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D.I.Kazakov[1], M.Yu.Kalmykov[2], I.N.Kondrashuk[3],
A.V.Gladyshev[4]

SOFTLY BROKEN FINITE SUPERSYMMETRIC GRAND UNIFIED THEORY

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[1]E-mail: kazakovd@thsun1.jinr.dubna.su
[2]E-mail: kalmykov@thsun1.jinr.dubna.su
[3]E-mail: ikond@thsun1.jinr.dubna.su
[4]E-mail: gladysh@thsun1.jinr.dubna.su, Physics Department, Moscow State University, Russia

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1 Introduction

During recent times there is a considerable interest in the Minimal Supersymmetric Standard Model [1] and in SUSY GUTs [2]. It is because of the remarkable unification of the gauge couplings in these models [3], which leads to predictions of the SUSY spectrum in the energy region within the reach of future accelerators [4]. The detailed analysis performed by various groups [5, 6] is based on the SUSY GUT scenario with soft supersymmetry breaking due to the supergravity mechanism and is different only in details. It takes into account two-loop renormalization group equations and one-loop corrections to the Higgs potential as well as the heavy and light threshold effects and various experimental constraints. Maybe the most remarkable fact is that all the requirements can be fulfilled simultaneously and are consistent with very few free parameters. The predicted mass spectrum is concentrated in the $10^2 - 10^3$ GeV region and is not very much model dependent. This leads to the conclusion that the MSSM and SUSY GUTs provide us with a very promising scenario that can be checked experimentally.

Of course, several problems remain unsolved. Besides the unknown explicit mechanism of SUSY breaking parametrized by soft terms with five free parameters [7], some problems of the Standard Model still remain. Namely, the quark mass spectrum and the mixing of the generations remain the biggest puzzles. And though some progress has been made in these directions, there is not a commonly accepted solution yet. One of the most interesting attempts of this kind is the one discussed in ref. [8] where the values of the Yukawa couplings and the Kobayashi-Maskawa mixing matrix at the unification scale are given in the form of the so-called *textures* and then evolve to the observed values at low energies. The textures themselves are chosen for reasons of maximal simplicity and symmetry while the needed parameters are fitted. The related idea explores the possibility of determining the Yukawa couplings by the infra-red stable fixed point structure of the theory lying beyond the Standard Model [9].

Another approach is based on a wider symmetry like $SO(10)$ [10]. In this case the masses of the heaviest generation arise from a single renormalizable Yukawa interaction, while the lighter masses are generated by nonrenormalizable operators of the Grand unified theory.

There are naturally many attempts to consider some non-minimal models that provide wider possibilities. Among them the so-called Next-to-minimal SSM [11] that allows one to relate some soft breaking terms to the vacuum expectation value of the singlet Higgs field. However, this model does not touch the problems of the quark mass spectrum and flavour mixing mentioned above.

Without denying these possibilities, we would like to suggest an alternative approach that naturally arises in attempts to construct SUSY GUTs free from ultraviolet divergences [12, 13, 14].

In the standard minimal SUSY GUT scenario the theory possesses both the supersymmetry and the unified gauge symmetry at the unification scale with soft

SUSY breaking terms arising from supergravity. At this scale all quarks and leptons are massless and all their superpartners have the same mass. Going down to lower energies the superpartners' masses run according to the RG equations, split due to different interactions and, thus, give us the mass spectrum. This is accompanied also by the radiative spontaneous symmetry breaking, which leads to the reconstruction of the vacuum state. The latter, according to the usual Higgs mechanism, provides us with masses for quarks, leptons and $SU(2)$ gauge bosons and additional mass terms to their superpartners.

Since the Standard Model exploits the minimal version of the Higgs mechanism with only one Higgs doublet to provide masses to all quarks and leptons simultaneously, the mass spectrum is given by that of the Yukawa couplings. In the MSSM one needs at least two doublets. One doublet provides masses to up quarks; while the other, to down quarks and leptons. Thus, we have two vacuum expectation values and their ratio $\tan\beta \equiv v_2/v_1$ is a free parameter of the model. It is usually fitted from the experimental constraints; however, on the other hand, the value of $\tan\beta$ can be found from the minimization of the potential for neutral Higgses, if the parameters are known, and differs from unity. Thus, we can get a hierarchy if the potential has an asymmetric minimum [15], though it is not essential in case the Yukawa couplings remain arbitrary.

This is not the case, however, in finite SUSY GUT models where the Yukawa couplings at the GUT scale can be calculated and appear to be degenerate with respect to generations. On the contrary, the number of Higgs doublets increases, each being adjusted to a certain flavour so that the mass spectrum is given by the spectrum of the v.e.v.s of the Higgs fields rather than by that of the Yukawa couplings.

The finite models, though non-minimal, still remain almost as rigid as the minimal one and are distinguished by their ultraviolet properties being absolutely UV finite to all orders of perturbation theory [12, 13]. Let us remind the main properties of a finite SUSY GUT:

- the number of generations is fixed by the requirement of finiteness,
- the representations and the number of the Higgs fields are fixed,
- all the Yukawa couplings are expressed in terms of the gauge one,
- various realistic possibilities are given by $SU(5), SU(6), SO(10)$ and $E(6)$ gauge groups with few generations. An abelian subgroup is not allowed.

The other attractive feature of a finite model is that if the gauge symmetry is not broken, the parameters, including the soft terms, are not running. This means that the couplings, masses, etc at the GUT scale have some absolute values. If they are governed by some symmetry, it does not matter whether we impose this symmetry at the GUT or at the Planck scale.

It should be mentioned, that the attractiveness of UV finite models without gravity is often called into question since, being renormalizable, SUSY GUTs are quite satisfactory in the practical sense. However, the motivation for SUSY itself is mainly due to the cancellation of quadratic divergences which allows one to preserve the hierarchy of the Higgs masses in SUSY GUTs. The finite model is the next step in this direction when not only quadratic but logarithmic divergences also cancel.

Below we consider a particular finite SUSY GUT model that is based on the $SU(5)$ gauge group and is one of the simplest models of this type deviating only slightly from the minimal SUSY GUT. It should be stressed that this model is almost unique among possible finite models, if one requires spontaneous symmetry breaking to take place via the Higgs mechanism with elementary Higgs fields. The other possible choice is $SU(6)$, but anyway the symmetry breaking takes place along the $SU(5)$ pattern. Higher groups inevitably explore composite Higgs fields [14].

On the other hand, if one accepts $SU(5)$, the number of generations is exactly three without any option. The singlets are not allowed due to finiteness; hence the right handed neutrino is excluded. Thus, the finiteness hypothesis happens to be very rigid and provides us with a unique selection of a possible GUT distinguished by its mathematical properties.

The paper is organized as follows. Sect.2 is devoted to a general review of the $SU(5)$ supersymmetric finite unified theory. We consider the simplest R-symmetric and B-L conserving superpotential and give an explicit solution to the conditions of one-loop finiteness for the Yukawa couplings. The soft SUSY breaking is considered in Sect.3. Going along the same line we suppose that the soft SUSY breaking terms are also finite above the GUT threshold, which leads to the universality condition at the Planck scale with some of the soft parameters being fixed. In Sect.4, the spontaneous breaking of SU(5) is discussed. The fine-tuning procedure which reduces the unified model to the MSSM below M_{GUT} is proposed. At the first step we are left with three pairs of Higgs doublets, one to each generation. They develop vacuum expectation values defining the Yukawa couplings of the low energy theory. Then, at the next step, minimizing the Higgs potential we separate the light pair of Higgses identified with that of the MSSM. Heavy fields decouple at high energies. In Sect.5, we analyze the compatibility of our model with various experimental constraints such as the values of the heavy quark masses, the proton lifetime, absence of the flavour changing neutral currents, etc. Finally, in Sect.6 the main attractive features of our model and its general status are summarized. The Appendix contains the derivation of the solution to the Higgs potential minimization conditions.

2 Unified Finite Theory

The model is a supersymmetric $SU(5)$ gauge theory whose field content and interactions are completely defined by the requirement of UV finiteness. From this point of view the finite model is even more rigid than the minimal one. The SUSY breaking is achieved via the supergravity mechanism in a usual way; however, the enlarged Higgs sector requires more parameters.

Field Content

Matter fields :	$\Psi_i - \overline{5}$	of $SU(5)$	$i = 1,2,3$	– generations
	$\Lambda_i - 10$	–/–		
Higgs fields :	$\Phi_a - 5$	–/–	$a = 1,2,3,4$	
	$\overline{\Phi}_a - \overline{5}$	–/–		
	$\Sigma - 24$	–/–		

Lagrangian

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{SUSY} + \mathcal{L}_{Breaking}, \\ \mathcal{L}_{SUSY} &= \mathcal{L}_{Gauge} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Mass}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \mathcal{L}_{Yukawa} = &\ y_1 \Psi_i K_{ij} \overline{\Phi}_i \Lambda_j + y_1' \Psi_i \overline{\Phi}_4 \Lambda_i + \frac{y_2}{8} \Phi_i \Lambda_i \Lambda_i + \frac{y_2'}{8} \Phi_4 \Lambda_i \Lambda_i \\ &+ y_3 \overline{\Phi}_i S_{ij} \Sigma \Phi_j + y_3' \overline{\Phi}_4 \Sigma \Phi_4 + \frac{y_4}{3} \Sigma^3, \end{aligned} \tag{2}$$

and

$$\mathcal{L}_{Mass} = \overline{\Phi}_i M_{ij} \Phi_j + \overline{\Phi}_4 M \Phi_4 + \frac{M_0}{2} \Sigma^2, \tag{3}$$

Here the matrices K and S are unitary:

$$K^+ K = 1, \quad S^+ S = 1,$$

K being the Cabbibo-Kobayashi-Maskawa mixing matrix and S playing the same role in the Higgs sector. As we show below (Sect.4), the matrix S contains all information about the quark masses hierarchy at the GUT scale.

Yukawa couplings: The requirement of UV finiteness is formulated already at the one-loop level. Besides the field content of the model it defines also the superfield Yukawa couplings in terms of the gauge one:

$$Y_i \equiv \frac{\overline{y}_i^2}{16\pi^2} = c_i \frac{g^2}{16\pi^2} \equiv c_i \widetilde{\alpha}_G.$$

For the Lagrangian, eq.(2), the RG equations for the Yukawa couplings are [12, 13]:

$$
\begin{aligned}
\frac{dY_1}{dt} &= Y_1\left[10Y_1 + 6Y_1' + 3Y_2 + 3Y_2' + \frac{24}{5}Y_3 - \frac{42}{5}\tilde{\alpha}_G\right],\\
\frac{dY_1'}{dt} &= Y_1'\left[6Y_1 + 18Y_1' + 3Y_2 + 3Y_2' + \frac{24}{5}Y_3 - \frac{42}{5}\tilde{\alpha}_G\right],\\
\frac{dY_2}{dt} &= Y_2\left[4Y_1 + 4Y_1' + 9Y_2 + 6Y_2' + \frac{24}{5}Y_3 - \frac{48}{5}\tilde{\alpha}_G\right],\\
\frac{dY_2'}{dt} &= Y_2'\left[4Y_1 + 4Y_1' + 6Y_2 + 15Y_2' + \frac{24}{5}Y_3 - \frac{48}{5}\tilde{\alpha}_G\right], \qquad (4)\\
\frac{dY_3}{dt} &= Y_3\left[4Y_1 + 3Y_2 + \frac{63}{5}Y_3 + Y_3' + \frac{21}{5}Y_4 - \frac{49}{5}\tilde{\alpha}_G\right],\\
\frac{dY_3'}{dt} &= Y_3'\left[12Y_1' + 9Y_2' + 3Y_3 + \frac{53}{5}Y_3' + \frac{21}{5}Y_4 - \frac{49}{5}\tilde{\alpha}_G\right],\\
\frac{dY_4}{dt} &= Y_4\left[9Y_3 + 3Y_3' + \frac{63}{5}Y_4 - 15\tilde{\alpha}_G\right].
\end{aligned}
$$

Here $t \equiv log(Q^2/M^2)$.

The finiteness solution contains one free parameter c:

$$
\begin{aligned}
&c_1 = c, \quad c_1' = \tfrac{3}{5} - c, \quad c_2 = \tfrac{4}{3}c, \quad c_2' = \tfrac{4}{3}(\tfrac{3}{5} - c),\\
&c_3 = \tfrac{5}{6}(\tfrac{3}{5} - c), \quad c_3' = -\tfrac{5}{2}(\tfrac{2}{5} - c), \quad c_4 = \tfrac{15}{14}
\end{aligned}
$$

Since $c_i \geq 0$, the parameter c is restricted by the inequality $\frac{2}{5} \leq c \leq \frac{3}{5}$. In particular cases we have:

$$
\begin{aligned}
&c = \tfrac{2}{5}, \quad c_1 = \tfrac{2}{5}, \quad c_1' = \tfrac{1}{5}, \quad c_2 = \tfrac{8}{15}, \quad c_2' = \tfrac{4}{15}, \quad c_3 = \tfrac{1}{6}, \quad c_3' = 0, \quad c_4 = \tfrac{15}{14};\\
&c = \tfrac{3}{5}, \quad c_1 = \tfrac{3}{5}, \quad c_1' = 0, \quad c_2 = \tfrac{4}{5}, \quad c_2' = 0, \quad c_3 = 0, \quad c_3' = \tfrac{1}{2}, \quad c_4 = \tfrac{15}{14}.
\end{aligned}
$$

In what follows we take the case $c = \frac{2}{5}$. Remind that $g_{SUSY} = \sqrt{2}g_{Non-SUSY}$. Later on we will use $g_{Non-SUSY} \equiv g_G$ everywhere.

These relations for the Yukawa couplings are valid in one- and two-loop orders and have to be corrected in higher loops [12]. The corrections are finite and can be expressed either in terms of the series in the renormalized gauge coupling, or in the regularization parameter (for instance, ε in dimensional regularization) for the bare coupling [14].

3 Soft SUSY Breaking via Supergravity

We accept a common procedure of the soft supersymmetry breaking via a supergravity mechanism when supersymmetry is broken in a hidden sector that couples to the observable world only via gravity. It is natural then to assume that the universal soft terms arise at the Planck scale. To determine their evolution down to the GUT scale one has to apply the RG equations of a particular GUT model.

In general, this may lead to considerable splitting between mass parameters [16], which results in uncertainties in the low energy predictions. Since in our model the soft parameters, like all the couplings, are not running, they are the same at the GUT scale and at the Planck one and have the universal form.

$$
\begin{aligned}
-\mathcal{L}_{soft} &= m^2_{\bar\phi}|\bar\phi_i|^2 + m^2_\phi|\phi_i|^2 + m^2_{\bar 4}|\bar\phi_4|^2 + m^2_4|\phi_4|^2 + m^2_\Sigma|\Sigma|^2 + m^2_5|\psi_i|^2 + m^2_{10}|\lambda_i|^2\\
&+ \left[B_\Sigma\frac{M_0}{2}\Sigma^2 + B_\phi\bar\phi_i M_{ij}\phi_j + B_4\bar\phi_4 M\phi_4\right. \qquad (5)\\
&+ A_1 y_1\psi_i K_{ij}\bar\phi_i\lambda_j + A_1' y_1'\psi_i\bar\phi_4\lambda_i + A_2\frac{y_2}{8}\phi_i\lambda_i\lambda_i + A_2'\frac{y_2'}{8}\phi_4\lambda_i\lambda_i\\
&\left.+ A_3 y_3\bar\phi_i S_{ij}\Sigma\phi_j + A_3' y_3'\bar\phi_4\Sigma\phi_4 + A_4\frac{y_4}{3}\Sigma^3 + \frac{1}{2}M_5\lambda_\alpha\lambda_\alpha + h.c.\right].
\end{aligned}
$$

where ϕ, ψ, λ, and Σ are the scalar components of the corresponding matter superfields and λ_α are the gauginos.

RG Equations for the soft terms are:

$$
\begin{aligned}
\frac{dm^2_{10}}{dt} &= \left[3Y_2(m^2_\phi + 2m^2_{10} + A_2^2) + 3Y_2'(m^2_4 + 2m^2_{10} + A_2'^2)\right.\\
&\left.+2Y_1(m^2_{\bar\phi} + m^2_{10} + m^2_5 + A_1^2) + 2Y_1'(m^2_{\bar 4} + m^2_{10} + m^2_5 + A_1'^2) - \frac{72}{5}\tilde{\alpha}_G M_5^2\right],\\
\frac{dm^2_5}{dt} &= \left[4Y_1(m^2_{\bar\phi} + m^2_{10} + m^2_5 + A_1^2) + 4Y_1'(m^2_{\bar 4} + m^2_{10} + m^2_5 + A_1'^2) - \frac{48}{5}\tilde{\alpha}_G M_5^2\right],\\
\frac{dm^2_\Sigma}{dt} &= \left[\frac{21}{5}Y_4(3m^2_\Sigma + A_4^2) + 3Y_3(m^2_{\bar\phi} + m^2_\phi + m^2_\Sigma + A_3^2)\right.\\
&\left.+Y_3'(m^2_{\bar 4} + m^2_4 + m^2_\Sigma + A_3'^2) - 20\tilde{\alpha}_G M_5^2\right],\\
\frac{dm^2_{\bar\phi}}{dt} &= \left[4Y_1(m^2_{\bar\phi} + m^2_{10} + m^2_5 + A_1^2) + \frac{24}{5}Y_3(m^2_{\bar\phi} + m^2_\phi + m^2_\Sigma + A_3^2) - \frac{48}{5}\tilde{\alpha}_G M_5^2\right],\\
\frac{dm^2_\phi}{dt} &= \left[3Y_2(m^2_\phi + 2m^2_{10} + A_2^2) + \frac{24}{5}Y_3(m^2_{\bar\phi} + m^2_\phi + m^2_\Sigma + A_3^2) - \frac{48}{5}\tilde{\alpha}_G M_5^2\right],\\
\frac{dm^2_{\bar 4}}{dt} &= \left[12Y_1'(m^2_{\bar 4} + m^2_{10} + m^2_5 + A_1'^2) + \frac{24}{5}Y_3'(m^2_{\bar 4} + m^2_4 + m^2_\Sigma + A_3'^2) - \frac{48}{5}\tilde{\alpha}_G M_5^2\right],\\
\frac{dm^2_4}{dt} &= \left[9Y_2'(m^2_4 + 2m^2_{10} + A_2'^2) + \frac{24}{5}Y_3'(m^2_{\bar 4} + m^2_4 + m^2_\Sigma + A_3'^2) - \frac{48}{5}\tilde{\alpha}_G M_5^2\right],\\
\frac{dM}{dt} &= M\left[6Y_1' + \frac{9}{2}Y_2' + \frac{24}{5}Y_3' - \frac{24}{5}\tilde{\alpha}_G\right],\\
\frac{dM_{ij}}{dt} &= M_{ij}\left[2Y_1 + \frac{3}{2}Y_2 + \frac{24}{5}Y_3 - \frac{24}{5}\tilde{\alpha}_G\right],\\
\frac{dM_0}{dt} &= M_0\left[3Y_3 + Y_3' + \frac{21}{5}Y_4 - 10\tilde{\alpha}_G\right],\\
\frac{dM_5}{dt} &= 0.
\end{aligned}
$$

The RGEs for the trilinear SSB parameters A_i and for the quadratic terms B_i can be obtained from the RGEs of the corresponding Yukawa couplings Y_i and mass parameters by the replacement [16]

$$\frac{dY_i}{dt} = Y_i\left[a_{ij}Y_j - b_i\tilde{\alpha}_G\right] \Rightarrow \frac{dA_i}{dt} = \left[a_{ij}Y_jA_j + b_i\tilde{\alpha}_G M_5\right],$$

$$\frac{dM_i}{dt} = M_i\left[a'_{ij}Y_j - b'_i\tilde{\alpha}_G\right] \Rightarrow \frac{dB_i}{dt} = 2\left[a'_{ij}Y_jA_j + b'_i\tilde{\alpha}_G M_5\right],$$

so that if $A_i = -M_5$, neither A_i, nor B_i are running in the finite model.

The condition of finiteness for the soft terms has the following solution:

$$\begin{aligned} m^2_{\bar\phi} &= m^2_{\bar 4} = m^2_{\bar\phi}, \\ m^2_{\phi} &= m^2_4 = \frac{2}{3}M_5^2 - m^2_{\bar\phi}, \\ m^2_{10} &= \frac{1}{6}M_5^2 + \frac{1}{2}m^2_{\bar\phi}, \\ m^2_5 &= \frac{5}{6}M_5^2 - \frac{3}{2}m^2_{\bar\phi}, \\ m^2_\Sigma &= \frac{1}{3}M_5^2, \end{aligned}$$

which is independent of the parameter c. If one assumes $m_5 = m_{10}$, one gets:

$$m^2_{\bar\phi} = m^2_\phi = m^2_{10} = m^2_5 = m^2_\Sigma = \frac{1}{3}M_5^2. \tag{6}$$

Thus, the requirement of finiteness naturally leads to the universality of the soft breaking terms at the GUT scale.

If the finiteness conditions are satisfied, the mass parameters M_0, M_{ij}, M and M_5 are not running.

4 Reduction to the MSSM

4.1 Spontaneous breaking of SU(5)

We follow the standard approach when the GUT symmetry is broken spontaneously in a usual way by the vacuum expectation value of the Higgs superfield Σ. For this purpose we minimize the superpotential

$$W_\Sigma = \frac{y_4}{3}\Sigma^3 + \frac{M_0}{2}\Sigma^2,$$

with the result

$$\langle\Sigma\rangle = \begin{pmatrix} V &&&& \\ & V &&& \\ && V && \\ &&& -\frac{3}{2}V & \\ &&&& -\frac{3}{2}V \end{pmatrix},$$

where $V \sim \dfrac{M_0}{y_4} \sim 10^{16}$ Gev.

After breaking of $SU(5)$ the Σ field obtains the mass of an order of the GUT scale ($\sim 10^{16}$ Gev) and decouples, while the quintets Φ and $\overline{\Phi}$ split into doublets and triplets. Their mass terms look like

$$y_3\overline{\Phi}_i S_{ij}\langle\Sigma\rangle\Phi_j + \overline{\Phi}_i M_{ij}\Phi_j = \overline{\Phi_i}\begin{pmatrix} y_3S_{ij}V + M_{ij} & \\ & -\frac{3}{2}y_3S_{ij}V + M_{ij}\end{pmatrix}\Phi_j.$$

and

$$y'_3\overline{\Phi}_4\langle\Sigma\rangle\Phi_4 + \overline{\Phi}_4 M\Phi_4 = \overline{\Phi}_4\begin{pmatrix} y'_3V + M & \\ & -\frac{3}{2}y'_3V + M\end{pmatrix}\Phi_4.$$

In the first case, depending on the details of the fine tuning procedure, we have several possibilities. Namely, one can have both the triplets and doublets to be heavy or one of them to be light according to the choice of the matrices S_{ij} and M_{ij}. In the latter case, since $y'_3 = 0$, there is no fine tuning and one has both triplet and doublet to be heavy. All the heavy fields of the theory decouple below the GUT scale.

The requirement of finiteness leads to the unitarity of the matrix S. One can represent an arbitrary unitary matrix in the following form:

$$S = \bar{X}\begin{pmatrix} e^{i\theta_1} & 0 & 0 \\ 0 & e^{i\theta_2} & 0 \\ 0 & 0 & e^{i\theta_3}\end{pmatrix}X^T = \bar{X}DX^T, \quad \bar{X}^T\bar{X} = I, \quad X^TX = I,$$

where X and $\bar{X}$ are some real orthogonal matrices and D is a unitary diagonal matrix. As can be shown, one common phase can be absorbed into the redefinition of the fields. Therefore, in what follows we put $\theta_3 = 0$.

While the unitarity of S is dictated by finiteness, the mass matrix M_{ij} is absolutely arbitrary. Our choice of M is caused by the following requirements:

i) the presence of light Higgs doublets and decoupling of the Higgs triplets;

ii) the absence of Goldstone bosons that may appear if the continuous global flavour symmetry in the Higgs sector is spontaneously broken;

iii) the reduction to the Standard Model at low energies.

To fulfil these requirements we choose the matrix M_{ij} in the form:

$$M = \bar{X}(RI + T'D)X^T, \tag{7}$$

and perform the following fine-tuning procedure:

$$T = T' - \frac{3}{2}y_3V, \quad R \sim T \sim V, \quad R + T = \mu \sim 10^3\ Gev. \tag{8}$$

Note that since in the finite model none of the parameters is running above the GUT scale, the fine-tuning here is more meaningful than in the other GUTs.

To argue that this choice of M satisfies all the afore-mentioned requirements, we analyze the theory below M_{GUT} where $SU(5)$ is spontaneously broken. After decoupling of the heavy triplets, the effective $SU(3)\times SU(2)\times U(1)$ invariant superpotential is:

$$\begin{aligned}\mathcal{L}_{Yukawa} &= \sqrt{\frac{2}{5}}g\Psi_i K_{ij}\overline{\Phi}_i\Lambda_j + \frac{1}{8}\sqrt{\frac{8}{15}}g\Phi_i\Lambda_i\Lambda_i \Rightarrow \\ &\Rightarrow \left(\sqrt{\frac{2}{5}}gQ_j^b K_{ij}\overline{H}_i^a D_i + \sqrt{\frac{2}{5}}gL_i^b\overline{H}_i^a E_i + \sqrt{\frac{8}{15}}gQ_i^b H_i^a U_i\right)\epsilon_{ab}, \qquad (9)\\ \mathcal{L}_{Mass} &= \overline{\Phi}_i M'_{ij}\Phi_j \Rightarrow \overline{H}_i M'_{ij} H_j = \overline{H}_i (X(RI+TD)X^T)_{ij}H_j \qquad (10)\end{aligned}$$

where $M' = M - \frac{3}{2}y_3 VS$, $a,b = 1,2$ are the $SU(2)$ indices and $\epsilon_{12}=1$.

Three pairs of the Higgs doublets have the following quantum numbers:

$$\overline{H}_i(1,2,-\frac{1}{2}) = \begin{pmatrix}\overline{H}_i^0\\ \overline{H}_i^-\end{pmatrix}, \quad H_i(1,2,\frac{1}{2}) = \begin{pmatrix}H_i^+\\ H_i^0\end{pmatrix}.$$

The soft SUSY breaking terms below M_{GUT} take the following form:

$$-\mathcal{L}_{Breaking} = m_0^2\sum_i|\varphi_i|^2 + \frac{1}{2}\left(m_{1/2}\sum_k\lambda_k\lambda_k + h.c.\right) \qquad (11)$$

$$+\left(A_D y_D\widetilde{q}_j^b K_{ij}\overline{H}_i^a\widetilde{d}_i + A_L y_L\widetilde{l}_i^b\overline{H}_i^a\widetilde{e}_i + A_U y_U\widetilde{q}_i^b H_i^a\widetilde{u}_i + B\overline{H}_i^a M'_{ij}H_j^b + h.c.\right)\epsilon_{ab},$$

where we have introduced the notation: $M_5 = m_{1/2}$, φ_i are all the squark and slepton fields and λ_k are the gauginos.

The last term deserves special comment. It contains the mixing of the Higgses in the generation space similar to the quark mixing via the Kobayashi-Maskawa matrix K. This matrix will play the key role in constructing the quark mass spectrum.

The boundary conditions at the GUT scale are:

$$m_0^2 = \frac{1}{3}m_{1/2}^2, \quad A_U = A_D = A_L = -m_{1/2}, \quad B\equiv B_\Phi = -m_{1/2}. \qquad (12)$$

The last equality follows from the fine-tuning requirement for the soft terms at the GUT scale.

Therefore, we end up with the following set of free parameters:

- 3 gauge couplings α_i,
- Mixing matrices K_{ij} and S_{ij},
- Mass terms $m_{1/2}, R, T$.

4.2 The Higgs Potential

The tree level scalar Higgs potential consists of the SUSY part of the Lagrangian and the soft terms

$$V(\overline{H}_i, H_i) = V_{SUSY} + V_{Soft}, \qquad (13)$$

where

$$\begin{aligned}V_{SUSY} &= \overline{H}_i^*\overline{m}_{ij}\overline{H}_j + H_i^* m_{ij}H_j + \frac{g^2+g'^2}{8}(|\overline{H}_i|^2 - |H_i|^2)^2\\ &+\frac{g^2}{4}\left[|\overline{H}_i^*\overline{H}_j|^2 - |\overline{H}_i^*\overline{H}_i|^2 + |H_i^*H_j|^2 - |H_i^*H_i|^2 + 2|\overline{H}_i^*H_j|^2\right] \qquad (14)\end{aligned}$$

with $m_{ij} = (M'^+M')_{ij} = (X((R^2+T^2)I + RT(D^*+D))X^T)_{ij}$, $\overline{m}_{ij} = (M'M'^+)_{ij} = (X((R^2+T^2)I + RT(D^*+D))X^T)_{ij}$ and the soft terms are given by eq(11).

Combining these equations, one gets the following scalar potential:

$$\begin{aligned}V(\overline{H}_i, H_i) &= (m_\phi^2 + R^2 + T^2)|\overline{H}_i|^2 + RT\overline{H}_i^*\left(X(D^*+D)X^T\right)_{ij}\overline{H}_j\\ &+ (m_\phi^2 + R^2 + T^2)|H_i|^2 + RTH_i^*\left(X(D^*+D)X^T\right)_{ij}H_j\\ &+ B\left(\overline{H}_i^a\left(X(RI+TD)X^T\right)_{ij}H_j^b\epsilon_{ab} + h.c.\right) + \frac{g^2+g'^2}{8}(|\overline{H}_i|^2 - |H_i|^2)^2\\ &+ \frac{g^2}{4}\left[|\overline{H}_i^*\overline{H}_j|^2 - |\overline{H}_i^*\overline{H}_i|^2 + |H_i^*H_j|^2 - |H_i^*H_i|^2 + 2|\overline{H}_i^*H_j|^2\right]\end{aligned}$$

Due to our fine-tuning procedure (7,8), this potential still contains the heavy Higgs fields with the masses of an order of the GUT scale. To separate these states, we perform the rotation in the Higgs sector and introduce the new fields $H = XH'$, $\overline{H} = X\overline{H}'$. Doing this, one can rewrite the potential as

$$\begin{aligned}V(\overline{H}'_i, H'_i) &= (m_\phi^2 + R^2 + T^2)|\overline{H}'_i|^2 + RT\overline{H}_i'^*(D^*+D)_{ij}\overline{H}'_j\\ &+ (m_\phi^2 + R^2 + T^2)|H'_i|^2 + RTH_i'^*(D^*+D)_{ij}H'_j \qquad (15)\\ &+ B\left(\overline{H}_i'^a(RI+TD)_{ij}H_j'^b\epsilon_{ab} + h.c.\right) + \frac{g^2+g'^2}{8}(|\overline{H}'_i|^2 - |H'_i|^2)^2\\ &+ \frac{g^2}{4}\left[|\overline{H}_i'^*\overline{H}'_j|^2 - |\overline{H}_i'^*\overline{H}'_i|^2 + |H_i'^*H'_j|^2 - |H_i'^*H'_i|^2 + 2|\overline{H}_i'^*H'_j|^2\right]\end{aligned}$$

The potential (15) is a simple generalization of that of the MSSM [1] but differs from the latter by the extension of the Higgs sector. The electroweak symmetry breaking and the Higgs sector of the broken theory in the models with the Higgs potential of this type have been analyzed in detail in ref. [17]. This potential is positively definite and has no minima different from zero at the GUT scale due to supersymmetry like in the MSSM. However, it develops the non-trivial minima radiatively, thus leading to the radiatively induced spontaneous breaking of the electroweak symmetry, just like in the standard scenario. The parameters of the potential evolve to the low energy values according to the renormalization group equations.

When evolving to low energies the relations between different parameters of the potential change, and, under some conditions, the Higgs fields gain nonzero v.e.v's. From the physical point of view, we are interested in the minima that are achieved on the Higgs field configurations that are gauge equivalent to the neutral real ones, namely $<\overline{H}'_i>= U\begin{pmatrix}\overline{v}_i\\0\end{pmatrix}$, $<H'_i> = U\begin{pmatrix}0\\v_i\end{pmatrix}$, where U is some $SU(2)$ matrix. In this case, the tree level minimization equations take the following form:

$$\frac{1}{2}\frac{\delta V}{\delta \overline{H}_i} = \mathcal{M}^2_{1ij}\overline{v}_j + B\mathcal{M}'_{ij}v_j + \frac{g^2+g'^2}{4}(\overline{v}_k^2 - v_k^2)\overline{v}_i = 0,$$
$$\frac{1}{2}\frac{\delta V}{\delta H_j} = v_i\mathcal{M}^2_{2ij} + \overline{v}_i B\mathcal{M}'_{ij} - \frac{g^2+g'^2}{4}(\overline{v}_k^2 - v_k^2)v_j = 0, \qquad (16)$$

where

$$\mathcal{M}_1^2 = \begin{pmatrix} m^2_{\overline{\phi}} + R^2 + T^2 + 2RT\cos\theta_1 & 0 & 0 \\ 0 & m^2_{\overline{\phi}} + R^2 + T^2 + 2RT\cos\theta_2 & 0 \\ 0 & 0 & m^2_{\overline{\phi}} + (R+T)^2 \end{pmatrix},$$

$$\mathcal{M}_2^2 = \begin{pmatrix} m^2_{\phi} + R^2 + T^2 + 2RT\cos\theta_1 & 0 & 0 \\ 0 & m^2_{\phi} + R^2 + T^2 + 2RT\cos\theta_2 & 0 \\ 0 & 0 & m^2_{\phi} + (R+T)^2 \end{pmatrix},$$

$$\mathcal{M}' = \begin{pmatrix} R + T\cos\theta_1 & 0 & 0 \\ 0 & R + T\cos\theta_2 & 0 \\ 0 & 0 & R+T \end{pmatrix}.$$

The Yukawa part of the superpotential expressed in terms of the new Higgs fields looks like:

$$\mathcal{L}_{Yukawa} = \left(y_D Q^b_j K_{ij} X_{ik} \overline{H}'^a_k D_i + y_L L^b_i X_{ik} \overline{H}'^a_k E_i + y_U Q^b_i X_{ik} H'^a_k U_i\right)\epsilon_{ab}.$$

Due to the fine-tuning convention (7,8) the only real and positive solution of eqs.(16) which gives the v.e.v's of an order of M_Z is (the details are given in the Appendix):

$$v_i = \begin{pmatrix}0\\0\\u\end{pmatrix}, \quad \overline{v}_i = \begin{pmatrix}0\\0\\\overline{u}\end{pmatrix}.$$

Here u and $\overline{u}$ are the same as v_1 and v_2 of the MSSM

$$u = \sqrt{\left(m_1^2 + m_2^2 \pm \sqrt{(m_1^2+m_2^2)^2 - 4B^2\mu^2}\right) F_{\pm}(B^2\mu^2)},$$
$$\overline{u} = -\,\mathrm{sign}(\mu)\sqrt{\left(m_1^2 + m_2^2 \mp \sqrt{(m_1^2+m_2^2)^2 - 4B^2\mu^2}\right) F_{\pm}(B^2\mu^2)},$$

where

$$F_{\pm}(B^2\mu^2) = \frac{1}{g^2+g'^2}\frac{\pm(m_1^2 - m_2^2) - \sqrt{(m_1^2+m_2^2)^2 - 4B^2\mu^2}}{\sqrt{(m_1^2+m_2^2)^2 - 4B^2\mu^2}}$$

and $m_1^2 = m^2_{\overline{\phi}} + \mu^2$, $m_2^2 = m^2_{\phi} + \mu^2$, $\mu = R + T$. One takes "+" sign when $m_1^2 > m_2^2$ and "−" sign in the opposite case. u and $\bar{u}$ obey the usual equations of the MSSM

$$\bar{u} = v\cos\beta, \quad u = v\sin\beta, \quad v^2 = \frac{4}{g^2+g'^2}\frac{m_1^2 - m_2^2\tan^2\beta}{\tan^2\beta - 1}, \quad \sin 2\beta = -2\frac{B\mu}{m_1^2+m_2^2}. \qquad (17)$$

For this minimum the Higgs doublets $\bar{H}'_3$ and H'_3 remain light and are associated with the usual fields H_1 and H_2 of the MSSM [1]. As for the doublets $\bar{H}'_{1,2}$ and $H'_{1,2}$, due to our fine-tuning procedure (7,8) they obtain masses of order of M_{GUT} and decouple below the GUT scale. This is also true for their superpartners.

Re-expressing the original superfields H_i and $\bar{H}_i$ through H'_i and $\bar{H}'_i$:

$$H_i = X_{i1}H'_1 + X_{i2}H'_2 + X_{i3}H'_3, \quad \bar{H}_i = \bar{X}_{i1}\bar{H}'_1 + \bar{X}_{i2}\bar{H}'_2 + \bar{X}_{i3}\bar{H}'_3$$

and discarding the heavy first two terms, we obtain the MSSM with

$$H_1 = \bar{H}'_3, \quad H_2 = H'_3$$

and the usual Yukawa potential

$$\mathcal{L}_{Yukawa} = \left(y_D \bar{n}_i Q^b_j K_{ij} H^a_1 D_i + y_L \bar{n}_i L^b_i H^a_1 E_i + y_U n_i Q^b_i H^a_2 U_i\right)\epsilon_{ab}, \qquad (18)$$

where

$$\bar{n}_i = \bar{X}_{i3}, \quad n_i = X_{i3}, \quad \bar{n}_i^2 = 1, \quad n_i^2 = 1.$$

Due to the degeneracy of the Yukawa couplings the Higgs v.e.v's play the key role in the creation of the quark and lepton mass spectrum. As it appears, the hierarchy of the up quark masses is defined by the vector n_i, while the down quark hierarchy is defined by $\bar{n}_i$. After decoupling of the heavy Higgs fields we end up with the Minimal Supersymmetric Standard Model, where the Yukawa couplings are given by the vacuum expectation values of the Higgs fields

$$y_i^U = n_i y^U, \quad y_i^D = \bar{n}_i y^D, \quad y_i^L = \bar{n}_i y^L, \qquad (19)$$

and $y^U = \frac{4}{\sqrt{15}}g_{GUT}$, $y^D = y^L = \frac{2}{\sqrt{5}}g_{GUT}$ at the GUT scale.

As usual, the quark and lepton masses are defined as eigenvalues of the corresponding mass matrices at low energies. To find them one has to run down the Yukawa matrices taking into account the initial conditions (19) and the generation mixing due to the Kobayashi-Maskawa matrix K. As a result, one gets new nondiagonal Yukawa matrices which again have to be diagonalized to extract the eigenvalues and the low energy Kobayashi-Maskawa mixing matrix.

We would like to stress that in this procedure all the initial information about the quark mass hierarchy is contained in the Higgs sector of the finite unified theory, namely, in the unitary Higgs mixing matrix S.

5 Experimental Constraints

The finite model described above appears to be a mathematically very rigid one. To argue its viability we perform a brief analysis of a compatibility of this model with existing experimental constraints, namely the unification of the gauge couplings, heavy quark and lepton masses, the lower experimental limit on the proton lifetime and the absence of the flavour changing neutral currents (FCNC).

Unification of the gauge couplings

Due to the heaviness of all the extra particles in the Finite model compared to the MSSM and the reduction of the Finite model to the MSSM at low energies, the unification of the gauge couplings takes place exactly in the same manner [3]. The numerical analysis is close to that of the MSSM with large $\tan\beta$ with the RG equations having exactly the same form as in MSSM [5, 6]. The only difference is that due to the finiteness requirement the initial values of the Yukawa couplings are fixed and the soft terms are more restricted compared to the MSSM and hence one has less freedom to fulfill all the requirements simultaneously.

Heavy quarks and lepton masses

One of the motivations for the finite model has been some possible impact on the quark spectrum. Contrary to the MSSM, where the Yukawa couplings are absolutely arbitrary, in our case they are fixed at the GUT scale. So, using the RG equations for the Yukawa matrices, which coincide with those of the MSSM, with the necessary thresholds, one can get their values at lower energies and, therefore, predict the values of the quark and lepton masses.

However, unfortunately, the requirement of finiteness does not fix all the arbitrariness in the Yukawa matrices, namely the Kobayashi-Maskawa matrix K and the vectors n_i and $\bar{n}_i$ remain arbitrary. Practically, one can make some predictions only for the third generation since the vectors n_i and $\bar{n}_i$ are aligned almost along the third axis in the generation space and one can take them in the first approximation to be equal to $n_i = \bar{n}_i = (0,0,1)$ and ignore the light generations. Adjusting the soft parameters and taking into account the light thresholds, one

can arrive at the experimental values of heavy quark and lepton masses. In doing this one has to take into account the difference between the running and the pole masses [18]. Since the starting values of the bottom and top Yukawa couplings, namely y^D and y^U, respectively, are close to each other, the large value of $\tan\beta$ is needed to explain the big difference between the top and bottom masses. The difference between the bottom quark and τ-lepton masses is due to the different renormalization factors.

Predictions of the heavy masses in the finite models and, first of all, the predictions of the top quark mass have first been considered in detail in the papers [19]. Our analysis is not much different from them.

Proton Decay

The lower experimental limit on the proton lifetime is a very rigid criterion of the viability of any GUT model. In the supersymmetric unified theories the proton decay takes place via dimension-five operators that are generated due to the exchange of heavy higgsino colour triplets. In the minimal SUSY $SU(5)$ model it has been analyzed in Ref.[20]. The preferred decay mode proves to be $p \to \bar{\nu} K^+$ [20, 21]. The amplitude of the proton decay is proportional to:

$$B_p \sim \frac{2\alpha_2}{\alpha_3 sin(2\beta)}\left(\frac{m_{\tilde g}}{m_{\tilde q}^2}\right)\frac{3M_{GUT}}{M_{H_3}}10^6, \tag{20}$$

where M_{H_3} is the mass of the heavy colour triplet. Experimentally one has [21]:

$$B_p < (293 \pm 42)\, Gev^{-1}.$$

This constraint can be easily satisfied for low $\tan\beta$, however, for large $\tan\beta$, which is the case of the finite model, one has problems due to the presence of the small factor of $\sin 2\beta$ in the denominator of eq.(20).

In our model, in addition to the dimension-five operators analogous to the minimal model generated by the exchange of the fourth colour triplet, there are additional ones generated by the exchange of the three colour triplets adjusted to each generation. They are mixed via the matrix M_{ij}. To find their contribution, one has to perform diagonalization by rotation of these three colour triplets with the help of the same matrices X and X that were used for their doublet counterparts. Then, proceeding along the lines of Ref.[21, 22] we will get the amplitude of the proton decay in complete analogy with the minimal model:

$$B_p \sim \frac{2\alpha_2}{\alpha_3 sin(2\beta)}\left(\frac{m_{\tilde g}}{m_{\tilde q}^2}\right)3M_{GUT}\left(\frac{1}{M_{H_3}^{(4)}} + 2\frac{X_{21}X_{21}}{M_{H_3}^{(1)}} + 2\frac{X_{22}X_{22}}{M_{H_3}^{(2)}} + 2\frac{X_{23}X_{23}}{M_{H_3}^{(3)}}\right)10^6. \tag{21}$$

Since the masses of all the colour triplets are of the same order of magnitude, one can roughly put

$$M_{H_3}^{(1)} \sim M_{H_3}^{(2)} \sim M_{H_3}^{(3)} \sim M_{H_3}^{(4)} \sim 3M_{GUT},$$

and eq.(21) becomes

$$B_p \sim \frac{2\alpha_2}{\alpha_3 sin(2\beta)} \left(\frac{m_{\tilde{g}}}{m_{\tilde{q}}^2}\right) (1 + 2(\tilde{X}X^T)_{22})\, 10^6.$$

Now taking into account that the product of two orthogonal matrices can always be written as $(\tilde{X}X^T)_{22} = \cos\theta$, we get

$$B_p \sim \frac{2\alpha_2}{\alpha_3 sin(2\beta)} \left(\frac{m_{\tilde{g}}}{m_{\tilde{q}}^2}\right) (1 + 2\cos\theta)\, 10^6. \tag{22}$$

One can easily see from eq.(22) that the additional factor $(1 + 2\cos\theta)$, which we gain in comparison to the minimal SUSY $SU(5)$ model can be used to compensate the smallness of $\sin 2\beta$ in the denominator and in this way to avoid the problem with the proton decay in our model in the case of large $\tan\beta$.

<u>FCNC</u>

The usual problem with the flavour changing neutral currents (FCNC) in the multihiggs models is that due to the flavour mixing in the Yukawa vertices of a general type one cannot avoid the FCNC already at the tree level [23]. The other source of the FCNC is the radiative corrections due to the Higgs mixing. Fortunately, both the mechanisms of FCNC do not cause any trouble in our model. The reason is that, first, the superpotential, eq.(2), is chosen in a way that the Yukawa matrices are diagonal (or become diagonal after the CKM rotation) in the generation space. This property of the superpotential is not changed by the radiative corrections. And, second, since the finite model coincides with the MSSM below the GUT scale, possible additional one-loop contributions to the FCNC different from the MSSM are strongly suppressed. Thus, we face the same problems with FCNC as in the MSSM.

After recent measurement of the branching fraction of the inclusive decay $b \to s\gamma$ [24] special attention has been attracted to this decay. The experimental value is very close to the prediction of the SM which is given by the so-called "penguin" diagrams [25]. This means that an additional contribution from SUSY particles should be suppressed, which leads to a new constraint on the parameters. The situation in the finite model does not differ from the MSSM with large $\tan\beta$. One can meet the needed requirement imposing rather severe constraints on the soft breaking terms. In this case the gluino contribution may be essential.

6 Summary

In conclusion, we summarize the main features of the model. First, the requirement of a general ultraviolet finiteness of the unified theory singles out almost a unique model and makes the theory very rigid. The Yukawa couplings appear to be polynomial functions in terms of a unique $SU(5)$ gauge coupling. Following the paper [16], we impose the universality conditions for the soft supersymmetry breaking terms at the Planck scale and extend the requirement of finiteness to them. This requirement makes the number of free parameters of the theory smaller than one has in the minimal model. To avoid the problem with the gauge couplings unification, which is usual for the theory with enlarged Higgs sector [26], we reduce our model to the MSSM below the GUT scale by the special fine-tuning procedure. This fine-tuning is valid in the unified theory and does not depend on the scale due to the finiteness of the latter, which makes the choice of parameters more meaningful in our model. The low-energy part of the theory, being the exact copy of the MSSM, bears an imprint of the high-energy unified theory, thus resulting in the following relations for the Yukawa couplings at the tree level:

$$y_i^U = n_i y^U, \quad y_i^D = \bar{n}_i y^D, \quad y_i^L = \bar{n}_i y^L.$$

They reduce the hierarchy of the Yukawa couplings in the MSSM to the hierarchy of the vacuum expectation values given by the projections of the vectors n_i and $\bar{n}_i$. These vectors, in their turn, are completely defined by the Higgs sector of the unified theory, namely by the Higgs mixing matrix.

Our main conclusion is that the finite supersymmetric Grand Unified theory, being mathematically very rigid and unique, has passed all the preliminary tests and is proved to be consistent. Being combined with the soft supersymmetry breaking via supergravity, it naturally generates the MSSM with some constraints on its parameters. The novel feature of the model is the presence of additional heavy Higgs particles and the mixing matrix in the Higgs sector which plays the crucial role in the creation of the hierarchy of the Higgs field v.e.v's and via the Higgs mechanism the hierarchy of quark and lepton masses. Another property of this matrix, which we do not discuss here, is its possible contribution to the CP-violation due to the presence of phase factor in S analogous to that in the Kobayashi-Maskawa matrix.

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Appendix

In this appendix we present the explicit solution of the minimization equations (16). As one can see, eqs.(16) contain nonlinearity as a quadratic combination $(\overline{v}_k^2 - v_k^2)$ originated from the potential (15). This is the key property of the system which allows us to solve it analytically. As the first step, let us rewrite eqs.(16) in the matrix form denoting this quadratic combination by x:

$$\begin{aligned} &(\mathcal{M}_1^2 + xI)\overline{v} + \mathcal{M}' v = 0, \\ &(\mathcal{M}_2^2 - xI)v + \mathcal{M}'^T \overline{v} = 0, \\ &x = \frac{g^2 + g'^2}{4}\left(\overline{v}^2 - v^2\right), \end{aligned} \qquad (A1)$$

where v and $\overline{v}$ are the real vectors in the generation space

$$\overline{v} = \begin{pmatrix} \overline{v}_1 \\ \overline{v}_2 \\ \overline{v}_3 \end{pmatrix}, \quad v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}.$$

It is obvious that if eqs.(A1) have a nontrivial solution, the condition

$$det \begin{pmatrix} \mathcal{M}_1^2 + xI & \mathcal{M}' \\ \mathcal{M}'^T & \mathcal{M}_2^2 - xI \end{pmatrix} = 0 \qquad (A2)$$

should be satisfied. Eq.(A2) is the sixth order equation with respect to x; however, it can be easily factorized and solved. Namely, due to the diagonal structure of the matrices $\mathcal{M}_i$ and $\mathcal{M}'$, one has

$$\left[((\mathcal{M}_1^2)_{11} + x)((\mathcal{M}_2^2)_{11} - x) - (\mathcal{M}'_{11})^2\right]\left[((\mathcal{M}_1^2)_{22} + x)((\mathcal{M}_2^2)_{22} - x) - (\mathcal{M}'_{22})^2\right]$$
$$\left[((\mathcal{M}_1^2)_{33} + x)((\mathcal{M}_2^2)_{33} - x) - (\mathcal{M}'_{33})^2\right] = 0,$$

which gives three solutions, respectively,

$$x_i = \frac{1}{2}\left((m_2^2)_i - (m_1^2)_i \pm \sqrt{((m_1^2)_i + (m_2^2)_i)^2 - 4(\mu_i)^2}\right), \quad i = 1,\ 2,\ 3, \qquad (A3)$$

where we have introduced the notation similar to the MSSM:

$$(m_1^2)_i = (\mathcal{M}_1^2)_{ii}, \quad (m_2^2)_i = (\mathcal{M}_2^2)_{ii}, \quad \mu_i = \mathcal{M}'_{ii}.$$

For each of the three x_i given above, the system (A1) is factorized to three independent subsystems, but only one of them has zero determinant and, consequently, nontrivial solution. Hence, there exist three different independent solutions

$$v_1 = \begin{pmatrix} u_1 \\ 0 \\ 0 \end{pmatrix}, \ \overline{v}_1 = \begin{pmatrix} \overline{u}_1 \\ 0 \\ 0 \end{pmatrix}; \ v_2 = \begin{pmatrix} 0 \\ u_2 \\ 0 \end{pmatrix}, \ \overline{v}_2 = \begin{pmatrix} 0 \\ \overline{u}_2 \\ 0 \end{pmatrix};$$
$$v_3 = \begin{pmatrix} 0 \\ 0 \\ u_3 \end{pmatrix}, \ \overline{v}_3 = \begin{pmatrix} 0 \\ 0 \\ \overline{u}_3 \end{pmatrix}.$$

Here u_i and $\overline{u}_i$ are defined as

$$u_i = \sqrt{\left((m_1^2)_i + (m_2^2)_i \pm \sqrt{((m_1^2)_i + (m_2^2)_i)^2 - 4B^2\mu_i^2}\right) F_\pm(B^2\mu_i^2)}. \qquad (A4)$$

$$u_i = -\,\mathrm{sign}(B\mu_i)\sqrt{\left((m_1^2)_i + (m_2^2)_i \mp \sqrt{((m_1^2)_i + (m_2^2)_i)^2 - 4B^2\mu_i^2}\right) F_\pm(B^2\mu_i^2)}. \qquad (A5)$$

where

$$F_\pm(B^2\mu_i^2) = \frac{1}{g^2 + g'^2}\,\frac{\pm((m_1^2)_i - (m_2^2)_i) - \sqrt{((m_1^2)_i + (m_2^2)_i)^2 - 4B^2\mu_i^2}}{\sqrt{((m_1^2)_i + (m_2^2)_i)^2 - 4B^2\mu_i^2}}$$

The arbitrariness in the choice of the sign in eqs. (A4) and (A5) originating from (A2) is fixed in the following way: we take the upper sign if $(m_1^2)_i > (m_2^2)_i$ and the lower sign in the opposite case.

The quantities u_i, u_i are real and positive by definition. In order to get the right-hand sides of eqs. (A4) and (A5) to be real and positive and to have the potential bounded from below in the direction of vanishing quartic terms in (15), the following conditions should be satisfied:

$$\begin{aligned} &(m_1^2)_i + (m_2^2)_i > 2|B\mu_i|, \\ &(m_1^2)_i (m_2^2)_i < B^2\mu_i^2, \end{aligned} \qquad (A6)$$

where the soft breaking parameter B is of an order of $10^2 - 10^3$ Gev.

Since due to our fine-tuning procedure (7,8) the quantities $(m_1^2)_1$, $(m_1^2)_2$, $(m_2^2)_1$, $(m_2^2)_2$, μ_1^2 and μ_2^2 are of an order of M_{GUT}^2, while $(m_1^2)_3$, $(m_2^2)_3$ and μ_3^2 are of an order of M_Z^2, eq.(A6) can be satisfied for the third solution only. This explains our choice of the vacuum solution.

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