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EMISSION OF TWO HARD PHOTONS IN LARGE-ANGLE BHABHA SCATTERING

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1 Introduction

The large–angle Bhabha process is well suited for the determination of the luminosity $\mathcal{L}$ at e^+e^- colliders of the intermediate energy range $\sqrt{s} = 2\varepsilon \sim 1\text{GeV}$ [1,2]. As far as 0.1% accuracy is needed in the determination of $\mathcal{L}$ the corresponding requirement

$$\left|\frac{\Delta\sigma}{\sigma}\right| \leq 10^{-3} \tag{1}$$

on the Bhabha cross section theoretical description appears. $\Delta\sigma$ is the unknown uncertainty in the cross section due to higher order radiative corrections. A great attention was paid to this process during the last decades [3]. The Born cross section with weak interactions taken into account as well as the radiative corrections to it, including the emission of a single virtual photon, soft and hard real one, where studied in detail [4]. Both contributions, the one reinforced by "the large logarithmic multiplier" $\mathrm{L} = \ln(s/m^2)$ (where $s = (p_+ + p_-)^2 = 4\varepsilon^2$ is thetotal center–of–mass (CM) energy square, m is the electron mass), and the one without L are to be kept in frames (1): $\alpha L/\pi$, α/π. As for the corrections in the second order of the perturbation theory, they are necessary in the leading and next–to–leading approximations and take the following orders respectively:

$$(\frac{\alpha}{\pi})^2 L^2, \quad (\frac{\alpha}{\pi})^2 L. \tag{2}$$

The total two–loop ($\sim (\alpha/\pi)^2$) correction could be constructed from: 1) the two–loop corrections arising from the emission of two virtual photons; 2) the one–loop corrections to a single real (soft and hard) photon emission; 3) the ones arising from the emission of two real photons; 4) the virtual and real e^+e^- pair production. As for the corrections in third order of the perturbation theory, only the leading ones proportional to $(\alpha L/\pi)^3$ are to be taken into account. Their calculation can be performed by means of the electron structure functions method [4].

In this paper we consider the emission of two real hard photons:

$$e^+(p_+) + e^-(p_-) \to e^+(q_+) + e^-(q_-) + \gamma(k_1) + \gamma(k_2). \tag{3}$$

The relevant contribution to the "experimental" cross section has the following form

$$\sigma_{\text{exp}} = \int d\sigma \Theta_+ \Theta_-, \tag{4}$$

where Θ_+ and Θ_- are the experimental restrictions providing the simultaneous detection of both scattered electron and positron. At first that means that their energy fractions should be larger then a certain (small) quantity $\varepsilon_{\text{th}}/\varepsilon$, ε_{th} is the energy threshold of the detectors. The second condition restricts their angles in respect to the beam axes, they should be larger then a certain finite value ψ_0 ($\psi_0 \sim 35°$ in the experimental conditions accepted in [1]):

$$\pi - \psi_0 > \theta_-, \theta_+ > \psi_0, \quad \theta_\pm = \widehat{\mathbf{q}_\pm \mathbf{p}_-}, \tag{5}$$

where $\theta_\pm$ are the polar angles of the scattered leptons in respect to the beam axes ($\mathbf{p}_-$). We accept the condition on the energy threshold of the charged particles registration:

$$q^0_\pm > \varepsilon_{\text{th}}, \qquad \varepsilon_{\text{th}} \ll \varepsilon.$$

Both photons are assumed to be hard, their minimal energy

$$\omega_{\min} = \Delta\varepsilon, \qquad \Delta \ll 1, \tag{6}$$

could be considered as the threshold of the photon registration.

The main ($\sim (\alpha L/\pi)^2$) contribution to the total cross section (5) arises from the collinear region: when both emitted photons move within narrow cones along the charged particle momenta (they may go along the same particle). So we will distinguish 16 kinematical regions:

$$\widehat{\mathrm{ak}_1} \text{ and } \widehat{\mathrm{ak}_2} < \theta_0, \qquad \widehat{\mathrm{ak}_1} \text{ and } \widehat{\mathrm{bk}_2} < \theta_0, \tag{7}$$
$$\frac{m}{\varepsilon} \ll \theta_0 \ll 1, \qquad a \neq b, \quad a, b = p_-, p_+, q_-, q_+ .$$

The summed over spin states matrix element module square in the regions (7) have the form of the Born ones multiplied by the so called collinear factors. The contribution to the cross section of each region has also the form of $2 \to 2$ Bhabha cross sections in the Born approximation multiplied by factors of the form

$$d\sigma_i^{\mathrm{coll}} = d\sigma_{0i}\big[a_i(x_j, y_j)\ln^2(\frac{\varepsilon^2\theta_0^2}{m^2}) + b_i(x_j, y_j)\ln(\frac{\varepsilon^2\theta_0^2}{m^2})\big], \tag{8}$$

where $x_j = \omega_j/\varepsilon$, $y_1 = q_-^0/\varepsilon$, $y_2 = q_+^0/\varepsilon$ are the energy fractions of the photons and of the scattered electron and positron. The dependence on the auxiliary parameter θ_0 will be canceled in the sum of the contributions of the collinear and semi-collinear regions. The last region corresponds to the kinematics, when only one of the photons is emitted inside the narrow cone $\theta_1 < \theta_0$ along one of the charged particle momenta and the second photon is emitted outside of any such a cone along charged particles ($\theta_2 > \theta_0$):

$$d\sigma_i^{\mathrm{SC}} = \frac{\alpha}{\pi}\ln(\frac{4\varepsilon^2}{m^2})d\sigma_{0i}^{\gamma}(k_2), \tag{9}$$

where $d\sigma_{0i}^{\gamma}$ has the known form of the single hard bremsstrahlung cross section in the Born approximation [5].

We show below explicitly that the result of the integration over the single hard photon emission in eq. (9) in the kinematical region $\theta_2^i > \theta_0$ (θ_2^i is the emission angle of the second hard photon in respect to the direction of one of the four charged particles) has the following form

$$\int d\sigma_{0i}^{\gamma}(k_2) = -2\ln(\theta_0^2)a_i(x, y)d\sigma_0^i + d\tilde{\sigma}^i. \tag{10}$$

The collinear factors in the double bremsstrahlung process were firstly considered in papers of the CALKUL collaboration [6]. Unfortunately they have rather complicate form, which is less convenient for further analytical integration in comparison with the expressions given below. Calculation of the collinear factors may be considered as a generalization of the quasi-real electron method [7] for the case of a multiple bremsstrahlung. Another generalization is needed for the calculations of the cross section of the process $e^+e^- \to e^+e^-e^+e^-$. We will consider it in a separate paper.

It is interesting to note that the collinear factors for the kinematical region of the two hard photons emission along the projectile and the scattered electron are found to be the same as for the electron–proton scattering process considered by one of us (N.P.M.) in paper [8].

There are 40 tree level Feynman diagrams which describe the double bremsstrahlung process in e^+e^- collisions. The expression for the differential cross section in terms of helicity amplitudes was computed about ten years ago [6,9]. It has a very complicated form. We note that the contribution from the kinematical region in which the angles (in the CM system) between any two final particles are large compared with m/ε has the magnitude of the order

$$\frac{\alpha^2 r_0^2 m^2}{\pi^2\varepsilon^2} \sim 10^{-36}\mathrm{cm}^2, \tag{11}$$

(r_0 is the classical electron radius). So, the corresponding events will have poor statistics at the colliders with the luminosity $\mathcal{L} \sim 10^{31} - 10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1}$. More probable are the cases of double bremsstrahlug imitating the processes $e^+e^- \to e^+e^-$ or $e^+e^- \to e^+e^-\gamma$. That corresponds to the emission of one or two photons along charged particles momenta.

2 Kinematics in the collinear region

It is convenient to introduce in the collinear region new variables and transform the phase volume of the final state in the following way (here and further we will work in the CM system):

$$\int d\Gamma = \int \frac{d^3q_-d^3q_+d^3k_1d^3k_2}{16q_-^0q_+^0\omega_1\omega_2(2\pi)^8}\delta^4(\eta_1p_- + \eta_2p_+ - \lambda_1q_- - \lambda_2q_+)$$
$$= \frac{m^4\pi^2}{4(2\pi)^6}\int\limits_{\Delta}^{1}dx_1\int\limits_{\Delta}^{1}dx_2x_1x_2\int\limits_0^{2\pi}\frac{d\phi}{2\pi}\int\limits_0^{z_0}dz_1\int\limits_0^{z_0}dz_2\int d\Gamma_q, \tag{12}$$
$$\int d\Gamma_q = \int \frac{d^3q_-d^3q_+}{4q_-^0q_+^0(2\pi)^2}\delta^4(\eta_1p_- + \eta_2p_+ - \lambda_1q_- - \lambda_2q_+),$$
$$z_{1,2} = (\frac{\theta_{1,2}\varepsilon}{m})^2, \qquad \phi = \widehat{\mathrm{k}_{1\perp}\mathrm{k}_{2\perp}}, \qquad x_i = \frac{\omega_i}{\varepsilon}, \qquad z_0 \gg 1, \qquad \Delta = \frac{\varepsilon_{\mathrm{th}}}{\varepsilon}$$

where θ_i ($i = 1, 2$) is the polar angle of the i–photon emission in respect to the momentum of the charged particle which emitted the photon; $\eta_\pm$, $\lambda_\pm$ depend on the specific emission kinematics, they are given in table 1.

Table 1. η_i and λ_i for the different collinear kinematics.

	p_-p_-	q_-q_-	p_+p_+	q_+q_+	p_-p_+	q_-q_+	p_-q_-	p_+q_+	p_-q_+	p_+q_-
η_1	y	1	1	1	$1-x_1$	1	$1-x_1$	1	$1-x_1$	1
η_2	1	1	y	1	$1-x_2$	1	1	$1-x_1$	1	$1-x_1$
λ_1	1	$\frac{1}{y}$	1	1	1	$\frac{1}{1-x_1}$	$1+\frac{x_2}{y_1}$	1	1	$1+\frac{x_2}{y_1}$
λ_2	1	1	1	$\frac{1}{y}$	1	$\frac{1}{1-x_2}$	1	$1+\frac{x_2}{y_2}$	$1+\frac{x_2}{y_2}$	1

The columns of the table correspond to a certain choice of the kinematics in the following way: p_-p_- means the emission of both photons along the projectile electron, p_+q_- means that the first of the photons goes along the projectile positron and the second — along the scattered electron and so on. The contributions from 6 remaining kinematical regions (when the photons in the last 6 columns are interchanged) could be found by the simple substitution $x_1 \leftrightarrow x_2$. We will use the momentum conservation law

$$\eta_1 p_- + \eta_2 p_+ = \lambda_1 q_- + \lambda_2 q_+ , \tag{13}$$

and the following relations coming from it:

$$\begin{aligned} &\eta_1 + \eta_2 = \lambda_1 y_1 + \lambda_2 y_2, \qquad \lambda_1 y_1 \sin\theta_- = \lambda_2 y_2 \sin\theta_+, \\ &\eta_1 - \eta_2 = \lambda_1 y_1 \cos\theta_- + \lambda_2 y_2 \cos\theta_+, \quad \theta_- = \widehat{\mathbf{q}_-\mathbf{p}_-}, \\ &\theta_+ = \widehat{\mathbf{q}_+\mathbf{p}_-}, \quad y_{1,2} = \frac{q^0_{1,2}}{\varepsilon}. \end{aligned} \tag{14}$$

One can find from eq. (13) (taking η_i, λ_i, $c = \cos\theta_-$ as the known quantities) that

$$\sin\theta_+ = \sin\theta_- \frac{2\eta_1\eta_2}{\eta_1^2 + \eta_2^2 + (\eta_2^2 - \eta_1^2)c}, \tag{15}$$

$$\lambda_1 y_1 = \frac{2\eta_1\eta_2}{\eta_1 + \eta_2 + (\eta_2 - \eta_1)c}, \qquad \lambda_2 y_2 = \frac{\eta_1^2 + \eta_2^2 + (\eta_2^2 - \eta_1^2)c}{\eta_1 + \eta_2 + (\eta_2 - \eta_1)c}.$$

Each contribution from 16 ones to the cross section of process (3) can be expressed in terms of the corresponding Born–like cross section multiplied by its collinear factor:

$$\begin{aligned} \mathrm{d}\sigma_{\mathrm{coll}} &= \frac{1}{2!}\left(\frac{\alpha}{4\pi}\right)^2 x_1 x_2 \sum_{(\eta,\lambda)} \overline{K}(\eta,\lambda)\,\mathrm{d}\tilde{\sigma}_0(\eta,\lambda)\,\mathrm{d}x_1\mathrm{d}x_2, \\ \mathrm{d}\tilde{\sigma}_0(\eta,\lambda) &= \frac{2\alpha^2}{s} B(\eta,\lambda)\,\mathrm{d}I(\eta,\lambda), \\ \mathrm{d}I_i(\eta,\lambda) &= \int \frac{\mathrm{d}^3q_-\mathrm{d}^3q_+}{q_-^0 q_+^0}\delta^4(\eta_1 p_- + \eta_2 p_+ - \lambda_1 q_- - \lambda_2 q_+) \\ &= \frac{4\pi\eta_1\eta_2\mathrm{d}c}{\lambda_1^2\lambda_2^2[c(\eta_2-\eta_1)+\eta_1+\eta_2]^2}, \\ B(\eta,\lambda) &= \left(\frac{\tilde{s}^2 + \tilde{t}^2 + \tilde{s}\tilde{t}}{\tilde{s}\tilde{t}}\right)^2, \qquad \tilde{s} = (\eta_1 p_- + \eta_2 p_+)^2 = 4\varepsilon^2\eta_1\eta_2 = s\eta_1\eta_2, \\ \tilde{t} &= (\eta_1 p_- - \lambda_1 q_-)^2 = -\tilde{s}\frac{\eta_1(1-c)}{\eta_1+\eta_2+(\eta_2-\eta_1)c}, \quad \tilde{s} + \tilde{t} + \tilde{u} = 0, \\ \overline{K}(\eta,\lambda) &= m^4 \int\limits_0^{z_0}\mathrm{d}z_1 \int\limits_0^{z_0}\mathrm{d}z_2 \int\limits_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\mathcal{K}(\eta,\lambda), \end{aligned} \tag{16}$$

the sum over (η,λ) means the sum over all 16 collinear kinematical regions and the correspondent (η,λ) could be found from table 1. $\mathcal{K}_i(\eta,\lambda)$ are listed below:

$$\begin{aligned} &\mathcal{K}(p_-p_-) = \frac{2}{y}\mathcal{A}(A_1,A_2,A,x_1,x_2), \quad \mathcal{K}(q_-q_-) = 2y\mathcal{A}(B_1,B_2,B,\frac{-x_1}{y},\frac{-x_2}{y}), \\ &\mathcal{K}(p_+p_+) = \frac{2}{y}\mathcal{A}(C_1,C_2,C,x_1,x_2), \quad \mathcal{K}(q_+q_+) = 2y\mathcal{A}(D_1,D_2,D,\frac{-x_1}{y},\frac{-x_2}{y}), \\ &\mathcal{A}(A_1,A_2,A,x_1,x_2) = -\frac{yA_2}{A^2A_1} - \frac{yA_1}{A^2A_2} + \frac{1+y^2}{x_1x_2A_1A_2} + \frac{r_1^3 + yr_2}{AA_1x_1x_2} \\ &\quad + \frac{r_2^3 + yr_1}{AA_2x_1x_2} + \frac{2m^2(y^2+r_1^2)}{AA_1^2x_2} + \frac{2m^2(y^2+r_2^2)}{AA_2^2x_1}, \end{aligned} \tag{17}$$

$$\begin{aligned} &\mathcal{K}(p_-p_+) = 2K_1K_2, \quad \mathcal{K}(p_-q_+) = -2K_1K_3, \quad \mathcal{K}(p_+q_-) = -2K_4K_5, \\ &\mathcal{K}(q_-q_+) = 2K_6K_7, \quad \mathcal{K}(p_-q_-) = -2K_1K_5, \quad \mathcal{K}(p_+q_+) = -2K_4K_3, \\ &K_1 = \frac{g_1}{A_1x_1r_1} + \frac{2m^2}{A_1^2}, \quad K_2 = \frac{g_2}{C_2x_2r_2} + \frac{2m^2}{C_2^2}, \quad K_3 = \frac{g_4}{D_2x_2t_2} - \frac{2m^2}{D_2^2}, \\ &K_4 = \frac{g_1}{C_1x_1r_1} + \frac{2m^2}{C_1^2}, \quad K_5 = \frac{g_3}{B_2x_2t_1} - \frac{2m^2}{B_2^2}, \quad K_6 = \frac{g_1}{B_1x_1} - \frac{2m^2}{B_1^2}, \\ &K_7 = \frac{g_2}{D_2x_2} - \frac{2m^2}{D_2^2}, \qquad r_1 = 1 - x_1, \quad r_2 = 1 - x_2, \\ &g_1 = 1 + r_1^2, \quad g_2 = 1 + r_2^2, \quad g_3 = y_1^2 + t_1^2, \\ &g_4 = y_2^2 + t_2^2, \quad t_1 = y_1 + x_1, \quad t_2 = y_2 + x_2, \\ &y = 1 - x_1 - x_2, \end{aligned} \tag{18}$$

y_1, y_2 are the energy fractions of the scattered electron and positron defined in eq. (15).

Expressions (18) agree with the results of paper [6] by exception a more simple form of $\mathcal{K}(q_-q_+)$; as for eq. (17) it has an evident advantage in comparison to the corresponding formulae given in paper [6]. Let us note that the remaining factors $\mathcal{K}(p,q)$ could be obtained from the ones given in eq. (18) using the relations of the following type

$$\mathcal{K}(p_-q_-)(x_1,x_2,A_1,B_2) = \mathcal{K}(q_-p_-)(x_2,x_1,A_2,B_1). \tag{19}$$

Note also that the terms of the kind

$$\frac{m^4}{B_2^2C_1^2} \tag{20}$$

do not give logarithmically reinforced contributions, we will omit them below. The denominators of the propagators entering eqs. (17, 18) are:

$$\begin{aligned} A_i &= (p_- - k_i)^2 - m^2, \qquad A = (p_- - k_1 - k_2)^2 - m^2, \\ B_i &= (q_- + k_i)^2 - m^2, \qquad B = (q_- + k_1 + k_2)^2 - m^2, \\ C_i &= (-p_+ + k_i)^2 - m^2, \qquad C = (-p_+ + k_1 + k_2)^2 - m^2, \\ D_i &= (q_+ + k_i)^2 - m^2, \qquad D = (q_+ + k_1 + k_2)^2 - m^2. \end{aligned} \tag{21}$$

For the further integration it is useful to rewrite the denominators in terms of the photons energy fractions $x_{1,2}$ and their emission angles. In the case of the emission of both photons along p_- we would have

$$\begin{aligned} \frac{A}{m^2} &= -x_1(1+z_1) - x_2(1+z_2) + x_1x_2(z_1+z_2) + 2x_1x_2\sqrt{z_1z_2}\cos\phi, \\ \frac{A_i}{m^2} &= -x_i(1+z_i), \end{aligned} \tag{22}$$

where $z_i = (\varepsilon\theta_i/m)^2$, ϕ is the azimuthal angle between the planes containing the space vector pairs $(\mathbf{p}_-, \mathbf{k}_1)$ and $(\mathbf{p}_-, \mathbf{k}_2)$. In the same way one can obtain in the case $k_1, k_2 \| q_-$:

$$\begin{aligned} \frac{B}{m^2} &= \frac{x_1}{y_1}(1 + y_1^2z_1) + \frac{x_2}{y_1}(1 + y_1^2z_2) + x_1x_2(z_1+z_2) + 2x_1x_2\sqrt{z_1z_2}\cos\phi, \\ \frac{B_i}{m^2} &= \frac{x_i}{y_1}(1 + y_1^2z_i). \end{aligned} \tag{23}$$

Then we perform the elementary azimuthal angle integration and the integration over z_1, z_2 within the logarithmical accuracy using the procedure suggested in paper [8]:

$$\overline{a} = m^4 \int\limits_0^{z_0} \mathrm{d}z_1 \int\limits_0^{z_0} \mathrm{d}z_2 \int\limits_0^{2\pi} \frac{\mathrm{d}\phi}{2\pi} a. \tag{24}$$

The list of the relevant integrals is given in Appendix A. In this way one obtains the differential cross section in the collinear region:

$$\begin{aligned}
\mathrm{d}\sigma_c = & \frac{\alpha^4 L}{4\pi^2 s}\frac{\mathrm{d}^3q_+\mathrm{d}^3q_-}{q_+^0 q_-^0}\frac{\mathrm{d}x_1\mathrm{d}x_2}{x_1x_2}(1+\mathcal{P}_{1,2})\{\frac{1}{yr_1^2}[\frac{1}{2}(L+2l)g_1g_5 \\
& +(y^2+r_1^4)\ln\frac{x_2r_1^2}{x_1y}+x_1x_2(y-x_1x_2)-2r_1g_5][B_{p_-p_-}\delta_{p_-p_-} \\
& +B_{p_+p_+}\delta_{p_+p_+}]+\frac{1}{yr_1^2}[\frac{1}{2}(L+2l+4\ln y)g_1g_5+(y^2+r_1^4)\ln\frac{x_1r_1^2}{x_2y} \\
& +x_1x_2(y-x_1x_2)-2r_1g_1]\times[B_{q_-q_-}\delta_{q_-q_-}+B_{q_+q_+}\delta_{q_+q_+}] \\
& +B_{p_-p_+}\delta_{p_-p_+}[(L+2l)\frac{g_1g_2}{r_1r_2}-2\frac{g_1}{r_1}-2\frac{g_2}{r_2}]+B_{q_-q_+}\delta_{q_-q_+}[(L+2l \\
& +2\ln(r_1r_2))\frac{g_1g_2}{r_1r_2}-2\frac{g_1}{r_1}-2\frac{g_2}{r_2}]+[B_{p_-q_-}\delta_{p_-q_-}+B_{p_+q_-}\delta_{p_+q_-}] \\
& \times[(L+2l+2\ln y_1)\frac{g_1g_3}{r_1y_1t_1}-2\frac{g_1}{r_1}-2\frac{g_3}{y_1t_1}]+[B_{p_+q_+}\delta_{p_+q_+} \\
& +B_{p_-q_+}\delta_{p_-q_+}][(L+2l+2\ln y_2)\frac{g_1g_4}{r_1y_2t_2}-2\frac{g_1}{r_1}-2\frac{g_4}{y_2t_2}]\}.
\end{aligned} \tag{25}$$

We used above symbol $\mathcal{P}_{1,2}$ for the interchange operator ($\mathcal{P}_{1,2}f(x_1,x_2)=f(x_2,x_1)$). We used also the notations (see also eq. (18)):

$$l=\ln\theta_0^2, \quad g_5=y^2+r_1^2, \tag{26}$$

where θ_0 is the collinear parameter. Symbol $\delta_{p,q}$ corresponds to the specific conservation law of the kinematical situation defined by the pair p,q (see table 1): $\delta_{p,q}=\delta^4(\eta_2p_++\eta_1p_--\lambda_1q_--\lambda_2q_+)$. Besides that, we imply that the first photon is emitted along momentum p and the second — along momentum q ($p,\ q=p_-,\ p_+,\ q_-,\ q_+$). These δ-functions could be accounted in the integration as that was done in the expression for $\mathrm{d}I(\eta,\lambda)$ (see eq. (16)). And, finally, we define

$$B_{p,q}=(\frac{\eta_2s}{\lambda_1t}+\frac{\lambda_1t}{\eta_2s}+1)^2, \qquad t=(p_--q_-)^2. \tag{27}$$

3 Contribution of the semi–collinear region

We will suggest for definiteness that the photon with momentum k_2 moves inside a narrow cone along the momentum direction of one of the charged particles, while the other photon moves in any direction outside such a cone along any charged particle. This choice allows us

to omit the statistical factor 1/2!. The quasireal electron method [7] may be used to obtain the cross section:

$$\begin{aligned}
\mathrm{d}\sigma^{\mathrm{SC}} = & \frac{\alpha^4}{32s\pi^4}\frac{\mathrm{d}^3q_-\mathrm{d}^3q_+\mathrm{d}^3k_1}{q_-^0q_+^0k_1^0}V\frac{\mathrm{d}^3k_2}{k_2^0}\{\frac{\mathcal{K}_{p_-}}{p_-k_2}\delta_{p_-}R_{p_-} \\
+ & \frac{\mathcal{K}_{p_+}}{p_+k_2}\delta_{p_+}R_{p_+}+\frac{\mathcal{K}_{q_-}}{q_-k_2}\delta_{q_-}R_{q_-}+\frac{\mathcal{K}_{q_+}}{q_+k_2}\delta_{q_+}R_{q_+}\}.
\end{aligned} \tag{28}$$

We omitted in eq. (28) the terms of the kind $m^2/(p_-k_2)^2$, because their contribution does not contain the large logarithm L. The quantities entering eq. (28) are presented below:

$$\begin{aligned}
V = & \frac{s}{k_1p_+\cdot k_1p_-}+\frac{s'}{k_1q_+\cdot k_1q_-}-\frac{t'}{k_1p_+\cdot k_1q_+}-\frac{t}{k_1p_-\cdot k_1q_-} \\
+ & \frac{u'}{k_1p_+\cdot k_1q_-}+\frac{u}{k_1q_+\cdot k_1p_-}.
\end{aligned} \tag{29}$$

V is the known accompanying radiation factor. $\mathcal{K}_i$ are the single photon emission collinear factors:

$$\begin{aligned}
\mathcal{K}_{p_-} = & \mathcal{K}_{p_+}=\frac{k_2}{x_2r_2}, \quad \mathcal{K}_{q_-}=\frac{y_1^2+(y_1+x_2)^2}{x_2(y_1+x_2)}, \\
\mathcal{K}_{q_+} = & \frac{y_2^2+(y_2+x_2)^2}{x_2(y_2+x_2)}.
\end{aligned} \tag{30}$$

Quantities R_i reads:

$$\begin{aligned}
R_{p_-} &= R[sr_2,,tr_2,ur_2s',t',u'], \quad R_{p_+}=R[sr_2,t,u,s',t'r_2,u'r_2], \\
R_{q_-} &= R[s,t\frac{t_1}{y_1},u,s'\frac{t_1}{y_1},t',u'\frac{t_1}{y_1}], \quad R_{q_+}=R[s,t,u\frac{t_2}{y_2},s'\frac{t_2}{y_2},t'\frac{t_2}{y_2},u'],
\end{aligned} \tag{31}$$

where function R has the form [10]:

$$\begin{aligned}
& R[s,t,u,s',t',u']=\frac{1}{ss'tt'}[ss'(s^2+s'^2)+tt'(t^2+t'^2)+uu'(u^2+u'^2)], \\
& s=(p_++p_-)^2, \quad s'=(q_++q_-)^2, \quad t=(p_--q_-)^2, \\
& t'=(p_+-q_+)^2, \quad u=(p_--q_+)^2, \quad u'=(p_+-q_-)^2.
\end{aligned} \tag{32}$$

And finally we defined

$$\begin{aligned}
\delta_{p_-} &= \delta^4(p_-r_2+p_+-q_+-q_--k_1), \\
\delta_{p_+} &= \delta^4(p_-+p_+r_2-q_+-q_--k_1), \\
\delta_{q_-} &= \delta^4(p_-+p_+-q_+-q_-\frac{y_1+x_2}{y_1}-k_1), \\
\delta_{q_+} &= \delta^4(p_-+p_+-q_+\frac{y_2+x_2}{y_2}-q_--k_1).
\end{aligned} \tag{33}$$

Performing the integration over the angular variables of the collinear photon we obtain

$$\begin{aligned}
\mathrm{d}\sigma^{\mathrm{SC}} = & \frac{\alpha^4L}{16s\pi^4}\frac{\mathrm{d}^3q_-\mathrm{d}^3q_+\mathrm{d}^3k_1}{q_-^0q_+^0k_1^0}\mathrm{d}x_2V\{\mathcal{K}_{p_-}[R_{p_-}\delta_{p_-}+R_{p_+}\delta_{p_+}] \\
& +\frac{1}{y_2}\mathcal{K}_{q_+}R_{q_+}\delta_{q_+}+\frac{1}{y_1}\mathcal{K}_{q_-}R_{q_-}\delta_{q_-}\}.
\end{aligned} \tag{34}$$

In order to see that the sum of cross sections (25) and (34)

$$d\sigma^{\gamma\gamma} = d\sigma^{\rm coll} + \int dO_1 \left(\frac{d\sigma^{\rm sc}}{dO_1}\right) \tag{35}$$

does not depend on the auxiliary parameter θ_0 it is convenient to represent the terms entering eq. (34) in the form:

$$\begin{aligned} VR_{p_-}\delta_{p_-} &= \frac{1}{k_1p_-}v_{p_-p_-}\delta_{p_-p_-} + \frac{1}{k_1p_+}v_{p_-p_+}\delta_{p_-p_+} + \frac{1}{k_1q_-}v_{p_-q_-}\delta_{p_-q_-} \\ &+ \frac{1}{k_1q_+}v_{p_-q_+}\delta_{p_-q_+} + [VR_{p_-}\delta_{p_-}]^f, \\ [VR_{p_-}\delta_{p_-}]^f &\equiv VR_{p_-}\delta_{p_-} - \sum_i \frac{1}{k_1q_i}v_{p_-q_i}\delta_{p_-q_i}, \qquad q_i = p_-,\, p_+,\, q_-,\, q_+, \end{aligned} \tag{36}$$

and so on in the same way for the other terms from eq. (34). Integrating $[VR_{p_-}\delta_{p_-}]^f$ over the angular variables we may integrate over the whole phase volume for k_1, i.e. we will obtain a finite contribution in the limit $\theta_0 \to 0$. Using the explicit expressions for quantities

$$v_{p_iq_j} = (VR_{p_i}\, k_1q_j)|_{k_1q_j \to 0}, \tag{37}$$

which are listed in Appendix B, we can see the cancelation of terms $L \cdot l$ from eq. (25) with terms

$$L\frac{k_1^0q_i^0}{2\pi}\int \frac{dO_1}{k_1q_i} \sim -L \cdot l, \tag{38}$$

which appear from 16 regions in the semi–collinear kinematics.

Physical results are the sum of the obtained differential cross-section integrated in the experimentally accessible region

$$\Delta < x_1, x_2 < 1, \qquad \theta_0 < \theta_-,\, \theta_+ < \pi - \theta_0 \tag{39}$$

with the contributions of virtual and real soft photon emission corrections. It should not depend on Δ.

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Appendix A

We present here the list of integrals (see eqs. (21 – 24)):

$$\overline{\frac{A_2}{A^2A_1}} = \frac{L_0}{x_1x_2r_1^2}\left[\frac{1}{2}L_0 + \ln\frac{x_2r_1^2}{x_1y} - 1 + \frac{x_1x_2}{y}\right],$$

$$\begin{aligned} &\overline{\frac{1}{AA_1}} = \frac{L_0}{x_1x_2r_1}\left[\frac{1}{2}L_0 + \ln\frac{x_2r_1^2}{x_1y}\right], \qquad \overline{\frac{m^2}{AA_1^2}} = -\frac{L_0}{x_1^2x_2r_1}, \\ &\overline{\frac{1}{A_1A_2}} = \frac{L_0^2}{x_1x_2}, \qquad \overline{\frac{1}{A_1B_2}} = -\frac{L_0}{y_1x_1x_2}(L_0 + 2\ln y_1), \\ &L_0 = \ln z_0 \equiv L + l, \qquad l = \ln\theta_0^2. \end{aligned} \tag{40}$$

The remaining integrals could be obtained using simple substitutions defined eqs. (21 – 24).

Appendix B

We put here the total list of quantities $v_{p_iq_j}$ entering eq. (37):

$$\begin{aligned} v_{p_-p_-} &= \frac{4(y^2+r_1^2)}{yx_1r_2}B_{p_-p_-}, & v_{p_+p_+} &= \frac{4(y^2+r_2^2)}{yx_1r_2}B_{p_+p_+}, \\ v_{q_-q_-} &= \frac{4(1+r_1^2)}{x_1r_1}B_{q_-q_-}, & v_{q_+q_+} &= \frac{4y(1+r_1^2)}{x_1r_1}B_{q_+q_+}, \\ v_{p_+q_+} &= \frac{4(y_2^2+(y_2+x_1)^2)}{x_1(y_2+x_1)}B_{p_+q_+}, & v_{p_-q_+} &= \frac{4(y_2^2+(y_2+x_1)^2)}{x_1(y_2+x_1)}B_{p_-q_+}, \\ v_{p_-q_-} &= \frac{4(y_1^2+(y_1+x_1)^2)}{x_1(y_1+x_1)}B_{p_-q_-}, & v_{p_+q_-} &= \frac{4(y_1^2+(y_1+x_1)^2)}{x_1(y_1+x_1)}B_{p_+q_-}, \\ v_{q_+q_-} &= \frac{4(1+r_1^2)}{x_1}B_{q_-q_+}, & v_{p_+p_-} &= \frac{4(1+r_1^2)}{x_1r_1}B_{p_-p_+}, \\ v_{p_-p_+} &= \frac{4(1+r_1^2)}{x_1r_1}B_{q_-q_+}, & v_{q_-q_+} &= \frac{4(1+r_1^2)}{x_1}B_{p_-p_+}, \\ v_{q_-p_-} &= \frac{4(1+r_1^2)}{x_1r_1}B_{p_-q_-}, & v_{q_+p_+} &= \frac{4(1+r_1^2)}{x_1r_1}B_{p_+q_+}, \\ v_{q_+p_-} &= \frac{4(1+r_1^2)}{x_1r_1}B_{p_-q_+}, & v_{q_-p_+} &= \frac{4(1+r_1^2)}{x_1r_1}B_{p_+q_-}. \end{aligned} \tag{41}$$

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