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CONSERVED CURRENTS FOR UNCONVENTIONAL SUPERSYMMETRIC COUPLINGS OF SELF-DUAL GAUGE FIELDS

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1. In the standard maximally supersymmetric $N=4$ Yang-Mills theory [1], both the self-dual $(1,0)$ as well as the anti-self-dual $(0,1)$ parts of the Yang-Mills field strength are contained in the *same* supermultiplet. This is not the case in lower N theories, where these two halves of the field strength live in separate irreducible representations of the supersymmetry algebra which, although conjugate in Minkowski space, are independent in spaces having other signatures. This means that pure super Yang-Mills theories with $N \leq 3$ admit super self-dual restriction, i.e. systems of equations which include the Yang-Mills self-duality condition $f_{\dot\alpha\dot\beta}=0$, which are invariant under the N-extended super Poincaré algebra, and which imply the full set of super Yang-Mills equations. The standard $N=4$ theory does not admit such a super self-dual restriction. However there does exist a rather remarkable $N=4$ supersymmetric extension of the self-duality condition [2], which was inspired by string theory [3]. This self-dual theory is independent of the standard maximally supersymmetric Yang-Mills theory [1]. Not only are the equations of motion of the latter theory not implied, but the spectrum of fields differs. The $N=4$ self-dual theory contains an additional spin 1 field, independent of the Yang-Mills vector potential. It is remarkable that gauge invariance allows such a coupling to the vector potential. In standard Yang-Mills theory,

$$\epsilon^{\dot\alpha\dot\gamma}\partial_{\alpha\dot\gamma}f_{\dot\alpha\dot\beta}+\epsilon^{\beta\gamma}\partial_{\gamma\dot\beta}f_{\alpha\beta}= J_{\alpha\dot\beta} ,$$

the conserved vector current $J_{\alpha\dot\beta}$ provides *all* consistent spin 1 couplings [4]. In the absence of any gauge invariances beyond the Yang-Mills one, massless super Yang-Mills multiplets contain, with the exception of the gauge potential $A_{\alpha\dot\beta}$, only fields transforming according to the either the $(s,0)$ or the $(0,s)$ representations of the rotation group, and according to skew-symmetric representations of the internal $SL(N)$ automorphism group of the N-extended supersymmetry algebra. The super self-duality equations up to $N=3$ take the following N-independent forms

$$
\begin{aligned}
f_{\dot\alpha\dot\beta} &= 0 \\
\epsilon^{\alpha\gamma}\nabla_{\gamma\dot\beta}\lambda_{i\alpha} &= 0 \\
\nabla^{\alpha\dot\beta}\nabla_{\alpha\dot\beta}W_{ij} &= \{\lambda_i^\alpha, \lambda_{j\alpha}\} \\
\epsilon^{\dot\alpha\dot\beta}\nabla_{\gamma\dot\beta}\chi_{ijk\dot\alpha} &= [\lambda_{[i\alpha}, W_{jk]}],
\end{aligned}
\tag{16}
$$

where $\nabla_{\gamma\dot\beta} = \partial_{\gamma\dot\beta} + A_{\gamma\dot\beta}$ is the gauge-covariant derivative and we have scaled the gauge coupling constant to one; all fields are gauge algebra valued and are therefore linear in the coupling constant. They are skew-symmetric in the internal $sl(N)$ indices $i,j = 1,\ldots,N$, which we always write as subscripts [2]. This is not always the most economical description of the degrees of freedom, for instance, there is only one scalar for $N=2$, $W_{ij} \equiv \epsilon_{ij}W$, or for $N=3$, three scalars, $W_{ij} \equiv \epsilon_{ijk}W^k$, and one $(0,\frac{1}{2})$ spinor, $\chi_{ijk\dot\alpha} \equiv \epsilon_{ijk}\chi_{\dot\alpha}$. However, this notation, which we use throughout this paper, has the advantage of being N-independent.

The equations (16) imply the full equations of motion of the standard super Yang-Mills theories [1]. They also display an interesting nested structure [5]; the fields W_{ij} which exist for $N \geq 2$ do not occur in the $N<2$ self-duality equations, and the fields $\chi_{ijk\dot\alpha}$ which appear when $N=3$ do not occur in the lower N equations of motion. This nested structure, which led us previously to call this system a *self-dual matreoshka*, is crucial for our present discussion. For in fact the *matreoshka* is even larger because of the following.

If we allow the internal indices to range over four values, $i,j,k = 1,\ldots,4$, then equations (16) imply that the following vector current is covariantly conserved:

$$
J_{ijkl\alpha\dot\beta} = \{\lambda_{[i\alpha}, \chi_{jkl]\dot\beta}\} - [W_{[ij}, \nabla_{\alpha\dot\beta}W_{kl]}], \tag{17}
$$

$$
\text{i.e.}\quad \nabla^{\alpha\dot\beta}J_{ijkl\alpha\dot\beta} = 0 .
$$

This current affords the enhancement of the $N=3$ multiplet to an $N=4$ one by the addition of a spin 1 field $g_{ijkl\dot\alpha\dot\beta}$ satisfying the equation of motion

$$
\epsilon^{\dot\alpha\dot\beta}\nabla_{\alpha\dot\beta}g_{ijkl\dot\alpha\dot\gamma} = J_{ijkl\alpha\dot\gamma} ; \tag{18}
$$

all the previous equations of motion (16) remaining intact. This is precisely the $N=4$ self-dual theory presented by Siegel. The Lorentz-covariant functional

$$
S_{ijkl} = \int d^4x \ \mathrm{Tr}\ \left(f^{\dot\alpha\dot\beta}g_{ijkl\dot\alpha\dot\beta} + \chi^{\dot\alpha}_{[ijk}\nabla_{\alpha\dot\alpha}\lambda^\alpha_{l]} + \frac{1}{2}W_{[ij}\nabla^{\alpha\dot\beta}\nabla_{\alpha\dot\beta}W_{kl]} - W_{[ij}\{\lambda^\alpha_k, \lambda_{l]\alpha}\} \right), \tag{19}
$$

[2] All our (skew-)symmetrisations are with weight one. For instance, $[\lambda_{[i\alpha}, W_{jk]}] \equiv [\lambda_{i\alpha}, W_{jk}] + [\lambda_{j\alpha}, W_{ki}] + [\lambda_{k\alpha}, W_{ij}]$

is an $sl(4)$ singlet and provides an action $S = \epsilon^{ijkl}S_{ijkl}$ for the $N=4$ theory [2]. The two conserved vector currents, from the equations of motion of the two spin 1 fields $A_{\alpha\dot\beta}$ and $g_{\dot\alpha\dot\beta}$, are manifestly independent:

$$
\begin{aligned}
j^{(1)}_{\alpha\dot\beta} &= -\partial^{\dot\alpha}_{\alpha}[A^{\gamma}_{(\dot\alpha}, A_{\gamma\dot\beta)}] , \\
j^{(2)}_{ijkl\alpha\dot\beta} &= (J_{ijkl\alpha\dot\beta} - \epsilon^{\dot\alpha\dot\gamma}[A_{\alpha\dot\gamma}, g_{ijkl\dot\alpha\dot\beta}]) .
\end{aligned}
\tag{20}
$$

Conservation of the latter, i.e. that

$$
\partial^{\alpha\dot\beta}j^{(2)}_{ijkl\alpha\dot\beta} = 0 ,
$$

corresponds to the global gauge-invariance of the functional (19), whereas conservation of the former can be interpreted as a consequence of the global gauge-invariance of the pure Yang-Mills functional when the self-duality conditions are satisfied.

The field equations (16,18), with internal indices now taken to range over five values, similarly imply the covariant constancy of an $N=5$ current

$$
J_{ijklm\alpha\dot\alpha\dot\beta} = [\lambda_{[i\alpha}, g_{jklm]\dot\alpha\dot\beta}] + \frac{2}{3}[\nabla_{\alpha(\dot\alpha}W_{[ij}, \chi_{klm]\dot\beta)}] - \frac{1}{3}[W_{[ij}, \nabla_{\alpha(\dot\alpha}\chi_{klm]\dot\beta)}] , \tag{21}
$$

affording the enhancement of the system (16,18) by the spin $\frac{3}{2}$ equation

$$
\epsilon^{\dot\alpha\dot\beta}\nabla_{\alpha\dot\beta}\psi_{ijklm\dot\alpha\dot\gamma\dot\delta} = J_{ijklm\alpha\dot\gamma\dot\delta} . \tag{22}
$$

The gauge-covariant conservation of $J_{ijklm\alpha\dot\gamma\dot\delta}$ implies the existence of a non-gauge-covariant divergence-free spin-vector current,

$$
j_{ijklm\alpha\dot\gamma\dot\delta} = J_{ijklm\alpha\dot\gamma\dot\delta} - \epsilon^{\dot\alpha\dot\beta}[A_{\alpha\dot\beta}, \psi_{ijklm\dot\alpha\dot\gamma\dot\delta}] .
$$

In turn, an $N=6$ spin 2 field can be introduced, with equation of motion

$$
\begin{aligned}
\epsilon^{\dot\alpha\dot\beta}\nabla_{\alpha\dot\beta}C_{ijklmn\dot\alpha\dot\gamma\dot\delta\dot\eta} = {} & \{\lambda_{[i\alpha}, \psi_{jklmn]\dot\gamma\dot\delta\dot\eta}\} + \tfrac{1}{6}\{\chi_{[ijk(\dot\gamma}, \nabla_{\alpha\dot\delta}\lambda_{lmn]\dot\eta)}\} \\
& + \tfrac{1}{2}[\nabla_{\alpha(\dot\gamma}W_{[ij}, g_{klmn]\dot\delta\dot\eta)}] - \tfrac{1}{6}[W_{[ij}, \nabla_{\alpha(\dot\gamma}g_{klmn]\dot\delta\dot\eta)}].
\end{aligned}
\tag{23}
$$

Again, the right-hand-side is a covariantly conserved current. This pattern continues; and it actually continues *ad infinitum*, essentially because the $(N-1)$ extended system nestles within the N-extended system completely intact, and provides a conserved source current for a new spin $\frac{(N-2)}{2}$ field of dimension $-\frac{N}{2}$. This is a further unconventional feature of these self-dual theories: The dimensions of our fields depend linearly on the spin, whereas in conventional field theories all bosons have dimension -1 and all fermions have dimension $-\frac{3}{2}$. It is tempting to speculate on the significance of the infinite number of local conserved currents in the infinite N limit of a theory with infinitely many spins reminicent of string theories.

The higher spin–vector conserved currents $J_{i_1 \dots i_N \alpha \dot\alpha_1 \dots \dot\alpha_{(N-2)}}$ may be obtained from the vector currents (17) by performing supersymmetry transformations. They therefore also essentially owe their existence to the gauge–invariant functionals S_{ijkl}, which are extremised by solutions of the equations of motion even for $N > 4$, in spite of the fact that higher spin fields manifestly do not contribute to them. The supersymmetry transformations of the $N = 6$ equations are given by

$$
\begin{aligned}
\delta A_{\alpha\dot\beta} &= -\bar\eta^{i}_{\dot\beta}\lambda_{i\alpha} \\
\delta\lambda_{j\alpha} &= \eta^{\beta}_{j} f_{\alpha\beta} + 2\bar\eta^{i\dot\beta}\nabla_{\alpha\dot\beta}W_{ij} \\
\delta W_{jk} &= \eta^{\alpha}_{[j}\lambda_{k]\alpha} + \bar\eta^{i\dot\beta}\chi_{ijk\dot\beta} \\
\delta\chi_{jkl\dot\alpha} &= \eta^{\alpha}_{[j}\nabla_{\alpha\dot\alpha}W_{kl]} + \bar\eta^{i\dot\beta}(g_{ijkl\dot\alpha\dot\beta} + \epsilon_{\dot\alpha\dot\beta}[W_{i[j}, W_{kl]}]) \\
\delta g_{jklm\dot\alpha\dot\beta} &= \eta^{\alpha}_{[j}\nabla_{\alpha(\dot\alpha}\chi_{klm]\dot\beta)} \\
&\quad + \bar\eta^{i\dot\gamma}(\psi_{ijklm\dot\alpha\dot\beta\dot\gamma} + \epsilon_{\dot\gamma(\dot\alpha}(\tfrac{2}{3}[W_{i[j}, \chi_{klm]\dot\beta)}] - \tfrac{1}{3}[W_{[jk}, \chi_{lm]i\dot\beta)}])) \qquad (24) \\
\delta\psi_{jklmn\dot\alpha\dot\beta\dot\gamma} &= \eta^{\alpha}_{[j}\nabla_{\alpha(\dot\alpha}g_{klmn]\dot\beta\dot\gamma)} + \bar\eta^{i\dot\delta}C_{ijklmn\dot\alpha\dot\beta\dot\gamma\dot\delta} - \bar\eta^{i}_{(\dot\alpha}(\tfrac{1}{2}[W_{i[j}, g_{klmn]\dot\beta\dot\gamma)}] \\
&\quad + \tfrac{1}{6}[W_{[jk}, g_{lmn]i\dot\beta\dot\gamma)}] + \tfrac{1}{6}[\chi_{i[jk\dot\beta}, \chi_{lmn]\dot\gamma)}]) \\
\delta C_{jklmnp\dot\alpha\dot\beta\dot\gamma\dot\delta} &= \eta^{\alpha}_{[j}\nabla_{\alpha(\dot\alpha}\psi_{klmnp]\dot\beta\dot\gamma)} \\
&\quad - \bar\eta^{i}_{(\dot\alpha}(\tfrac{2}{5}[W_{i[j}, \psi_{klmnp]\dot\beta\dot\gamma\dot\delta)}] - \tfrac{1}{10}[W_{[jk}, \psi_{lmnp]i\dot\beta\dot\gamma\dot\delta)}] \\
&\quad - \tfrac{1}{15}[\chi_{[jkl\dot\beta}, g_{mnp]i\dot\gamma\dot\delta)}] - \tfrac{1}{10}[\chi_{i[jk\dot\beta}, g_{lmnp]\dot\gamma\dot\delta)}]) .
\end{aligned}
$$

These are most conveniently obtained from the superspace formulation of these theories, where superfield versions of the functionals S_{ijkl} exist, generalising the $N = 4$ superspace actions [2, 6]. Our arbitrarily extended self-duality equations may be compactly expressed in terms of the chiral superspace curvature constraints

$$
\begin{aligned}
\{\nabla_{i\dot\alpha}, \nabla_{j\dot\beta}\} &= \epsilon_{\dot\alpha\dot\beta} f_{ij} \\
[\nabla_{i\dot\alpha}, \nabla_{\beta\dot\beta}] &= \epsilon_{\dot\alpha\dot\beta} f_{i\beta} \qquad (25) \\
[\nabla_{\alpha\dot\alpha}, \nabla_{\beta\dot\beta}] &= \epsilon_{\dot\alpha\dot\beta} f_{\alpha\beta} ,
\end{aligned}
$$

where $\nabla_{i\dot\alpha} = \frac{\partial}{\partial\vartheta_{i\dot\alpha}} + A_{i\dot\alpha}$, $\nabla_{\alpha\dot\alpha} = \frac{\partial}{\partial x_{\alpha\dot\alpha}} + A_{\alpha\dot\alpha}$ are chiral superspace gauge-covariant derivatives. We shall present a proof of the equivalence of these constraints to the equations of motion (16, 18, 22, 23) in a separate publication.

2. For $N = 0$ self-duality the stress tensor vanishes identically

$$T_{\alpha\dot\alpha,\beta\dot\beta} \equiv \mathrm{Tr}\, f_{\dot\alpha\dot\beta} f_{\alpha\beta} \equiv 0.$$

This remains true for all supersymmetrisations up to $N = 3$. However, the appearance of the invariant functional (19) for $N = 4$ resurrects the stress tensor. For $N \geq 4$, in fact, there exist $\binom{N}{4}$ second rank traceless (i.e. satisfying

$\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}T_{ijkl\alpha\dot\alpha,\beta\dot\beta} = 0$) conserved tensors, corresponding to this number of invariant functionals S_{ijkl} :

$$
\begin{aligned}
T_{ijkl\alpha\dot\alpha,\beta\dot\beta} = \mathrm{Tr}\,(& g_{ijkl\dot\alpha\dot\beta} f_{\alpha\beta} + \nabla_{\alpha\dot\beta}\lambda_{[i\beta}\chi_{jkl]\dot\alpha} - \lambda_{[i\alpha}\nabla_{\beta\dot\alpha}\chi_{jkl]\dot\beta} \\
& + \tfrac{1}{2}\lambda_{[i\beta}\nabla_{\alpha\dot\alpha}\chi_{jkl]\dot\beta} - \tfrac{1}{2}\nabla_{\alpha\dot\alpha}\lambda_{[i\beta}\chi_{jkl]\dot\beta} \\
& - \tfrac{1}{3}\nabla_{(\alpha\dot\alpha}W_{[ij}\nabla_{\beta)\dot\beta}W_{kl]} + \tfrac{1}{3}W_{[ij}\nabla_{\alpha\dot\alpha}\nabla_{\beta\dot\beta}W_{kl]} \\
& + \tfrac{2}{3}\epsilon_{\dot\beta\dot\alpha}\epsilon_{\beta\alpha}\{\lambda^{\gamma}_{[i}, \lambda_{j\gamma}\}W_{kl]}).
\end{aligned}
\qquad (26)
$$

In fact there exist three second rank conserved tensors,

$$
\begin{aligned}
T^{(1)}_{ijkl\alpha\dot\alpha,\beta\dot\beta} &= \mathrm{Tr}\,(g_{ijkl\dot\alpha\dot\beta} f_{\alpha\beta} - \lambda_{[i\alpha}\nabla_{\beta\dot\alpha}\chi_{jkl]\dot\beta} - \nabla_{\beta\dot\alpha}W_{[ij}\nabla_{\alpha\dot\beta}W_{kl]}) \\
T^{(2)}_{ijkl\alpha\dot\alpha,\beta\dot\beta} &= \mathrm{Tr}\,(\tfrac{1}{2}\lambda_{[i\beta}\nabla_{\alpha\dot\alpha}\chi_{jkl]\dot\beta} - \tfrac{1}{2}\nabla_{\alpha\dot\alpha}\lambda_{[i\beta}\chi_{jkl]\dot\beta} + \nabla_{\alpha\dot\beta}\lambda_{[i\beta}\chi_{jkl]\dot\alpha} \\
&\quad + \epsilon_{\beta\alpha}\epsilon_{\dot\beta\dot\alpha}\{\lambda^{\gamma}_{[i}, \lambda_{j\gamma}\}W_{kl]}) \qquad (27) \\
T^{(3)}_{ijkl\alpha\dot\alpha,\beta\dot\beta} &= \mathrm{Tr}\,\tfrac{1}{3}(W_{[ij}\nabla_{\alpha\dot\alpha}\nabla_{\beta\dot\beta}W_{kl]} - \nabla_{\alpha\dot\alpha}W_{[ij}\nabla_{\beta\dot\beta}W_{kl]} \\
&\quad + 2\nabla_{\beta\dot\alpha}W_{[ij}\nabla_{\alpha\dot\beta}W_{kl]} - \epsilon_{\beta\alpha}\epsilon_{\dot\beta\dot\alpha}\{\lambda^{\gamma}_{[i}, \lambda_{j\gamma}\}W_{kl]}) ,
\end{aligned}
$$

of which the sum (26) is the unique traceless linear combination. (The Tr in these expressions denotes of course the gauge algebra trace). These gauge–invariant tensors have conserved superpartners. The lower rank conserved spin–tensors are

$$
\begin{aligned}
T_{ijk\alpha\dot\alpha,\beta} &= \mathrm{Tr}\,(2f_{\alpha\beta}\chi_{ijk\dot\alpha} - \nabla_{\alpha\dot\alpha}\lambda_{[i\beta}W_{jk]} + \lambda_{[i\beta}\nabla_{\alpha\dot\alpha}W_{jk]} \\
&\quad - 2\lambda_{[i\alpha}\nabla_{\beta\dot\alpha}W_{jk]}) \\
T^{(1)}_{ijklm\alpha\dot\alpha,\dot\beta} &= \mathrm{Tr}\,(4\lambda_{i\alpha}g_{jklm\dot\alpha\dot\beta} - \lambda_{[j}g_{klm]i\dot\alpha\dot\beta} - 4\chi_{i[jk\dot\alpha}\nabla_{\alpha\dot\beta}W_{lm]} \\
&\quad + 6\nabla_{\alpha\dot\beta}W_{i[j}\chi_{klm]\dot\alpha} - 5\epsilon_{\dot\alpha\dot\beta}W_{i[j}[\lambda_{k\alpha}, W_{lm]}]) \\
T^{(2)}_{ijklm\alpha\dot\alpha,\dot\beta} &= \mathrm{Tr}\,(\nabla_{\alpha\dot\alpha}W_{i[j}\chi_{klm]\dot\beta} - W_{i[j}\nabla_{\alpha\dot\alpha}\chi_{klm]\dot\beta} - 2\nabla_{\alpha\dot\beta}W_{i[j}\chi_{klm]\dot\alpha} \qquad (28) \\
&\quad + 2\epsilon_{\dot\alpha\dot\beta}W_{i[j}[\lambda_{k\alpha}, W_{lm]}]) \\
T^{(3)}_{ijklm\alpha\dot\alpha,\dot\beta} &= \mathrm{Tr}\,(\nabla_{\alpha\dot\alpha}\chi_{i[jk\dot\beta}W_{lm]} - \chi_{i[jk\dot\beta}\nabla_{\alpha\dot\alpha}W_{lm]} + 2\chi_{i[jk\dot\alpha}\nabla_{\alpha\dot\beta}W_{lm]} \\
&\quad + 2\epsilon_{\dot\alpha\dot\beta}W_{i[j}[\lambda_{k\alpha}, W_{lm]}]) \\
T_{ijkl\alpha\dot\alpha} &= \mathrm{Tr}\,(3\lambda_{i\alpha}\chi_{jkl\dot\alpha} + \lambda_{[j\alpha}\chi_{kl]i\dot\alpha} + 2\nabla_{\alpha\dot\alpha}W_{i[j}W_{kl]} - 2W_{i[j}\nabla_{\alpha\dot\alpha}W_{kl]}) .
\end{aligned}
$$

All these tensors satisfy the conservation law

$$\partial^{\alpha\dot\alpha}T_{i\dots m\alpha\dot\alpha,\dots} = 0$$

in virtue of the equations of motion, and they may be used to couple these self-dual gauge theories to gravity and supergravity.

3. The free (but massive) versions of the higher spin equations (18, 22, 23) were considered a long time ago by Dirac [7] and by Fierz [8]; and the problems of consistently coupling such fields to an external electromagnetic field were discussed. In the zero rest-mass limit, corresponding to the zero coupling limit of our equations,

$$\partial_\gamma^{\dot\alpha_1} f_{\dot\alpha_1 \ldots \dot\alpha_{2s}} = 0 \,, \tag{29}$$

there is no problem in consistently coupling an external *self-dual* Yang-Mills field to such spinors (this of course requires the background space to have signature (4,0) or (2,2)). The replacement of $\partial_{\alpha\dot\alpha}$ in (29) by the gauge–covariant derivative requires the satisfaction of precisely the self-duality equation $f_{\dot\alpha\dot\beta} = 0$ for consistency. Such a coupling of a zero rest–mass spinor to a *self-dual* vector potential appears to be the unique consistent coupling of the type which Dirac attempted to find. If a non-zero source current J is present, the further consistency condition for a minimal gauge coupling is the gauge–covariant constancy of the current. As we have seen supersymmetric self-dual Yang-Mills theory automatically provides such conserved currents. In fact all our higher spin equations (18, 22, 23) have the general form

$$\partial_\alpha^{\dot\alpha_1} \varphi_{i_1 \ldots i_{(2s+2)} \dot\alpha_1 \dot\alpha_2 \ldots \dot\alpha_{2s}} = j_{i_1 \ldots i_{(2s+2)} \alpha \dot\alpha_2 \ldots \dot\alpha_{2s}} \,, \quad s \geq 1, \tag{30}$$

where the current on the right is a functional of all fields of spin $\leq s$ (including the Yang-Mills vector potential). Now differentiating on the left,

$$\partial^{\alpha\dot\alpha_2} \partial_\alpha^{\dot\alpha_1} \varphi_{i_1 \ldots i_{(2s+2)} \dot\alpha_1 \dot\alpha_2 \ldots \dot\alpha_{2s}} = \frac{1}{2} \epsilon^{\dot\alpha_1 \dot\alpha_2} \partial^{\gamma\dot\beta} \partial_{\gamma\dot\beta} \varphi_{i_1 \ldots i_{(2s+2)} \dot\alpha_1 \dot\alpha_2 \ldots \dot\alpha_{2s}} \,,$$

clearly shows that the consistency condition for the linear equation (30) is precisely the conservation of the current on the right, since φ is symmetric in its dotted spinor indices whereas ϵ is skew–symmetric. This is analogous to the consistency requirement in conventional theories. For instance, for zero mass vector fields, consistency of the equation $\partial^\mu F_{[\mu\nu]} = J_\nu$ implies $\partial^\nu J_\nu = 0$ in virtue of the antisymmetry of $F_{\mu\nu}$.

The zero rest-mass Dirac-Fierz equation (29) has also been studied by Penrose [9] who discussed the possible geometrical significance of the spin 2 case. It remains an intruiguing open question whether any relation exists between our $N = 6$ theory and his considerations. We note, however, that our $N = 5$ theory is probably the unique supersymmetric theory in which a spin $\frac{3}{2}$ field is coupled to a vector field, without requiring a spin 2 coupling as well for consistency [10]. Traditional theorems forbidding higher–spin couplings do not apply to our systems since these self-dual theories have only one coupling constant (the Yang-Mills one) and only one type of gauge–invariance (also the Yang-Mills one). Even for $N = 5$ or $N = 6$ there is no further coupling constant and no additional gauge–invariance. The fields

$\psi_{\dot\alpha\dot\beta\dot\gamma}$ and $C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$, being gauge algebra valued, transform covariantly under Yang-Mills gauge transformations and have dimensions $-\frac{5}{2}$ and -3 respectively. It is these high negative dimensionalities which render it impossible to write down action functionals for these higher–spin fields. We should note, however, that *locally* supersymmetric versions of our higher spin theories are also possible, having spin 1, spin $\frac{3}{2}$, and spin 2 gauge–invariances, as well as both Yang-Mills and gravitational coupling constants. We are currently investigating such $N \geq 8$ self-dual supergravities.

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