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ON QCD Q^2 -EVOLUTION OF DEUTERON
STRUCTURE FUNCTION $F_2^D(x_D, Q^2)$ FOR $x_D > 1$

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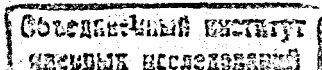
1 Introduction

The progress of perturbative Quantum Chromodynamics (QCD) in the description of the high energy physics of strong interactions is considerable [1]. Deep-inelastic scattering (DIS) of leptons provides the data on nucleon structure functions (SF) which could be used for the most precise comparison with QCD. The accuracy of SF data is sufficient to be sensitive to nuclear effects in DIS, e.g. the EMC effect [2]. A direct evidence of the nucleus structure in DIS has been obtained by measuring the carbon SF [3] in the kinematical region forbidden for scattering on a free nucleon (the Bjorken kinematical variable $x = Q^2/2(pq)$ is more than 1 in the range). The region is known as a cumulative region [4] and it is intensively investigated both experimentally and theoretically in proton-nucleus and nucleus-nucleus collisions [5, 6]. An important matter of the relativistic nuclear physics is to adduce arguments for applying QCD to analyse SF at $x_D > 1$. The deuteron as a simplest nuclear system is an excellent object to investigate the problem of relativistic theory of a deuteron.

Dependence of the deuteron structure functions F_2^D on x_D in the cumulative range has been studied by many authors. Different models (the few-nucleon correlation model [7], the flucton model [8]) are used to describe a high momentum component of the deuteron wave function. The microscopic picture for cumulative phenomena on the deuteron near the threshold based on perturbative QCD has been developed in [9].

The dependence of the nuclear structure function ratio $R^{C/D} = F_2^C/F_2^D$ on x_D has been considered in [10]. In the analysis the BCDMS experimental data on F_2^C [3] have been used. The deuteron SF has been calculated in the light-cone covariant approach [11] and it was found that the ratio $R^{C/D}(x_D)$ reveals an exponent growth $\sim \exp(\alpha \cdot x_D)$ for $x_D > 1$ with the slope parameter $\alpha \simeq 6.6$ and differs significantly from the ratio behaviour for $x_D < 1$. The latter is in good agreement with experimental data [12, 13, 14]. It is considered that the ratio $R^{C/D}$ for $x_D < 1$ describes quark distributions in a nucleon of a nucleus. The ratio $R^{C/D}$ for $x_D > 1$ describes quark distributions in a nucleus. In [15] similar exponent dependence of the inclusive pion backward cross section ratio $R^{A/D} \sim \rho_A^\pi(X)/\rho_D^\pi(X)$ on the cumulative number X [16] for the $p + A \rightarrow \pi(180^\circ) + \dots$ process was found. It was concluded that this dependence demonstrates the change of the regime from hadron to quark degrees of freedom. We can consider that the exponent growth of the similar ratio $R^{A/D}$ for different processes (deep-inelastic lepton scattering, deep-inelastic nuclear reactions) in the cumulative range is the general feature of quark interactions in the superdensity matter.

An important matter in the description of DIS is whether or not the Q^2 -evolution of the structure function is possible in the framework of QCD in the cumulative range. If it is possible, the QCD analysis could be used to study the transition regime and to determine quark distribution functions in nuclei.



In the present paper, the QCD analysis of data on the deep-inelastic deuteron structure function $F_2^D(x_D, Q^2)$ for $x_D < 1$ and $x_D > 1$ is performed. The data on the x_D and Q^2 -dependence of SF are simulated by the relativistic deuteron model [11]. The simulated data for $F_2^D(x_D, Q^2)$ for the range $x_D < 1$ are in good agreement with experimental data [12]. The experimental data for the deuteron SF in the range $x_D > 1$ for high Q^2 are absent. These data would be very interesting both to verify deuteron models and methods of describing relativistic nuclear systems and to study the high momentum component of the relativistic deuteron wave function (RDWF). The QCD analysis of data is based on the expansion method of SF over Jacobi polynomials [17]-[19]. The Q^2 -evolution of the moments of the structure function F_2^D is found as the solution of the corresponding renormalization equation. Effectivity of the method to control higher perturbative QCD corrections and to investigate sensitivity of the QCD scale parameter Λ were shown in [19]. It was found that the data on $F_2^D(x_D, Q^2)$ obey the Q^2 -evolution of SF in the framework of QCD both for $0.275 < x_D < 0.85$ and $1.1 < x_D < 1.4$ ranges in accordance with the criterium χ^2 very well. The value of the parameter Λ is determined to be the same for both the regions. We would like to note that the parameters of structure function parametrisations differ significantly for these ranges and this gives us some evidence for two regimes of the x_D -dependence of the structure function $F_2^D(x_D, Q^2)$.

2 Deuteron Structure Function $F_2^D(x_D, Q^2)$

The deuteron structure functions $W_{1,2}(\nu, Q^2)$ are related to the imaginary part of the forward scattering amplitude of a virtual photon on a deuteron $W_{\mu\nu}$ by the standard formula

$$W_{\mu\nu}^D = -(g_{\mu\nu} - q_\mu q_\nu / q^2) \cdot W_1^D + (p_\mu - q_\mu(pq)/q^2)(p_\nu - q_\nu(pq)/q^2) \cdot W_2^D / M^2. \quad (1)$$

Here q, p are momenta of a photon and a deuteron; M is the deuteron mass.

In the relativistic impulse approximation (RIA) the forward scattering amplitude of the virtual photon γ^* on the deuteron $A_{\mu\nu}^D$ is defined via a similar scattering amplitude on the nucleon $A_{\mu\nu}^N$ as follows

$$A_{\mu\nu}^D(q, p) = \int \frac{d^4 k_1}{(2\pi)^4 i} Sp\{A_{\mu\nu}^N(q, k_1) \cdot T(s_1, k_1)\}. \quad (2)$$

In the expression (2) $T(s_1, k_1)$ is the amplitude of the forward $N-D$ scattering and the conventional notations $Q^2 = -q^2 > 0$, $\nu \equiv (pq)$, $s_1 = (p - k_1)^2$ are used. The integration is carried out over the active nucleon momentum k_1 . The calculation of the imaginary part of the amplitude $A_{\mu\nu}^D$ gives us the possibility to put the nucleon spectator with the momentum $k = p - k_1$ on mass shell. Therefore the tensor $W_{\mu\nu}^D$ is expressed via the DNN vertex with one nucleon on mass shell. The vertex is

described by the function $\Gamma_\alpha(k_1)$ and depends on one variable k_1 . The vector index α characterizes the deuteron spin. With the relation between the RDWF and the vertex function $\Gamma_\alpha(k_1)$: $\psi_\alpha(k_1) = \Gamma_\alpha(k_1) \cdot (m + k_1)^{-1}$, the expression for the tensor $W_{\mu\nu}^{\alpha\beta}$ can be written as

$$W_{\mu\nu}^D = \int \frac{d^4 k}{(2\pi)^3} \delta(m^2 - k^2) \Theta(k_0) \Theta(p_+ - k_+) Sp\{\psi_\mu^N \cdot \psi^\alpha(k_1) \cdot (m + \hat{k}) \cdot \psi^\beta(k_1)\} \cdot \rho_{\alpha\beta}(p). \quad (3)$$

Here $\rho_{\alpha\beta}(p) = -(g_{\alpha\beta} - p_\alpha p_\beta / M^2) / 3$ is the deuteron polarization density matrix and the Θ -function and light-cone variables ($k_\pm = k_0 \pm k_3, k_\perp$) are used. The vertex function $\Gamma_\alpha(k_1)$ is defined via 4 scalar functions $a_i(k_1^2)$ ($i = 1-4$) and has the form

$$\Gamma_\alpha(k_1) = k_{1\alpha} [a_1(k_1^2) + a_2(k_1^2)(m + \hat{k}_1)] + \gamma_\alpha [a_3(k_1^2) + a_4(k_1^2)(m + \hat{k}_1)]. \quad (4)$$

The relativization procedure of deuteron wave function ψ_α has been proposed in [11, 20]. The scalar functions $a_i(k_1^2)$ have been constructed in the form of a sum of pole terms. Some pole positions and residues have been found from the comparison of our RDWF in the nonrelativistic limit with the known nonrelativistic deuteron wave function. For the latter the Paris wave function [21] was taken. Other parameters were fixed from the description of the static characteristics of the deuteron (an electric charge $G_e(0) = 1(e)$, magnetic $G_m(0) = \mu_D(e/2M_D)$, and quadrupole $G_Q(0) = Q_D(e/M_D^2)$ moments) in the relativistic impulse approximation.

The calculation of (3) in the light-cone variables gives the final expression for the deuteron SF $F_2^D \equiv \nu W_2^D$

$$F_2^D(x_D, Q^2) = \int_z^1 dx d^2 k_\perp p(x, k_\perp) \cdot F_2^N(z/x, Q^2), \quad (5)$$

where $x_D = 2z$, $0 < z < 1$. The nucleon SF $F_2^N = (F_2^p + F_2^n) / 2$ is defined by the proton and neutron ones. The positive function $p(x, k_\perp)$ describes the probability that the active nucleon carries away the fraction of the deuteron momentum $x = k_{1+} / p_+$ and the transverse momentum k_\perp in the infinite momentum frame. It is expressed via the vertex function $\Gamma_\alpha(k_1)$ as follows

$$p(x, k_\perp) \propto Sp\{\psi^\alpha(k_1) \cdot (m + \hat{k}) \cdot \psi^\beta(k_1) \cdot \hat{q} \cdot \rho_{\alpha\beta}(p)\}. \quad (6)$$

Note that in the approach used the distribution function $p(x, k_\perp)$ includes not only the usual S - and D -wave components of the deuteron but the P -component too. The latter describes the contribution of the $N\bar{N}$ -pair production.

In the RIA the deuteron SF F_2^D is defined by (5) as a sum of the proton and neutron SF integrated on the x and k_\perp . The NMC data [13] on the ratio $R_F^{D/p} =$

F_2^D/F_2^p and F_2^p and the relativistic deuteron model have been used to extract the neutron SF F_2^n . For the latter the parametrisation

$$F_2^n(x, Q^2) = (1 - 0.75x)(1 - 0.15\sqrt{x(1-x)}) \cdot F_2^p(x, Q^2) \quad (7)$$

has been obtained in [22]. The results for the absolute value of F_2^D calculated within the parametrisation of the neutron SF are in good agreement with experimental data both in low ([13]) and high ([12, 14]) x_D -ranges.

3 Proton Structure Function $F_2^p(x, Q^2)$

In our analysis we shall use the parametrisation of $F_2^p(x, Q^2)$ given in [13]. The parametrization describes the NMC, SLAC and BCDMS data very well. The verification of this fit in the region $0.006 < x < 0.6$ gives $\Lambda = 200 \text{ MeV}$. To parametrize $F_2^p(x, Q^2)$ at $0.55 < x < 1$, we have made the QCD fit of parametrisation [13] considering it as an "experimental points" at $0.275 < x < 0.55$ with the leading order nonsinglet evolution of the moments of $F_2^p(x, Q^2)$ with $\Lambda = 200 \text{ MeV}$. The SF at the fixed point $Q_0^2 = 10 \text{ (GeV/c)}^2$ was parametrized as

$$F_2^p(x, Q_0^2) = Ax^B(1-x)^C(1+\gamma x). \quad (8)$$

The parameters A, B, C and γ in Eq. (8) are free parameters and are determined by the fit of the data. Then, on the basis of expression (8) the values of SF for $0.55 < x < 1$ were calculated. To achieve agreement between QCD evolution results and parametrization [13], the former should be multiplied by the factor $R(x, Q^2)$ for $0.55 < x < 1$:

$$R(x, Q^2) = (1 + 17.611(x - 0.55)^{-(3+0.661\log(Q^2/Q_0^2))})\Theta(x - 0.55). \quad (9)$$

The modified parametrization of the proton SF in a wide range of the Bjorken variable x is used as an "experimental" input for the deuteron model described above to simulate the data on the deuteron SF. The data for the deuteron SF are simulated for $x_D = 0.1 - 1.8$ and $Q^2 = 17 - 230 \text{ (GeV/c)}^2$.

Figure 1 shows the x_D -dependence of $F_2^D(x_D, Q^2)$ at $Q^2 = 61.5 \text{ (GeV/c)}^2$. One can see that the SF falls down drastically for $x_D > 1$. The open points are BCDMS data for SF F_2^C for the $\mu + ^{12}\text{C} \rightarrow \mu' + \dots$ process [3]. The model-simulated data for $F_2^D(x_D, Q^2)$ are in a qualitative agreement with BCDMS data. We would like to note that the more detail calculations [10] of the dependence of the nuclear structure function ratio $R^{C/D} = F_2^C/F_2^D$ on x_D with RDWF [11] have shown that the ratio is similar to the experimental data for $0.01 < x_D < 1$ [23] and exponentially grows for $x_D > 1$.

4 Method and Results of Analysis

Now we can apply the nonsinglet QCD fit to the data on the deuteron SF simulated in the previous section. We have reduced the deuteron data to a standard interval of the scaling variable $0 < x < 1$ by putting $x = x_D/2$ and have applied the method of the QCD analysis based on the expansion of SF over the Jacobi polynomials [17]-[19]. The SF is presented as follows

$$F_2^{N_{max}}(x, Q^2) = x^\alpha(1-x)^\beta \sum_{n=0}^{N_{max}} \Theta_n^\beta(x) \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) M_{j+2}(Q^2), n = 2, 3, \dots \quad (10)$$

$$M_n(Q^2) = \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{d_n} M_n(Q_0^2) \quad (11)$$

where $d_n = -\gamma_{NS}^0/2\beta_0$, $\beta_0 = 11 - \frac{2}{3}N_f$, N_f is the number of flavours, $\gamma_{NS}^{(0)}$ is the anomalous dimension in the one-loop perturbative QCD approximation: $\gamma_{NS}^{(0)} = 8/3 [4S_1(n) - 3 - 2/(n(n+1))]$, where $S_1(n) = \sum_{j=1}^n 1/j$.

The moments $M_n(Q^2)$ of SF F_2^D are defined by

$$M_n(Q^2) = \int_0^1 dx x^{N-2} F_2^D(x, Q^2). \quad (12)$$

The parametrization form of SF similar to (8) is used at a fixed point Q_0^2 :

$$F_2^D(x, Q_0^2) = A(x_D/2)^B(1-x_D/2)^C(1+\gamma x_D/2). \quad (13)$$

The constants A, B, C, γ in (13) and the QCD scale parameter Λ are considered as free parameters to be determined at Q_0^2 .

Table 1. The results of the LO nonsinglet QCD fit of the deuteron SF. $N_f = 4$, $17 < Q^2 < 230 \text{ (GeV/c)}^2$, $N_{max} = 12$. The errors of F_2^D are put to be 10%.

	$0.275 \leq x_D \leq 0.85$	$1.1 \leq x_D \leq 1.1$
Λ [MeV]	268 ± 67	240 ± 44
A	8.39 ± 1.50	$(6.26 \pm 0.84)10^{-5}$
B	0.898 ± 0.065	-0.115 ± 0.015
C	8.659 ± 0.456	11.07 ± 0.07
γ	-2.034 ± 0.055	$3376. \pm 457.$
$\chi^2_{d.f.}$	12.0/121	31.4/68

The results of the fit are presented in Table 1. We have found that values of Λ are approximately the same in two different regions of x_D and not significantly differ from the input proton parameter $\Lambda = 200 \text{ GeV}$ [13]. The x_D -dependence of SF for two ranges $x_D = 0.225 - 0.85$ and $x_D = 1.1 - 1.4$, is determined separately. It is established that the shapes of SF are essential different (see Table 1). The obtained value of $\chi^2_{d.f.}$ corresponds to 10% for errors of "experimental" deuteron data.

Figure 2 shows the dependence of $F_2^D(x_D, Q^2)$ on Q^2 for $x_D = 1.1 - 1.8$. The results of the QCD fit for $x_D = 1.1 - 1.4$ are drawn by solid lines, and the results of direct calculation by formula (5) are shown by open points. Good agreement between the model-simulated data on $F_2^D(x_D, Q^2)$ and results of the QCD fit is observed.

We would like to note that a more general task is a simultaneous QCD fit of F_2^D in the whole range $x_D = 0. - 2$, but it requires a rather complicated x -parametrization of SF instead of (8).

5 Conclusion

The QCD analysis of data on the deep-inelastic deuteron SF $F_2^D(x_D, Q^2)$ was performed. The data were simulated by the relativistic deuteron model in the covariant approach in light-cone variables. It was found that the data on $F_2^D(x_D, Q^2)$ obey the Q^2 -evolution of SF in the framework of QCD both for $0.275 < x_D < 0.85$ and $1.1 < x_D < 1.4$ in accordance with the criterium χ^2 very well. The value of the parameter Λ was determined to be the same for both the regions. The parameters of structure function parametrizations were determined and it was found that they differ significantly for these ranges. These results give some evidence for two regimes of the x_D -dependence of the structure function $F_2^D(x_D, Q^2)$.

We would like to emphasize that the QCD Q^2 -evolution of SF in the cumulative range ($x_D > 1$) could be a crucial test for verification of both the nuclear models and the parton model, and thus measurements of SF in DIS for $x_D > 1$ should be performed with a high accuracy at CERN, DESY and FERMI LAB.

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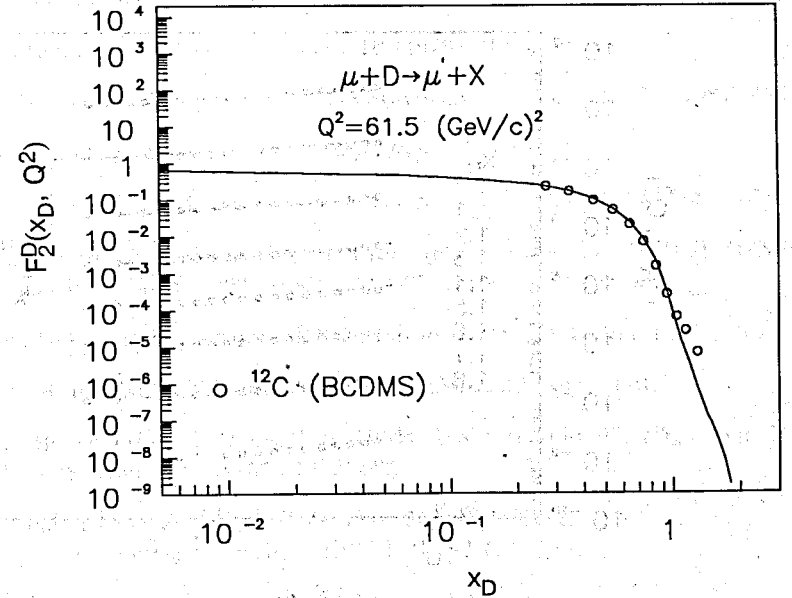


Figure 1: Deep-inelastic deuteron structure function $F_2^D(x_D, Q^2)$. Theoretical results have been obtained with the proton parametrization F_2^P taken from [13]. Experimental data for F_2^C are taken from [3]: $\circ - 61.5 \text{ (GeV/c)}^2$.

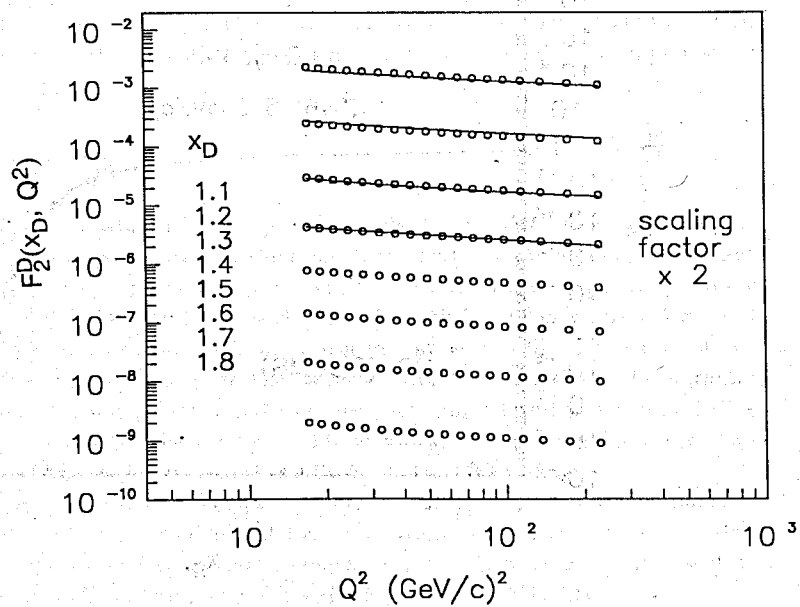


Figure 2: Deep-inelastic deuteron structure function $F_2^D(x_D, Q^2)$. Open points are generated "experimental" data; solid lines are results of the QCD fit of the data.

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