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ELECTROPRODUCTION OF ϕ-MESON FROM PROTON WITHIN RELATIVISTIC HARMONIC OSCILLATOR MODEL

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1. Introduction

Conventional phenomenological quark models describing low-energy properties of baryons treat nucleons as consisting of only up and down quarks. So it naturally comes as a big surprise when some recent measurements and theoretical analyses indicate a possible existence of a significant strange quark content in the nucleon. For example, new analysis of the sigma term [1] in pion-nucleon scattering suggests that about one third of the rest mass of the proton comes from $s\bar{s}$ pairs. The EMC measurement of the spin structure function of the proton in deep-inelastic muon scattering [2] has been interpreted as an indication of the strange quark sea $s\bar{s}$ strongly polarized opposite to the nucleon spin, leading to the conclusion that the total quark spin contributes little to the total spin of the proton. A similar conclusion has been drawn from the BNL elastic neutrino-proton scattering [3, 4]. These have prompted a set of new experimental proposals [5] to measure the neutral weak form factors of the nucleon which might be sensitive to the strange quarks inside the nucleon. An intriguing idea as proposed by the guest of honor of this Symposium, Professor Henley and his collaborators [6, 7], is to directly probe the strangeness content of the proton by the lepto- and photo-production of ϕ-meson that is essentially 100% $s\bar{s}$. The idea is to determine the amount of the $s\bar{s}$-admixture in the nucleon by isolating the contribution of direct knockout of $s\bar{s}$ cluster to the measured cross section. Henley *at, al.* [7] carried out a calculation in a non-relativistic quark model (NRQM) that contains some strangeness admixture in the proton. They found that the direct knockout contribution is comparable to the prediction of the vector-meson dominance model (VDM) of diffractive production [8, 9, 10] if a (10–20)% strange quark admixture is assumed.

The knockout contribution is closely related to the hadron form factors which in a non-relativistic quark model depend on the three-momentum transfer squared $\mathbf{q}^2 = -q^2 + \nu^2$, where q and ν are the four-momentum and energy transferred to the hadron system, respectively. In the considered kinematical regions of ϕ-production, the minimal value of $\mathbf{q}^2$ is about 3.6 GeV^2. It is clear that the momentum transfer in this region is too large to use the non-relativistic quark model because its predictions for hadron form factors are in poor agreement with experiment at $\mathbf{q}^2 \geq 0.3$ GeV^2. Furthermore, only the $s\bar{s}$-knockout from the proton was considered in Ref. [7] and the process of direct knockout of the uud-cluster was left out as it was argued to be suppressed.

In this talk, we report a calculation which improves the calculation of Ref. [7] by taking into account the relativistic feature of the process with the use of relativistic harmonic oscillator model (RHOM) [11] which describes the nucleon form factor in a wide kinematical region of the momentum transfer, up to 25 GeV^2, and reproduces the quark counting rule. We find that the direct knockout mechanism is comparable to the VDM diffractive production if only (1–2)% strange quark admixture is assumed. The contribution of direct uud-cluster knockout as well as the interference between the $s\bar{s}$-knockout and the uud-knockout is also calculated and discussed.

2. Kinematics and cross sections

The one-photon exchange diagram for ϕ electroproduction is shown in Fig. 1. The four momenta of the initial electron and proton, final electron and proton, the produced ϕ meson, and the virtual photon are denoted by k, p, k', p', q_ϕ, and q, respectively. In the laboratory frame, we write $k \equiv (E_e, \mathbf{k})$, $k' \equiv (E'_e, \mathbf{k}')$, $p \equiv (E, \mathbf{p})=(M_N, \mathbf{0})$, where M_N is the nucleon mass, $p' \equiv (E', \mathbf{p}')$, $q_\phi \equiv (\omega_\phi, \mathbf{q}_\phi)$, and $q \equiv (\nu, \mathbf{q})$, respectively, θ is the electron scattering angle defined by $\cos\theta = \mathbf{k}\cdot\mathbf{k}'/|\mathbf{k}|\,|\mathbf{k}'|$. The other invariant kinematical variables are $\nu \equiv p\cdot q/M_N = E_e - E'_e$, the minus of photon mass squared $Q^2 \equiv -q^2$, the four-momentum transfer squared to the proton $t \equiv (p-p')^2$, the proton-virtual-photon center-of-mass (CM) energy $W^2 = (p+q)^2$, and the total energy squared in the CM system $s = (p+k)^2$.

In terms of the conventional T-matrix elements T_{fi}, we find the triple-differential cross section of the ϕ electroproduction in the laboratory frame in the form of

$$\frac{d^3\sigma}{dW\,dQ^2dt} = \frac{WE'_eE'\omega_\phi}{4M_N^2E_e|\mathbf{k}||\mathbf{q}|}\frac{1}{(2\pi)^3}\overline{\overline{|T_{fi}|^2}}, \qquad (1)$$

where

$$\overline{\overline{|T_{fi}|^2}} = \int \overline{|T_{fi}|^2}\frac{d\varphi_{p'}}{2\pi}\frac{d\varphi_{k'}}{2\pi} \quad \text{and} \quad \overline{|T_{fi}|^2} = \frac{1}{4}\sum_{m's}|T_{fi}|^2, \qquad (2)$$

with $\varphi_{p',k'}$ the corresponding azimuthal angles.

The diffractive production has been widely used to describe the vector-meson photoproduction and electroproduction [8]. In the VDM of diffractive production process depicted in Fig. 2(a), the virtual photon turns into ϕ meson which then scatters diffractively with the proton through the exchange of a Pomeron. The triple-differential cross section predicted by VDM is [7]

$$\frac{d^3\sigma_{\rm diff}}{dW\,dQ^2dt} = \frac{d^2\sigma_{\rm diff}}{dW\,dQ^2}b_\phi\exp\left\{-b_\phi\left|t - t_-(Q^2)\right|\right\}, \qquad (3)$$

where the exponential slope b_ϕ in ϕ production is determined from the experiment and $t_-(Q^2)$ is the possible maximum value of t (t is always negative). The corresponding double-differential cross section $d^2\sigma_{\rm diff}/dW\,dQ^2$ predicted by the VDM is given as

$$\frac{d^2\sigma_{\rm diff}}{dW\,dQ^2} = (2\pi)\Gamma_W(Q^2,W)\sigma_{\rm diff}(Q^2,W), \qquad (4)$$

$$\sigma_{\rm diff}(Q^2,W) = \frac{\sigma(0,W)}{[1+(Q^2/M_\phi^2)]^2}\frac{p^*_\gamma(0)}{p^*_\gamma(Q^2)}(1+\epsilon R_\phi)\ \exp\{-b_\phi|t_-(Q^2)-t_-(0)|\}, \qquad (5)$$

where $\Gamma_W(Q^2,W)$ is the flux of transverse virtual photons in the laboratory frame, M_ϕ the ϕ mass and ϵ the virtual photon polarization parameter. The factor $(1+\epsilon R_\phi)$ corrects for the fact that photon has a longitudinal component for $Q^2 \neq 0$. The term $p^*_\gamma(0)/p^*_\gamma(Q^2)$, where $p^*_\gamma(Q^2)$ is the virtual photon flux in the photon-hadron CM frame, accounts for the Q^2-dependence of this flux. $\sigma(0,W)$ is the observed photoproduction cross section. The exponential factor arises from the fact that the physical range of t is reduced for $Q^2 > 0$. As in Refs. [7, 8, 9, 10], we will work with the quantities with $\Gamma_W(Q^2,W)$ factored out from the triple and double differential electroproduction cross sections, e.g., $\sigma(Q^2,W) = (d^2\sigma/dW\,dQ^2)/(2\pi\Gamma_W(Q^2,W))$.

Figure 2(b) corresponds to the process where an $s\bar{s}$ pair is directly knocked out by the photon and Fig. 2(c) to the direct uud knockout. It is also possible that the system would go through hadronic intermediate states like N and N^* before ϕ meson is emitted. Such processes may give important contributions and should be studied. Here we focus only on the direct knockout mechanisms and leave the others for future study.

The knockout amplitude T_{fi} in the one photon exchange approximation may be written in the most general form as

$$(2\pi)^4\delta(p+q-p'-q_\phi)T_{fi} = (2\pi)^4\delta(p+q-p'-q_\phi)\left\langle h_f|\hat{J}^h_\mu|h_i\right\rangle\frac{g^{\mu\nu}}{q^2}\left\langle k'|\hat{J}^e_\nu|k\right\rangle, \qquad (6)$$

where $\left\langle f|\hat{J}^{h,e}_\nu|i\right\rangle$ are the hadron and electron electromagnetic current matrix elements, respectively. The electron matrix element is given by

$$\left\langle k'|\hat{J}^e_\nu|k\right\rangle = \sqrt{\frac{M_e^2}{E_eE'_e}}j^e_\nu = \sqrt{\frac{M_e^2}{E_eE'_e}}\overline{u}_{m'}(k')e\gamma_\nu u_m(k) \equiv q^2\mathcal{A}_\nu, \qquad (7)$$

where M_e is the electron mass, $u_m(k)$ the Dirac spinor for the electron (m denotes the spin projection) normalized as $\overline{u}u = 1$, and $\mathcal{A}_\nu$ the "external" electromagnetic field. The hadron electromagnetic current matrix element depends on the model for the description of the initial and final hadron states $|h_{i,f}\rangle$ and the form of the electromagnetic current operator $\hat{J}^h_\mu$. Here we approximate $\hat{J}^h_\mu$ to be the sum of one-body current which enables one to represent the amplitude T_{fi} as

$$T_{fi} = T^{s\bar{s}}_{fi} + T^{uud}_{fi}, \tag{8}$$

where the first term describes the interaction of the electromagnetic field with the s and $\bar{s}$ quarks, i.e., the $s\bar{s}$-cluster knockout, while the second one corresponds to the uud-knockout.

3. Wave functions in the RHOM

Following Ref. [7] we write the constituent quark wave function of hadrons in the Fock space as

$$\begin{aligned} |q_\phi\rangle &= |[s\bar{s}]^1\rangle, \\ |p\rangle &= A|[uud]^{\frac{1}{2}}\rangle + B\left[a_0|\{[uud]^{\frac{1}{2}} \otimes [s\bar{s}]^0\}^{\frac{1}{2}}\rangle + a_1|\{[uud]^{\frac{1}{2}} \otimes [s\bar{s}]^1\}^{\frac{1}{2}}\rangle\right], \end{aligned} \tag{9}$$

where B^2 is the strangeness admixture of the proton and (a_0^2, a_1^2) are the spin-0 and spin-1 fraction of the $s\bar{s}$, respectively. The superscripts $(0,\frac{1}{2},1)$ represent the spin of each cluster. Normalization of the proton wave function gives $A^2 + B^2 = a_0^2 + a_1^2 = 1$. It is supposed that the quarks in clusters are in a relative $1s$-state with respect to the cluster CM. Parity consideration requires that $s\bar{s}$-cluster be in a relative p-wave *w.r.t.* the uud-cluster. Symbol $\otimes$ in Eq. (9) means the vector addition of the spins of the uud and $s\bar{s}$ clusters and their relative orbital angular momentum $\mathbf{L}$ ($\ell = 1$) in a proton. It is also assumed that each quark cluster in Eq. (9) is described by the spin-flavor wave function combined with the totally antisymmetric color SU(3) wave function to form a color singlet. The spatial structure of the hadron is determined by the effective confining quark-quark interactions, which, in the following, is approximated by the relativistic harmonic oscillator potential.

The RHOM as first used in Ref. [12] for the study of the proton form factors, enables one to take into account the Lorentz contraction effect of the composite particle wave function. This essential relativistic effect becomes important at large Q^2 and provides an explanation of the dipole-like Q^2 dependence of the elastic nucleon form factor. Because of this advantage, the RHOM has been widely used for the description of the hadron properties at large momentum transfers [12, 13, 14, 15, 16, 17], in spite of some inherent theoretical difficulties in the model [13, 18].

In RHOM, the spatial motion of a five-quark system is described by

$$\left(\sum_{i=1}^{5} \Box_i - \kappa \sum_{i\neq j=1}^{5} (x_i - x_j)^2 + V_0\right) \Psi = 0, \tag{10}$$

where κ and V_0 are the usual harmonic oscillator model parameters, and the four-vectors squared are defined as $x^2 = x_0^2 - \mathbf{x}^2$ and $\Box = \partial_0^2 - \boldsymbol{\nabla}^2$. The equation can be diagonalized using the relativistic Jacobian coordinates

$$\begin{aligned} &\xi_1 = \tfrac{1}{\sqrt{6}}(x_2 + x_3 - 2x_1), \quad \xi_2 = \tfrac{1}{\sqrt{2}}(x_3 - x_2), \quad \rho = \tfrac{1}{\sqrt{2}}(x_5 - x_4), \\ &\chi = \sqrt{\tfrac{2}{15}}(x_1 + x_2 + x_3) - \sqrt{\tfrac{3}{10}}(x_4 + x_5), \quad X = \tfrac{1}{\sqrt{5}}\sum_{i=1}^{5} x_i, \end{aligned} \tag{11}$$

where the labels $i = 1, 2, 3$ refer to particles in the uud-cluster and $i = 4, 5$ to the $s\bar{s}$-cluster. In terms of the Jacobian coordinates, Eq. (10) becomes

$$\left[\Box_X + \sum_{v's} (\Box_v - \omega_v^2 v^2) + V_0\right] \Psi = 0, \tag{12}$$

with $\omega_v^2 = 5\kappa$ and $v = (\xi_1, \xi_2, \rho, \chi)$. In the rest frame $p = (M_N, \mathbf{0})$, the ground state spatial wave function in the RHOM can be written as

$$\Psi = e^{-iM_N X_0/\sqrt{5}}\Psi_{\text{int},\lambda}(\xi_1, \xi_2, \rho, \chi), \quad \Psi_{\text{int},\lambda}(p; \xi_1, \xi_2, \rho, \chi) = \prod_{v's} \psi(p; v), \tag{13}$$

where

$$\begin{aligned} \psi(v) &= \left(\frac{\omega_v}{\pi}\right) \exp\left\{-\tfrac{1}{2}\omega_v(v_0^2 + \boldsymbol{v}^2)\right\}, \qquad (v = \xi_1, \xi_2, \rho), \\ \psi_\lambda(\chi) &= \sqrt{2\omega_\chi}\left(\frac{\omega_\chi}{\pi}\right) \chi_\lambda \exp\left\{-\tfrac{1}{2}\omega_\chi(\chi_0^2 + \boldsymbol{\chi}^2)\right\}, \end{aligned} \tag{14}$$

and

$$\chi_0 = \chi_z, \quad \chi_{\pm 1} = \mp\frac{1}{\sqrt{2}}(\chi_x \pm i\chi_y),$$

with λ being the projection of the clusters' relative orbital angular momentum $\ell = 1$.

The functions in Eq. (14) can be written covariantly in any arbitrary frame with $p = (E_p, \mathbf{p})$ using the identity

$$-(v_0^2 + \boldsymbol{v}^2) = v^2 - 2\left(\frac{p\cdot v}{M_N}\right)^2. \tag{15}$$

Note that in the covariant wave function of a state with momentum p_μ, the argument of the Hermite polynomial contains the component of the space-like four vector $v_\mu - (p \cdot v/M_N^2)p_\mu$.

With the spin-flavor degrees of freedom we can write the total intrinsic wave function of a five-quark system as

$$\Phi_{\text{int},m_J} = \alpha_{j_{s\bar{s}}} \sum_{\substack{j,m_j,\lambda,\\ m',m_{s\bar{s}}}} \left\langle \tfrac{1}{2}\, m'\, 1\, \lambda \middle| j\, m_j \right\rangle \left\langle j\, m_j\, j_{s\bar{s}}\, m_{s\bar{s}} \middle| \tfrac{1}{2}\, m_J \right\rangle \Psi_{\text{int},\lambda}\, \varphi^{uud}_{\frac{1}{2},m'}\, \varphi^{s\bar{s}}_{j_{s\bar{s}},m_{s\bar{s}}}, \tag{16}$$

where $\alpha_0 = 1$ and $\alpha_1 = \frac{1}{\sqrt{2}}$. Here $\varphi^{uud}_{\frac{1}{2},m'}$ and $\varphi^{s\bar{s}}_{j_{s\bar{s}},m_{s\bar{s}}}$ are the spin-flavor wave functions of the uud- and $s\bar{s}$-clusters, respectively. For simplicity we drop the color wave function. Intrinsic wave function of the three-quark component in a proton as well as the diquark ϕ meson in the final state have the similar structures.

The next point to be considered is the hadron electromagnetic current. The RHOM describes quarks as spinless particles. Therefore, the interaction of the quark spin with an external electromagnetic field has to be introduced into the model by hand. We use the following relativistic prescription for the electromagnetic current operator:

$$\hat{J}^h_\mu = \hat{J}^h_{\text{min},\mu} + \hat{J}^h_{\text{mag},\mu}, \tag{17}$$

where $\hat{J}^h_{\text{min},\mu}$ comes from the usual gauge "minimal substitution," $p_\mu \to p_\mu - eCA_\mu$, in the relativistic equation of motion (10), so that

$$\hat{J}^h_{\text{min},\mu} = C(n_q) \sum_{k=1}^{n_q} e_k [p_{k,\mu}\delta(x - x_k) + \delta(x - x_k) p_{k,\mu}], \tag{18}$$

where n_q is the number of quarks, and the constant C is determined by the normalization condition for elastic scattering hadronic current; i.e., $J^h_0(q) \to e$ as $q \to 0$. It gives $C(n_q) = n_q/2M_N$. The second term in Eq. (17) concerns with the interaction of the quark spins with the external electromagnetic field and has the similar form in both the relativistic and NR models as

$$\hat{J}^h_{\text{mag},\mu} = \left(0,\; i\sum_{k=1}^{n_q} {}_k\mu_k\, \delta(x - x_k) \frac{\boldsymbol{\sigma} \times \boldsymbol{\nabla}}{2M_N}\right), \tag{19}$$

where the magnetic moments μ_k's are chosen to be the same as in the NRQM [7], namely, $\mu_{u,d} = \mu = 3$ and $\mu_s = M_N/M_s \simeq 1.88$, where M_s is the s-quark mass.

4. The amplitudes

Given the proton wavefunction and the electromagnetic current, one can evaluate the T-matrix for the ϕ meson electroproduction. For the $s\bar{s}$ knockout process, the amplitude reads

$$T^{s\bar{s}}_{fi} = -2A^* B a_0 \left\langle \tfrac{1}{2}\, m_f\, 1\, \lambda \middle| \tfrac{1}{2}\, m_i \right\rangle I^{s\bar{s}}_\lambda \frac{ie_s\mu_s}{2M_N} [\mathbf{q} \times \boldsymbol{\mathcal{A}}]_{m_\phi}, \tag{20}$$

where m_ϕ is the spin projection in z-direction of the outgoing ϕ and $I^{s\bar{s}}_\lambda$ is given as

$$(2\pi)^4 \delta(p + q - p' - q_\phi) I^{s\bar{s}}_\lambda = \int dx_1 \ldots dx_5 e^{-iq\cdot x_4} \Psi^*_{p'}(x_1, x_2, x_3) \Psi^*_{q_\phi}(x_4, x_5) \Psi_{p,\lambda}(x_1, \ldots, x_5), \tag{21}$$

which leads to

$$I^{s\bar{s}}_\lambda = \Gamma^{s\bar{s}} F_{s\bar{s}} \psi_{s\bar{s},\lambda}(p'), \tag{22}$$

with

$$\begin{aligned} \Gamma^{s\bar{s}} &= \int d^4\xi_1 d^4\xi_2 \psi^{uud}_f(p'; \xi_1, \xi_2) \psi^{uud}_i(p; \xi_1, \xi_2), \\ F_{s\bar{s}} &= \int d^4\rho e^{-iq\cdot\rho/\sqrt{2}} \psi_\phi(q_\phi; \rho) \psi^{s\bar{s}}_i(p; \rho), \\ \psi_{s\bar{s},\lambda}(p') &= \int d^4\chi e^{i(\sqrt{5/6}p' - \sqrt{3/10}p)\cdot\chi} \psi_\lambda(p; \chi). \end{aligned} \tag{23}$$

Note that $F_{s\bar{s}}$ depends not only on $\mathbf{q}^2(Q^2, W)$ but also on t, because the intrinsic wave function depends on q_ϕ. $\Gamma^{s\bar{s}}$ also depends on t through the p'-dependence of the intrinsic uud cluster wave function in the final state.

From Eq. (20), the spin-averaged amplitude squared is obtained as

$$\overline{|T^{s\bar{s}}_{fi}|^2} = |\frac{2}{3} A B a_0 F_{s\bar{s}} \Gamma^{s\bar{s}} \tilde{V}_{s\bar{s}}(\mathbf{p}') \frac{e^2\mu_s}{Q^2}|^2 \cos^2\frac{\theta}{2} \left\{ f^{s\bar{s}}_1 + f^{s\bar{s}}_2 \tan^2\frac{\theta}{2} \right\}, \tag{24}$$

where $f^{s\bar{s}}_1 = Q^2/4M_N^2$ and $f^{s\bar{s}}_2 = \mathbf{q}^2/2M_N^2$. The momentum distribution $\tilde{V}(\mathbf{p}')$ is related to the momentum distribution of the outgoing proton:

$$|\tilde{V}_{s\bar{s}}(\mathbf{p}')|^2 = \frac{1}{3} \sum_\lambda |\psi_{s\bar{s},\lambda}(\mathbf{p}')|^2. \tag{25}$$

Note that the normalization of $|\tilde{V}(\mathbf{p}')|^2$ is different from the NR one because of the different normalization of the intrinsic spatial wave functions and the additional integration over the time variable in the RHOM [19]. In the NR case we have the usual physical normalization, i.e., one baryon number per unit volume. We keep this normalization in the relativistic case by renormalizing $\tilde{V} \to V = const. \times \tilde{V}$ with

$$\int \frac{d\mathbf{p}}{(2\pi)^3} |V(\mathbf{p})|^2 = 1. \tag{26}$$

The final form of the distribution $|V_{s\bar{s}}|^2$ is

$$\frac{1}{(2\pi)^3}|V_{s\bar{s}}(\mathbf{p})|^2 = \frac{v_{s\bar{s}}(\mathbf{p})}{\int d\mathbf{p} v_{s\bar{s}}(\mathbf{p})}, \quad v_{s\bar{s}}(\mathbf{p}) = \mathbf{p}^2 \exp\left\{-\frac{5}{3\omega_\chi}\left(\mathbf{p}^2 - d_{s\bar{s}} M_N \sqrt{\mathbf{p}^2 + M_N^2}\right)\right\} \tag{27}$$

with $d_{s\bar{s}} = 3/5$, which approaches the NRQM results for $\mathbf{p}^2 \ll M_N^2$ and $\omega_\chi = \frac{7}{6}\omega_\chi^{(NR)}$ [19].

Similarly, we obtain the amplitude of the uud knockout as

$$T_{fi}^{uud} = \frac{A^* B a_1}{\sqrt{2}} \sum_{j,m',\lambda} \left\langle \tfrac{1}{2}\, m'\, 1\, \lambda | j\, m_j \right\rangle \left\langle j\, m_j\, 1\, m_\phi | \tfrac{1}{2}\, m_i \right\rangle I_\lambda^{uud} \left\langle \varphi^{uud}_{\frac{1}{2},m_f} | \mathcal{F}_\mu^{uud} | \varphi^{uud}_{\frac{1}{2},m'} \right\rangle \mathcal{A}^\mu, \tag{28}$$

where the overlap integral I_λ^{uud} can be obtained from Eq. (21) by substituting x_1 for x_4. This gives $I_\lambda^{uud} = \Gamma^{uud} F_{uud} \psi_{uud,\lambda}(q_\phi)$ with

$$\begin{aligned}
\Gamma^{uud} &= \int d^4\rho \psi_\phi(q_\phi;\rho)\psi_i^{s\bar{s}}(p;\rho), \\
F_{uud} &= \int d^4\xi_1 d^4\xi_2 e^{i\sqrt{2/3}q\cdot\xi_1} \psi_f^{uud}(p';\xi_1,\xi_2)\psi_i^{uud}(p;\xi_1,\xi_2), \\
\psi_{uud,\lambda}(q_\phi) &= \int d^4\chi e^{-i(\sqrt{5/6}q_\phi - \sqrt{2/15}p)\cdot\chi} \psi_\lambda(p;\chi).
\end{aligned} \tag{29}$$

The four-vector operator $\mathcal{F}_\mu^{uud}$ reads

$$\mathcal{F}_\mu^{uud} = (\mathcal{F}_0, \mathcal{F}_\parallel + \mathcal{F}_\perp), \tag{30}$$

where

$$\begin{aligned}
\mathcal{F}_0 &= e f_0 = e\frac{5}{6}\left(1 + \frac{E' - \omega_\phi}{M_N}\right), \\
\mathcal{F}_\parallel &= e f_\parallel \frac{\mathbf{q}}{\mathbf{q}^2}, \qquad f_\parallel = e\frac{5(2\mathbf{p}'\cdot\mathbf{q} - \mathbf{q}^2 + 2\nu\mathbf{q}^2/E)}{6M_N}, \\
\mathcal{F}_\perp &= e\frac{5p'\sin\theta_{p'q}}{3M_N}(\hat{\mathbf{x}}\cos\varphi' + \hat{\mathbf{y}}\sin\varphi') + i\frac{\mu}{2M_N}(\boldsymbol{\sigma}\times\mathbf{q}),
\end{aligned} \tag{31}$$

with φ' the azimuthal angle of $\mathbf{q}_\phi$. $\mathcal{F}_\mu^{uud}$ is not gauge invariant and gives wrong results at small Q^2. We amend it with the following gauge invariant modification:

$$\widetilde{\mathcal{F}}_\mu^{uud} = \left(\mathcal{F}_0,\ \widetilde{\mathcal{F}}_\parallel + \mathcal{F}_\perp\right), \qquad \text{with} \qquad \widetilde{\mathcal{F}}_\parallel = e f_0 \frac{\nu\mathbf{q}}{\mathbf{q}^2}, \tag{32}$$

It leads to

$$\overline{\overline{|T_{fi}^{uud}|^2}} = |ABa_1 F_{uud} I_\rho^{uud} V_{uud}(\mathbf{q}_\phi)\frac{e^2}{Q^2}|^2 \cos^2\frac{\theta}{2}\left\{f_1^{uud} + f_2^{uud}\tan^2\frac{\theta}{2}\right\}, \tag{33}$$

where V_{uud} has the same form as Eq. (27) with the substitution $d_{s\bar{s}} \to d_{uud} = \frac{2}{5}$, and $f_{1,2}$ are

$$\begin{aligned}
f_1^{uud} &\simeq (\mu^2 Q^2/4M_N^2)[1 + \frac{f_0}{\mu^2}\frac{Q^2}{\mathbf{q}^2}] \simeq \mu^2 Q^2/4M_N^2, \\
f_2^{uud} &\simeq \mu^2\mathbf{q}^2/2M_N^2.
\end{aligned} \tag{34}$$

We see that in the RHOM the dominant contribution to the matrix element for the uud-knockout comes from the magnetic part of the hadronic current. This explains the successes of some calculations on hadron form factors that use the non-relativistic structure of the quark current operator with the proper form of its magnetic part.

Similarly to the previous cases we find the interference term as

$$\begin{aligned}
\overline{\overline{|T_{fi}^{int}|}} &= |AB|^2 a_0 a_1 \frac{4\mu\mu_s e^4}{3Q^4}\left|F_{uud}F_{s\bar{s}}\Gamma^{uud}\Gamma^{s\bar{s}}V_{uud}(\mathbf{q}_\phi)V_{s\bar{s}}(\mathbf{p}')\cos\theta_{p'q_\phi}\right| \\
&\quad \times c_{\rm int}\cos^2\frac{\theta}{2}\left\{f_1^{s\bar{s}} + f_2^{s\bar{s}}\tan^2\frac{\theta}{2}\right\},
\end{aligned} \tag{35}$$

where $c_{\rm int} \simeq 0.7$, numerically.

The main difference between the non-relativistic results of Ref. [7] and the relativistic ones comes from the different expressions for the overlap integrals F_{uud}, $F_{s\bar{s}}$, Γ^{uud}, and $\Gamma^{s\bar{s}}$. In the evaluation of overlap integrals, we assume that the oscillator parameters ω's are the effective ones, and take $\omega_{\xi,i} = \omega_{\xi,f} = r_p^{-2}$ and $\omega_{\rho,i} = \omega_{\rho,f} = \frac{3}{4}r_{s\bar{s}}^{-2}$. The resulting overlap integrals are

$$F_{uud}(q_1^2,t) = \left(1 - \frac{t}{2M_N^2}\right)^{-2}\exp\left\{-\frac{r_p^2 q_1^2}{6}\right\}, \tag{36}$$

$$F_{s\bar{s}}(q_2^2,t) = \left(\frac{M_\phi}{\omega_\phi}\right)\exp\left\{-\frac{r_{s\bar{s}}^2 q_2^2}{6}\right\}, \tag{37}$$

$$\Gamma^{uud} = \frac{M_\phi}{\omega_\phi}, \quad \Gamma^{s\bar{s}} = \left(1 - \frac{t}{2M_N^2}\right)^{-2}, \tag{38}$$

where

$$q_1^2 = \mathbf{q}^2\frac{E' - \nu}{E'} + \nu^2\left(1 - \frac{\mathbf{p}'^2 - \mathbf{q}_\phi^2}{E'\nu}\right), \quad q_2^2 = \frac{t\mathbf{q}^2}{2M_N\omega_\phi} + \nu^2\left(1 + \frac{\mathbf{p}'^2 - \mathbf{q}_\phi^2}{\omega_\phi\nu}\right). \tag{39}$$

From Eqs. (36-38), it is apparent that (i) the overlap integrals Γ^{uud} and $\Gamma^{s\bar{s}}$ are smaller than 1 over the entire range of t as in the NRQM and $\Gamma^{s\bar{s}}$ decreases with increasing $-t$ while Γ^{uud} increases. This brings some suppression of the RHOM amplitudes; (ii) the overlap integrals F_{uud} and $F_{s\bar{s}}$ depend on the effective momentum transfers q_i^2's, which are functions of t at fixed Q^2 and W. It can be seen from Eq. (39) that $q_1^2 \ll \mathbf{q}^2$ at $t \sim t_{\min}$ and $q_2^2 \ll \mathbf{q}^2$ at $t \sim t_{\max}$, which leads to the strong enhancement of the RHOM results in comparison with the NRQM ones.

5. Results and discussions

We fix the model parameters as follows: dimensional parameter r_p^2 is chosen to reproduce the empirical proton (uud) magnetic form factor up to $Q^2 \simeq 10$ GeV^2, which leads to $r_p = r_p^{exp}/c = 0.53$ fm, and, using the experimental value of $r_p^{exp}(=0.83$ fm), the scale factor is fixed as $c = 1.566$. It is interesting to note that numerically this factor is very close to the scale factor $c = 1.5$ used in the NRQM calculation [7]. Parameter $r_{s\bar{s}}$ is chosen to be $r_{s\bar{s}} = r_\phi/c$ and $r_\phi = 0.45$ fm, with the same scale factor as for the uud cluster. Parameter ω_X is determined for the distribution $V(\mathbf{p})$ to reduce to that of the NRQM at $|\mathbf{p}| \to 0$. The strangeness probability is taken to be $B^2 = 0.02$ assuming that $a_0^2 = a_1^2$.

Displayed in Figs. 3–5 are the RHOM predictions. For comparison, we show both relativistic and NRQM predictions of the Q^2 dependence of $\sigma(Q^2, W)$ [19] in Fig. 3. Here, the VDM cross section is given by solid line and the RHOM predictions on the $s\bar{s}$-knockout, uud-knockout, and the interference are by long-dashed, dash–triple-dotted, and dash–double-dotted lines, respectively, whereas those for the NRQM by dotted, dashed, and dash-dotted lines, respectively. We find that at large transfer momentum, i.e., $Q^2 > 0.5$ GeV^2, the RHOM prediction exceeds the NRQM result and the difference reaches several orders of magnitude. This is mainly due to the new functional dependence of the overlap integrals F_{uud} and $F_{s\bar{s}}$ in the RHOM. The "relativistic modification" of form factors is more crucial for F_{uud} than for $F_{s\bar{s}}$ because the dimensional parameter r in Eq. (36) is larger than that of Eq. (37). Only at small values of Q^2, the RHOM result for $s\bar{s}$ knockout is comparable or even smaller than that of the NRQM prediction. In this region the effect of enhancement in the form factor $F_{s\bar{s}}$ is smaller than the effect of suppression in the overlap integral $\Gamma^{s\bar{s}}$, as was discussed above. In addition, we find that the $s\bar{s}$ knockout prevail over the uud knockout at finite values of Q^2 and that the interference term is small.

In Figs. 4–5 we present the t-dependence of the cross section. At $Q^2 = 0.02$ GeV^2 (Fig.4) RHOM predicts strong enhancement of the uud knockout (dotted line), while the $s\bar{s}$ knockout (dashed line) is smaller than that of NRQM. This is due to the

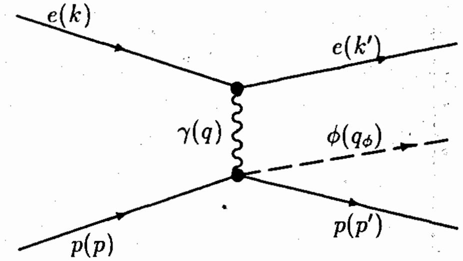

Figure 1: Feynman diagram for the one-photon exchange electroproduction of ϕ meson from the proton.

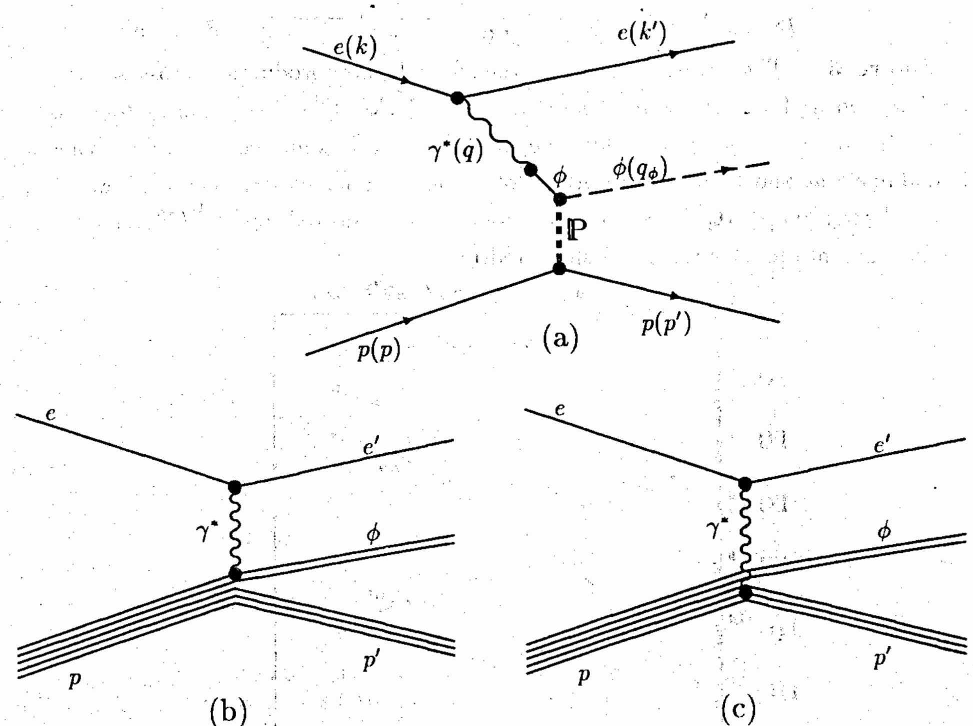

Figure 2: (a) Diffractive ϕ-meson production within vector meson dominance model by means of the Pomeron exchange; (b) $s\bar{s}$-knockout contribution to the electroproduction of ϕ-meson; (c) uud-knockout contribution.

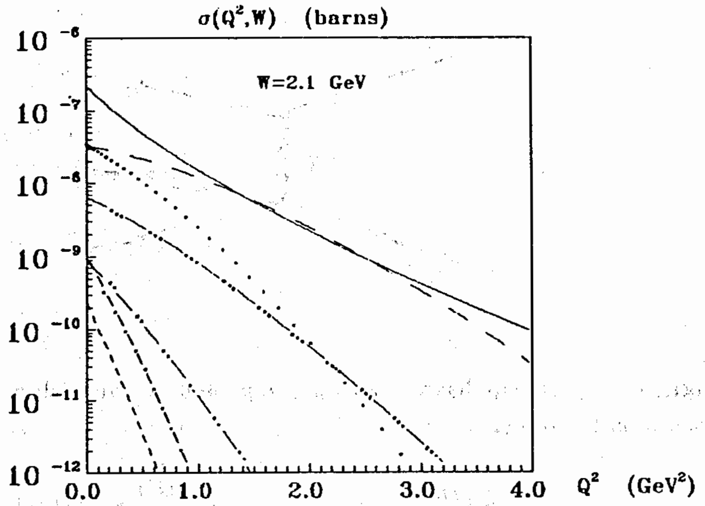

Figure 3: The Q^2-dependence of the virtual photoproduction cross section $\sigma(Q^2, W)$ with W=2.1 GeV in RHOM and in NRQM. The curves are as follows. The diffractive cross section – solid line; RHOM: $s\bar{s}$ knockout cross section – long dashed line, uud knockout – dash-dotted-dotted-dotted line, interference term – dash-dotted-dotted line; NRQM: $s\bar{s}$ knockout cross section – dotted line, uud knockout – dashed line, interference term – dash-dotted line.

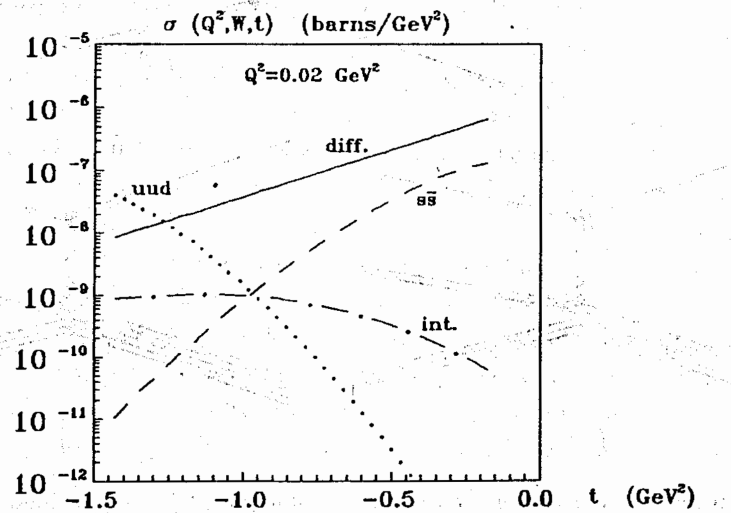

Figure 4: The t-dependence of the virtual photoproduction cross section $\sigma(Q^2, W, t)$ with W=2.1 GeV and Q^2 = 0.02 GeV² in RHOM. The curves are as follows. The diffractive cross section – solid line; $s\bar{s}$ knockout – dashed line; uud knockout – dotted line; interference term – dash-dotted line.

suppression of $\Gamma^{s\bar{s}}$, which increases with increasing $-t$. Therefore, at large $-t$ the uud knockout gives the dominant contribution. The interference term (dash-dotted line) is still negligible. At $Q^2 = 1$ GeV² (Fig. 5) the diffractive production (solid line) and $s\bar{s}$-knockout are close to each other whereas uud-knockout dominates at large $-t$. The contributions of the interference term to the total knockout cross section is not important. We find that the interference term gives no more than 10-15% correction to the total cross section.

By analyzing the cross sections, we conclude that the knockout contribution is comparable to the diffractive VDM ϕ meson production even with 1–2% strangeness probability in proton. Note that this result arises mainly from the Lorentz contraction effect and is not sensitive to the expressions of hadron electromagnetic current in the relativistic model.

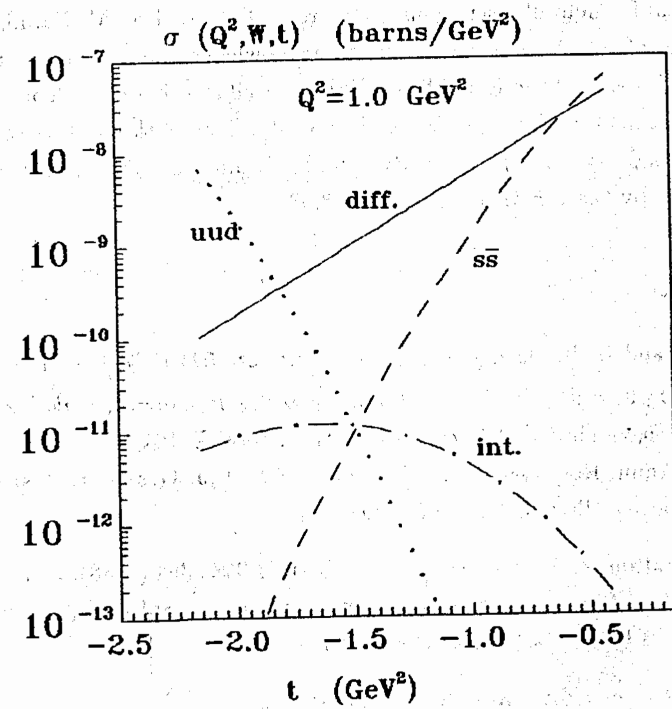

Figure 5: The t-dependence of the virtual photoproduction cross section $\sigma(Q^2, W, t)$ with W=2.1 GeV and Q^2 = 1.0 GeV² in RHOM. Notation is the same as in Fig. 4.

6. Summary

In summary, we re-estimate the electroproduction of the ϕ-meson from proton within the uud-$s\bar{s}$ cluster model. Our calculation is based on the RHOM which takes into account the main relativistic effect, namely, the Lorentz contraction of the composite particle wave function. Our results strongly support the proposal of Henley *et al.* [7], i.e., to use the electro- and photo-production of ϕ-meson for probing the $s\bar{s}$ content of the nucleon. Even with only 2% admixture of strange quarks, the RHOM predicts that the direct knockout mechanisms give comparable contribution to the diffractive production. The strong difference found in the t-dependence of the $s\bar{s}$- and uud-knockout cross sections could be useful for testing the $s\bar{s}$-uud cluster model of the proton and it can be checked in future experiments at CEBAF.

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Электророждение ϕ-мезонов на протонах
в модели релятивистского гармонического осциллятора

Анализируется электророждение ϕ-мезонов на протонах на основе $uud-s\bar{s}$-кластерной модели с целью возможного определения скрытой странности в протоне. Наше рассмотрение основано на модели релятивистского гармонического осциллятора, которая естественным образом учитывает лоренцевское сокращение продольного размера связанной системы. Найдено, что выход ϕ-мезонов при выбивании $s\bar{s}$-компоненты из нуклона сравним по величине с предсказанием дифракционного рождения в модели векторной доминантности в предположении, что скрытая странность составляет величину 1—2%. Причем сечения выбивания uud- и $\bar{s}$-кластеров имеют качественно разную зависимость от переданного протону момента и могут быть различены экспериментально.

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Препринт Объединенного института ядерных исследований. Дубна, 1995

Titov A.I., Yongseok Oh, Shin Nan Yang E2-95-48

Electroproduction of ϕ-Meson from Proton
within Relativistic Harmonic Oscillator Model

We analyze electroproduction of ϕ-meson from a proton within a $uud-s\bar{s}$ cluster model as a probe of the strangeness content of the proton. Our consideration is based on the relativistic harmonic oscillator quark model which takes into account the Lorentz-contraction effect of the hadron wave functions. We show that the knockout mechanisms are comparable to the vector meson-dominance model of diffractive production if only (1—2)% strange quark admixture is assumed. The uud- and $s\bar{s}$-knockout cross sections have a qualitatively different dependence on the four-momentum transfer squared to the proton and may be distinguished experimentally.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR, and at the National Taiwan University, Taipei, Taiwan.

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