all known hadronic resonances starting from light mesons and ending with bottomonium resonances. We have used a simplified version of the strength function method in this study.

Regular periodic structures in distributions of invariant masses of resonances are established. They have the period $\Delta m \approx 200$ MeV in regions of the light unflavored ψ and Υ mesons and $\Delta m \approx 100$ MeV for baryon resonances. Such regular behaviour of the invariant mass of resonances is due to an emergence of many-dimensional closed orbits where some states have regions of high amplitude as in the standard nuclear physics.

We found also that the charmonium and bottomonium systems might have "molecular type" states in three-meson (1^{--}) decay channels. They should play an essential role in understanding of mass distributions of the ψ and Υ mesons. The main characteristic feature of these states is that the relative momentum in any their binary subsystems is very low: ~ 100 MeV/c. Therefore this is the typical low energy phenomenon.

The asymptotic quantization condition can be obtained by applying the R-matrix[5] formalism to particle reactions[6]. According to these papers, one can assume that the resonating system having several two-particle decay channels is free at relative separation $r \geq r_0$ in the center of mass; hence the following logarithmic radial derivative of the internal wave functions can be introduced:

$$\left(\frac{r}{u_{in}}\frac{du_{in}}{dr}\right)\Bigg|_{r=r_0-0} = f \equiv \frac{1}{R} \quad . \tag{1}$$

which should be calculated in the framework of some microscopic models for example modern quark models.

For simplicity let us consider only systems with one dominating open channel. As has been argued in the papers[1-4], the decay of hadronic resonances can be considered in a full analogy with open classical electrodynamic resonators[7] and the mathematical formalism given in this excellent monograph can be used. Therefore the boundary conditions for the emitted waves must be written as follows (*the conditions of radiation*):

$$\left(\frac{r}{h_l^{(1)}(Pr)}\frac{dh_l^{(1)}(Pr)}{dr}\right)\Bigg|_{r=r_0+0} = f \quad . \tag{2}$$

where $h_l^{(1)}(Pr) = \sqrt{\frac{\pi Pr}{2}} H_{l+\frac{1}{2}}^{(1)}(Pr)$ are the spherical Riccati-Hankel functions. We assumed that $f = 0$ for the well isolated resonances. Such surface waves localized at $r = r_0$ have exponentially small absorption (for $r < r_0$) in full analogy with the waves in the "whispering gallery". This phenomenon is very close to the phenomenon of the full refraction of the waves on the boundary separating two media with different refraction properties. Rainbow effects[8] and open resonators [7] can be considered as other examples of such kind. It means that nuclear and hadronic resonances have the same physical origin: emergence of well-localized surface waves with wavelengths of order r_0.

The new quantization condition for asymptotic momenta P of decay products of a resonance was obtained in the framework of this conception (see for details ref.[9]):

$$Pr_0 = n + \gamma \tag{3}$$

here $Pr_0 = n + 1/2$ may be interpreted as a radial quantization and $Pr_0 = l$ may be considered as the well-known Bohr-Sommerfeld orbital quantization. It results in the Balmer-like mass formula used in our study:

$$m_n(R) = \sqrt{m_1^2 + P^2} + \sqrt{m_2^2 + P^2} = \sqrt{m_1^2 + (\frac{n+\gamma}{r_0})^2} + \sqrt{m_2^2 + (\frac{n+\gamma}{r_0})^2} + \Delta m_n \tag{4}$$

where γ=0 or 1/2, R labels the resonance, while the indices 1 and 2 refer to the constituents 1 and 2 observed in the 2-particle decay of the resonance $R \to 1+2$ respectively.

Formula (4) describes the gross structure of the resonance spectrum with reasonable accuracy because of the relation $\Delta m_n < \Gamma$ that is valid in all investigated cases of strong decays $R \to 1+2$. The leading term of the mass formula describes only the "center of gravity" position of the corresponding multiplets and thus the gross structure of the hadron and dibaryon resonances. The fine structure in each multiplet is determined by residual interactions and corresponding quantum numbers that are not contained in the approach[1-4]. Therefore the condition $\Delta m_n < \Gamma$ is to be considered as an empirical fact. Further we neglected the contribution of Δm_n to mass of resonances.

Let us consider the quantization of the hydrogen - like system. The kinetic energy of the system in nonrelativistic limit can be obtained from (4):

$$T = m_n(R) - m_1 - m_2 = \sqrt{m_1^2 + P^2} + \sqrt{m_2^2 + P^2} - m_1 - m_2 = \frac{P^2}{2m_{12}} = \frac{l^2}{2m_{12}r^2} \tag{5}$$

where m_{12} is the reduced mass of system and $l = Pr = m_{12}vr = n\hbar$ is the adiabatic invariant. The electrostatic force between the nucleus and the electron binds the hydrogen -like system. Equating the magnitude of the Coulomb force to the centrifugal acceleration (the classical equation of motion) we obtain

$$F = \frac{e^2}{r^2} = m_{12}a = \frac{m_{12}v^2}{r} = \frac{l^2}{m_{12}r^3} = \frac{n^2\hbar^2}{m_{12}r^3}. \tag{6}$$

The second and the last forms may be solved to determine the allowed values for r, yielding

$$r = \frac{l^2}{m_{12}e^2} = \frac{n^2\hbar^2}{m_{12}e^2} = n^2 a_0 \tag{7}$$

where a_0 is the Bohr radius by definition and is given by $a_0 = \hbar^2/m_{12}e^2$.

The total energy of the system is the sum of the kinetic and potential energy

$$E = T + V = \frac{m_{12}v^2}{2} - \frac{e^2}{r} = -\frac{e^2}{2r} = -\frac{e^2}{2n^2 a_0}. \tag{8}$$

Thus one concludes that the mass formula (4) for resonances in the nonrelativistic limit is reduced to the Bohr result or to the Balmer formula.

It is easy to obtain useful relations using (3) and (6) for $\gamma = 0$

$$\alpha = n\frac{\lambda_C}{\lambda_D} = n\frac{v}{c}, \quad \frac{r}{\lambda_D} = n, \tag{9}$$

where λ_C and λ_D are the lengths of Compton and de Broglie waves respectively. It means that the Compton and de Broglie waves play a fundamental role in the quantization of electronic orbit in hydrogen - like atom. Such quantization is possible only if the ratio of the Compton wave length to the de Broglie wave length (v/c) is commensurable with the fine structure constant α. The ratio (9) can be interpreted as a definition of a similitude parameter for hydrogen - like atom where r is the Bohr radius while $\lambda_D = \hbar/P = \hbar/mv$ is the de Broglie wave length.

The hypothesis of automodelity introduced in elementary particle physics in ref.[10] means that some physical observables are invariant in respect of the transformation of the momentum space $P_i \to \xi P_i$. Further development of the principles of similitude and automodelity were achieved in ref.[11] devoted to the relativistic theory of dynamical system. Let us rewrite (4)

$$m_n(R) = \sqrt{m_1^2 + P^2} + \sqrt{m_2^2 + P^2} = \hbar c\sqrt{\lambda(1)_C^{-2} + \lambda_D^{-2}} + \hbar c\sqrt{\lambda(2)_C^{-2} + \lambda_D^{-2}}. \tag{10}$$

Here λ_D/λ_C, r_0/λ_C are similitude parameters. The magnitudes of invariant masses of resonances at the given value of the similitude parameters remain fixed for different values of other parameters (at $r_0/\lambda_D \gg 1$). This is the automodelity of the second type. The invariant masses of resonances are changing as a homogeneous function

$$m_n(R) \to \xi m_n(R), \tag{11}$$

under scale transformation $P \to \xi P, m_i \to \xi m_i$. This is the formulation of the principle of automodelity for hadronic resonances decaying to two particles with the equal masses.

Subtracting $m_1 + m_2$ in (10) we obtain under the conditions $m_1 > P$ and $m_2 > P$ (in nonrelativistic limit) that the "excitation energy" E_n is

$$E_n(R) = \sqrt{m_1^2 + P^2} + \sqrt{m_2^2 + P^2} - m_1 - m_2 \approx \frac{P^2}{2m_{12}} = \frac{1}{2m_{12}}(\frac{n+\gamma}{r_0})^2, \tag{12}$$

where $m_{12} = m_1m_2/(m_1 + m_2)$ is a reduced mass. This expression is completely the same as the well known formula for the rotational energy of a diatomic molecule [12] in quasiclassical approach. Indeed quantity $m_{12}r_0$ plays a role of a moment of inertia of a molecule while $n + \gamma$ (if $\gamma = 1/2$) is a quasiclassical analog of total angular momentum of the molecule.

If $m_1 < P$ and $m_2 < P$ then

$$E_n(R) \approx 2P = 2\frac{n+\gamma}{r_0} \tag{13}$$

which is in full analogy with the formula of vibrational energy of nuclei within the molecule.

Thus the Lorenz-invariant mass formula (10) obtained from the resonance condition using Heisenberg uncertainty relation contains two limiting cases: 1) the rotational spectra and 2) the vibrational spectra. It is well-known in nuclear physics that pure elementary states (say, rotational, vibrational etc.) are model concepts in nuclei and are only approximately realized for the ground and low-lying parts of spectra in nuclei having large spectroscopic factors (branching ratios, see for details, ref. [13]). Such states played a decisive role in the development of modern nuclear physics. Similar situation could take place in particle physics.

We assumed as a working hypothesis that the resonances are the result of interplay between the "effective size" (r_0) and wavelength of the system (the automodelity of the second type by definition). We have demonstrated[1-4] that this hypothesis does not contradict with the existing experimental data; it is useful for systematic analysis of hadronic resonances and for predictions of new resonances. We have established the similarity of spectra between different systems. As example we show on Fig.1 experimental spectra of the D, D^*, D^{**},... and B, B^*, B^{**},... systems taken from talks M.Feindt, J.Bartlet and G.Sciolla given at Hadron'95 Conference [14]. As the second example we show on Fig.2 experimental spectra (1^- states) [15] of the $\rho, \omega, \phi, J/\psi$ and Υ systems. As final example we show on Fig.3 experimental spectra ($1^-, 2^+$ and 3^- states) of different hadron resonances.

We can conclude that the decay of a resonance into two particles obeys to the similitude principle according to which the Compton and de Broglie wavelengths have to be commensurable independently on a particular form of the interaction.

The appearance of the puzzling peaks in the pion-pion system (so called ABC particles) in reactions of type $a+b \rightarrow a'+b'+(\pi\pi)$ were observed in different laboratories (see review paper[16]). According to this review the existence of 4 resonance – like states for $\pi\pi$ system at 315, 455, 550 and 750 MeV are not excluded by experiments carried out so far. Such peaks were not observed in free pion - pion scattering. Note that the ABC particles were never predicted by theoretical models except of our prediction [4] of a resonance at 360 MeV (at least, we could not find corresponding references).

There are tremendous number of examples in the nuclear physics when two particles display resonances-like structure in presence of a third particle (particles) but no such structure appears when the third particle is absent. For example the proton – proton or proton – deuteron scattering at low energy does not display pecularities but in the company with a third particle ($p+n \rightarrow (pp)+\pi^-, d+d \rightarrow (pp)+nn, H+{}^6He \rightarrow \alpha+t$) resonance-like structures were observed[17]. Two-particle resonance – like structure does indeed depend on the physical properties of the third particle, dynamical conditions of reactions and etc. It is not surprising that experimental groups claimed the observation of different two particle resonances for the same pair of particles (but in different environment) using different reactions. This is a result of the coherent enhancement of amplitudes for resonances in different binary channels. The presence of a third particle change dramatically the resonance property of two particle subsystem.

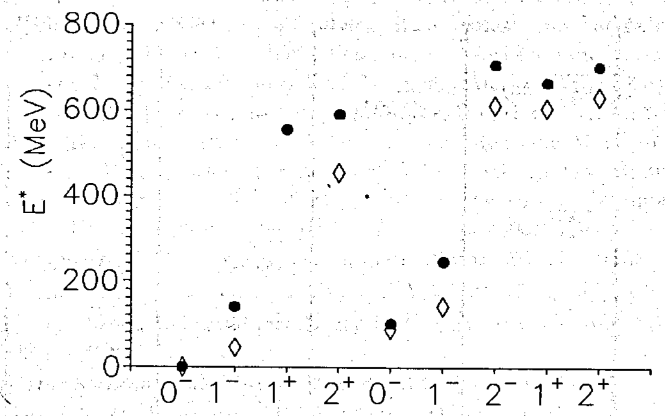

Fig.1 Spectra of the D, D^*, D^{**},... (circles) and B, B^*, B^{**},... (rhombus).

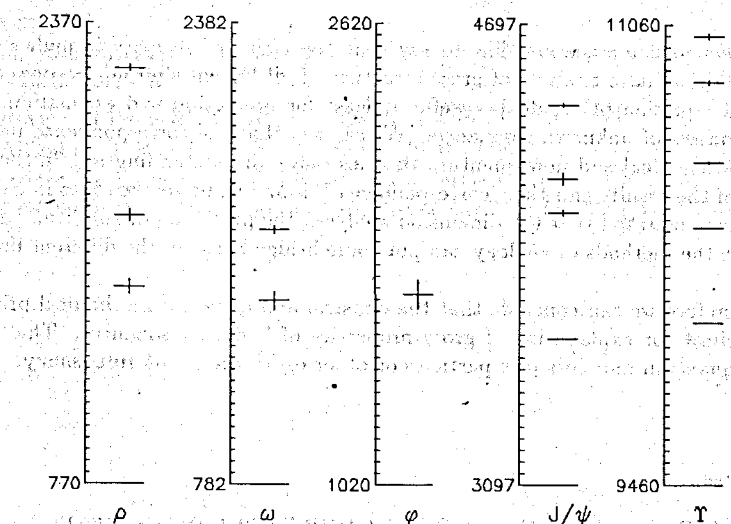

Fig.2 1^- states for ρ, ω, ϕ, J/ψ, Υ mesons.

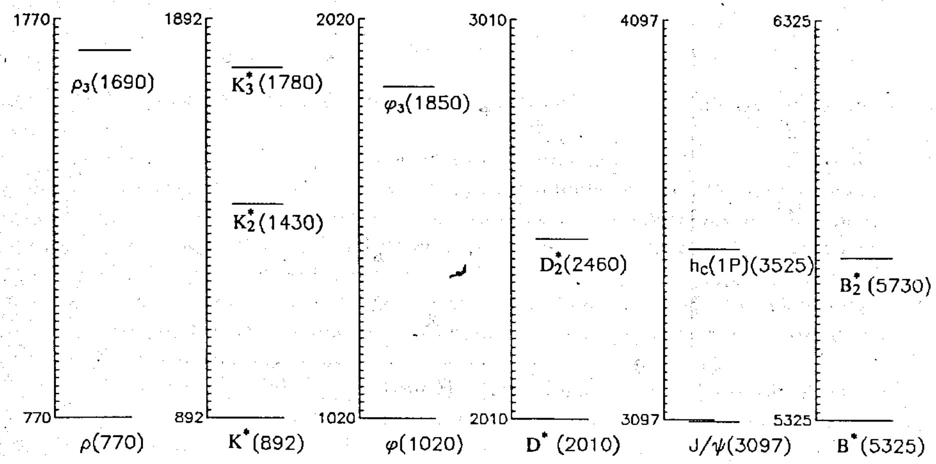

Fig.3 1^-, 2^+ and 3^- states for ρ, K^*, ϕ, D^*, J/ψ, B^* mesons.

As conclusion we would like to say that the Balmer-like mass formula (4) was applied for systematic analysis of gross structure of all known hadronic resonances. It means that equation (4) could be useful at least for prediction and estimation of the invariant masses of unknown resonances. We can say that the correspondence principle between old classical and new quantum theories plays an outstanding role in the interpretation of the results and this "correspondence" allows us to go even into fine details. We have demonstrated that the dimension analysis, the principles of similitude and automodelity, the methods of analogy can put some bridge between the different branches of physics.

Therefore we can conclude that the classical and quantum mechanical principles **are sufficient** for explanation of gross properties of hadron resonances. That means that new quantum numbers new particles or other exotics **are not necessary**.

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