ОБЪЕДИНЕННЫЙ
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Дубна

E2-95-44

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ONE-SPIN ASYMMETRIES IN PAIR PRODUCTION AND BREMSSTRAHLUNG PROCESSES

Submitted to «Physics Letters B»

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1995

1 Introduction

Determination of the polarization degree of longitudinally polarized leptons and circularly polarized photons at high energies is an actual problem in experiments with colliding polarized $e^+ e^-$, $e\,p$, $\gamma\,p$, $e\,\gamma$ beams [1]. In this paper we suggest as polarimeter processes the pair production process in photon–electron or photon–proton collisions and the bremsstrahlung process in electron–electron or electron–proton collisions. In both cases we assume the target to be unpolarized. Both considered processes have pure electromagnetic nature. The spin–momentum correlation in the differential cross section arises from the interference of the Born and one-loop contributions to the matrix element. The correlations of this kind were considered at first in 1959 year papers by H. Olssen and L. Maximon and then continued in a series of papers of 1963–1964 years [2]. The main attention was paid in that works to the case when the target was a nuclei of charge Z, and the parameter $Z\alpha \approx Z/137$, was assumed to be of the order of unity. They found the asymmetry parameter A in the form

$$\frac{\mathrm{d}\sigma(\vec{n}) - \mathrm{d}\sigma(-\vec{n})}{\mathrm{d}\sigma(\vec{n}) + \mathrm{d}\sigma(-\vec{n})} = \xi_2 A \sin\phi\, \vec{n}\vec{k}, \qquad \vec{n} = \frac{[\vec{q}_1 \times \vec{q}_2]}{|[\vec{q}_1 \times \vec{q}_2]|}, \tag{1}$$

here $\vec{k}$ is the ort along the photon momentum, ξ_2 is the degree of the photon circular polarization, $\phi = \widehat{\vec{q}_1\vec{q}_2}$ $(0 < \phi < \pi)$ is the azimuthal angle between momenta components $\vec{q}_1$, $\vec{q}_2$ of the particles of the created pair transverse to the projectile photon momentum. $A \sim 2Z\alpha \ln 2$ was found to be a rather large quantity. Such a large magnitude of the asymmetry comes mainly from the integration over the region where the transverse components of the outgoing particle momenta are of the order of the created pair mass $|\vec{q}_1| \sim |\vec{q}_2| \sim |\vec{q}_1 + \vec{q}_2| \sim m$. That means very small angles of the pair components in respect to the beam direction, which are unsuitable for high energy experiments.

The preferable experimental region of transverse momenta corresponds to the angles of created particles

$$\theta_i = \frac{|\vec{q}_i|}{\omega x_i}, \qquad i = 1, 2, \tag{2}$$

which should be accessible for a measurement ($\theta_i \gtrsim 10^{-2}$); here $x_{1,2}$ ($x_1 = x,\ x_2 = y,\ x + y = 1$) are the energy fractions of the electron and the positron from the pair respectively. ω is the initial photon energy (we work in the laboratory reference frame). In the case of the bremsstrahlung of a longitudinally polarized electron on a charged target the same angle range for the scattered electron and the emitted photon should be considered.

So, we will suggest further

$$\omega^2 \gg |\vec{q}_i{}^2| \gg m^2. \tag{3}$$

In that region we may use methods, developed to describe the processes of the formation of a jet moving close to the forward direction.

2 Processes

Consider first the process of a pair creation by a circularly polarized photon on an electron:

$$e^-(p) \;+\; \gamma(k_1) \;\to e^-(p') \;+\; e^-(q_1) \;+\; e^+(q_2)\,. \tag{4}$$

From 8 Feynman diagrams which describe it in the Born approximation the contribution of only two diagrams (Bethe–Heitler ones) survive in the high energy limit. Really, as was shown in the paper by E. Haug [3], the influence of other diagrams is of the order of some percents starting already from $\omega \gtrsim 100m$. Moreover, the exact contribution of that two chosen diagrams differs from its asymptotic ($\omega \gg m$) value less then by few percents for ω value exceeding $500m$.

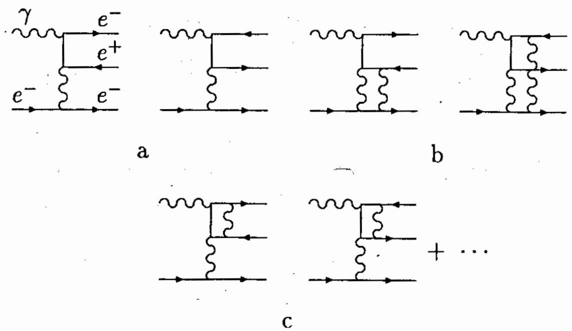

Fig. 1. The Bethe–Heitler diagrams and one–loop corrections for the pair production.

The representatives of the relevant in the chosen high energy limit (3) Feynman diagrams in the Born and the one–loop levels are drawn in fig. 1. Among the one–loop diagrams only those which have a non–zero imaginary part are to be considered. We divide the relevant one–loop diagrams into two types: the ladder or eiconal–like ones (fig. 1b type), and the diagrams with an interaction in the final state (fig. 1c type).

The similar situation takes place for the classification of the one–loop diagrams describing the bremsstrahlung process

$$e^+(p) \;+\; e^-(p_1) \;\to e^+(p') \;+\; e^-(p_2) \;+\; \gamma(k), \tag{5}$$

where the longitudinally polarized electron interacts with a charged target (with a positron here).

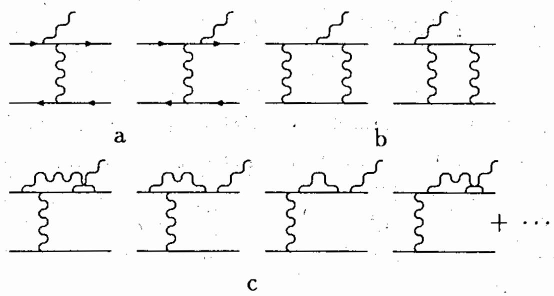

Fig. 2. The Born diagrams and one–loop corrections for the bremsstrahlung process.

3 Eiconal–like interactions

Consider at first the diagrams fig. 1b. Following paper [4] we introduce Sudakov variables

$$k = \alpha_k \tilde{p} + \beta_k k_1 + k_\perp, \quad \tilde{p} = p - k_1 \frac{m^2}{2p_2 k_1}, \quad \tilde{p}^2 = k_1^2 = 0 \tag{6}$$
$$q_j = \alpha_j \tilde{p} + \beta_j k_1 + q_{j\perp}\,, \quad p'^2 = p^2 = m^2, \quad q_1^2 = q_2^2 = m^2,$$

where $q_j (j = 1, 2)$ are the momenta of the particles from the created pair, $k_\perp k_1 = k_\perp p = q_{j\perp} k_1 = q_{j\perp} p = 0$. Note that $\beta_j \sim 1$, $\alpha_j \ll 1$, $\beta_1 + \beta_2 = 1$, and, as we will see further, $\alpha_k, \beta_k \ll 1$. One has in these variables

$$d^4 k = \frac{1}{2} S\, d^2 k_\perp\, d\alpha_k\, d\beta_k, \qquad S = 2pk_1 . \tag{7}$$

We represent $g^{\mu\nu}$ in a more convenient form

$$g^{\mu\nu} = 2\frac{p^\mu k_1^\nu + p^\nu k_1^\mu}{S} + g_\perp^{\mu\nu} . \tag{8}$$

Now if J_μ^{top} and J_ν^{bot} correspond to the top and the bottom currents respectively (see fig. 1a), then

$$J_\mu^{top} J_\nu^{bot} g^{\mu\nu} = (J^{top} p)(J^{bot} k_1)\frac{2}{S} + (J^{top} k_1)(J^{bot} p)\frac{2}{S} + J_\mu^{top} J_\nu^{bot} g_\perp^{\mu\nu} . \tag{9}$$

Because the vectors J_μ^{top} and J_ν^{bot} have the components of the order of unity along the momenta $\vec{k}_1$ and $\vec{p}$ respectively, the first term in eq. (9) is of the order S, the second $\sim 1/S$ and the third ~ 1. So, in our approximation we need to consider only the first term. Because $\alpha_k, \beta_k \ll 1$ we can replace $k^2 \to -k_\perp^2$ and $q^2 \to -q_\perp^2 > 0$. The matrix elements of the diagrams of the fig. 1b type could be presented in the form

$$M^{(1)} = -\frac{2ie^5 (2\pi i)^2}{(2\pi)^4 S} \int \frac{d^2 k_\perp}{k_\perp^2 (q-k)_\perp^2} J^{top} J^{bot}, \tag{10}$$
$$J^{bot} = \int \frac{d\beta_k}{2\pi i} J_{\rho\eta}^{bot} k_1^\rho k_1^\eta , \qquad J^{top} = \int \frac{d\alpha_k}{2\pi i} J_{\mu\nu}^{top} p^\mu p^\nu .$$

The integrals over α_k and β_k are independent and it is possible to integrate $(J_{\mu\nu}^{top} p^\mu p^\nu)$ and $(J_{\rho\eta}^{bot} k_1^\rho k_1^{eta})$ separately and to obtain the so called impact–factors. Consider at first J^{bot}, it is described by the diagrams fig. 3.

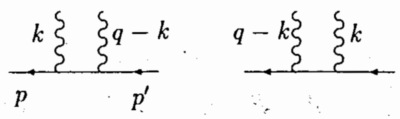

Fig. 3. The bottom blocks for the ladder diagrams.

$$J^{bot} = \int \frac{d\beta_k}{2\pi i} \bar{u}(p')\, \hat{k}_1 \left\{ \frac{\hat{p} - \hat{k} + m}{(p-k)^2 - m^2 + i\epsilon} + \frac{\hat{p}' + \hat{k} + m}{(p'+k)^2 - m^2 + i\epsilon} \right\} \hat{k}_1 u(p). \tag{11}$$

Pulling k_1 to the right, using eqs. (6), $k_1^2 = 0$ and representing the denominator we obtain

$$J^{bot} = \int \frac{d\beta_k}{2\pi i} \bar{u}(p') \left\{ \frac{S(1-\alpha_k)}{-S(\beta_k(1-\alpha_k) - i\epsilon)} + \frac{S(1+\alpha_k)}{S(\beta_k(1+\alpha_k) + i\epsilon)} \right\} \hat{k}_1 u(p). \tag{12}$$

Closing the contour of the integration over β in the bottom complex half–plane we obtain

$$J^{bot} = -\bar{u}(p')\, \hat{k}_1\, u(p). \tag{13}$$

Consider now the top block. It is described by the diagrams fig. 4a,b,c and by other three diagrams (a',b',c') which differ from fig. 4a,b,c ones only by the replacement $k \to (q-k)$.

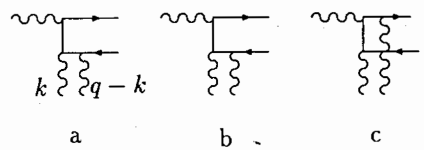

Fig. 4. The top blocks for the pair production ladder diagrams.

The detailed analysis of their contributions is given in [4] and the result has the form

$$M^{(1)} = -\frac{ie^5}{S(2\pi)^4} \int \frac{d^2 k_\perp}{k_\perp^2 (q-k)_\perp^2} \bar{u}(q_1)[2\beta_1 Q_\perp e_\perp^* + \hat{Q}_\perp \hat{e}_\perp^*]\hat{p} v(q_2) \bar{u}(p') \hat{k}_1 u(p) , \tag{14}$$

$$Q_\perp^\mu = \frac{q_{1\perp}^\mu}{q_{1\perp}^2 + m^2} + \frac{k_\perp^\mu - q_{1\perp}^\mu}{(k-q_1)_\perp^2 + m^2} - \frac{q_{2\perp}^\mu}{q_{2\perp}^2 + m^2} - \frac{k_\perp^\mu - q_{2\perp}^\mu}{(k-q_2)_\perp^2 + m^2}.$$

Note that integral (14) has no infrared and ultraviolet divergences.

Integral of the first term in Q_μ is simple: we introduce the "photon mass" parameter λ and obtain

$$I_1 = \int \frac{d^2k_\perp/\pi}{(k_\perp^2+\lambda^2)((k-q)_\perp^2+\lambda^2)} = \int_0^1 dx \int \frac{d^2k_\perp/\pi}{\{(k_\perp^2+\lambda^2)x + ((k-q)_\perp^2+\lambda^2)(1-x)\}^2} = \int_0^1 \frac{dx}{q^2(1-x)x+\lambda^2} = \frac{2}{q^2}\ln\frac{q^2}{\lambda^2}, \tag{15}$$

$q_\perp$ and $k_\perp$ are the two-dimensional euclidean vectors, we use further $q_\perp^2 \equiv q^2 > 0$. Consider now the vector integral

$$I^\mu = \int \frac{d^2k_\perp(k^\mu - q_1^\mu)/\pi}{(k_\perp^2-\lambda^2)((k-q)_\perp^2-\lambda^2)((k-q_1)_\perp^2-m^2)} \tag{16}$$

For facilitate further evaluations we combine the terms in the denominator using Feynman's formula

$$\frac{1}{abc} = 2\int_0^1 x\,dx \int_0^1 dy\{axy + bx(1-y) + c(1-x)\}^{-3} \tag{17}$$

For $a = (q_1-k)^2 - m^2$, $b = (q-k)^2 - \lambda^2$, $c = k^2 - \lambda^2$ we obtain

$$I^\mu = 2\int_0^1 x\,dx \int_0^1 dy \int \frac{d^2k_\perp}{\pi} \frac{k^\mu - q_1^\mu}{\{(k-xl)^2 - (Ax^2+Bx+C)\}^3}, \tag{18}$$

where

$$l = q_1 + q_2(1-y), \qquad A = l^2, \qquad C = \lambda^2 \tag{19}$$
$$B = q_1^2 y + q^2(1-y) + m^2 y - \lambda^2 y.$$

Performing the integrations over d^2k and dx using the tables [6] we put I^μ in the form

$$I^\mu = \int_0^1 \frac{dy}{B^2}\Big\{-q_1^\mu\big(\frac{B}{A+B+C} + L - 2\big) + q^\mu\big(\frac{B}{A} - \frac{B^2}{A(A+B+C)}\big)\Big\}, \tag{20}$$

where

$$L = \ln\Big(\frac{B+2A-\sqrt{\Delta}}{B+2A+\sqrt{\Delta}}\cdot\frac{B+\sqrt{\Delta}}{B-\sqrt{\Delta}}\Big) = \ln\Big(\frac{B^2}{C(A+B)}\Big), \qquad \Delta = B^2 - 4AC, \tag{21}$$

$q_1, q = q_1 + q_2$ are the two-dimensional euclidean vectors ($q_\perp^2 = q^2 > 0$, $q_1 = q_{1\perp}^2 > 0$). Using the integrals calculated within the power accuracy (we omit the terms of the order m^2/q^2 compared with the ones of the order of unity) we get

$$\int_0^1 \frac{dy}{B(A+B+C)} = \frac{1}{q^2q_2^2}\ln\frac{q_2^2}{\lambda^2} + \frac{1}{q_1^2q_2^2}\ln\frac{q_2^2}{m^2} + \frac{q^2-q_1^2}{q^2q_1^2q_2^2}\ln\frac{q_1^2}{q^2}, \tag{22}$$
$$\int_0^1 \frac{dy}{B^2}L = \frac{1}{q^2q_1^2}\big[2 + \ln\frac{q^2q_1^2}{q_2^2\lambda^2}\big], \qquad \int_0^1 \frac{dy}{B^2} = \frac{1}{q^2q_1^2}.$$

Finally we obtain for the contribution to the matrix element of the one-loop corrections originating from the ladder diagrams of the fig. 1b type the following:

$$M^{(1)} = -\frac{ie^5}{2\pi^2q^2S}\ln\frac{q_2^2}{q_1^2}\,\bar{u}(p')\,\hat{k}_1\,u(p)\,\bar{u}(q_1)\{2\beta_1(Q^0e) + \hat{Q}^0\hat{e}\}\,\hat{p}\,v(-q_2), \tag{23}$$
$$Q^0 = \frac{q_1}{q_1^2+m^2} + \frac{q_2}{q_2^2+m^2}, \qquad q_1 + q_2 = q = \vec{q}_{1\perp} + \vec{q}_{2\perp} = \vec{q}_\perp.$$

We see that $M^{(1)}$ is proportional to the Born matrix element:

$$M^{(0)} = -\frac{e^3}{2\pi^2 q^2 S}\, u(p')\,\hat{k}_1\, u(p)\,\bar{u}(q_1)\{2\beta_1(Q^0 e) + \hat{Q}^0\hat{e}\}\,\hat{p}\, v(-q_2). \tag{24}$$

and $M^{(1)}$ is the second term of the expansion of such an expression

$$M^{(\infty)} = M^{born} \exp\{-i\alpha \ln\frac{q_2^2}{\lambda^2} + i\alpha \ln\frac{q_1^2}{\lambda^2}\}, \tag{25}$$

where $M^{(\infty)}$ is the sum of all ladder diagrams.

In the bremsstrahlung case (5) we introduce the appropriate variables

$$\begin{aligned} &k_1 = \alpha\,\tilde{p}_2 + \beta\,\tilde{p}_1 + k_{1\perp}, \qquad p_1' = \alpha'\tilde{p}_2 + \beta'\tilde{p}_1 + p'_{1\perp}, \\ &k = \alpha_k\,\tilde{p}_2 + \beta_k\,\tilde{p}_1 + k_\perp, \qquad \tilde{p}_1 = p_1 - p_2\frac{m_1^2}{s}, \quad s = 2p_1p_2, \\ &\tilde{p}_2 = p_2 - p_1\frac{m_2^2}{s}, \quad p_1^2 = p_1'^{\,2} = m_1^2, \quad p_2^2 = m_2^2. \end{aligned} \tag{26}$$

The matrix element will have the form of eq. (10). The bottom block is the same as in the pair production case with the substitution $k_1 \to p_1$ (diagrams fig. 3). For the top block we have the following set of diagrams

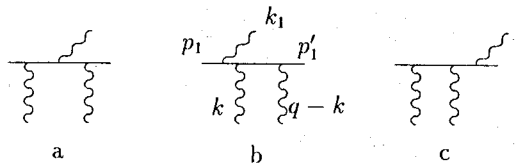

Fig. 5. The top blocks for bremsstrahlung ladder diagrams.

And ones more there are else three diagrams a', b', c' which differ from the diagrams fig. 5a,b,c by the replacement $k \leftrightarrow q-k$. The presence of all six contributions provide the convergence of the integration in the imaginary α_k plane of the impact factor J^{top}. Closing the contour of the integration in the upper half-plane we see that only the poles of the diagrams fig. 5a and fig. 5c' contribute. The corresponding integral over $d^2k_\perp$ coincides with I_1 in eq. (15). As the result we obtain

$$\begin{aligned} M^{(2)} = &\; i\frac{(4\pi\alpha)^{5/2}}{2\,S\,q_\perp^2\pi}\ln(\frac{q_\perp^2}{\lambda^2})\bar{u}(p_2')\,\hat{p}_1\,u(p_2)\,\bar{u}(p_1')\,\hat{p}_2\,\{\,2(\vec{e}_\perp\vec{Q}_{0\perp}^{brem}) \\ &+\beta_1\hat{e}_\perp\hat{Q}_0^{brem}\,\}\,u(p_1), \qquad \beta_1 = \frac{k_1^0}{p_1^0}, \\ \vec{Q}_0^{brem} = &-\frac{\vec{k}_{1\perp}}{\vec{k}_{1\perp}^2} + \frac{\vec{k}_{1\perp} - \beta_1\vec{q}_\perp}{(\vec{k}_{1\perp} - \beta_1\vec{q}_\perp)^2}, \qquad \vec{q}_\perp = \vec{k}_{1\perp} + \vec{p}\,'_{1\perp}. \end{aligned} \tag{27}$$

We see that it is proportional to the born matrix element.

$$\begin{aligned} M^{(born)} = &\;\frac{-2(4\pi\alpha)^{3/2}}{S\,q_\perp^2\pi}\bar{u}(p_2')\,\hat{p}_1\,u(p_2)\,\bar{u}(p_1')\,\hat{p}_2\,\{\,2\vec{e}_\perp\vec{Q}_{0\perp}^{brem} \\ &+\beta_1\hat{e}_\perp\hat{Q}_{0\perp}^{brem}\,\}\,u(p_1) \end{aligned} \tag{28}$$

Comparing the contributions of the ladder–type diagrams in the pair production and bremsstrahlung cases we see that they are not bound by crossing relations, which is a characteristic feature of the Born approximation. The absence of the infrared singularities in the eikonal phase of the pair production is due to the difference of the electron and positron phases, which both contain infrared singularities. The infrared singularities remains in the bremsstrahlung case — it is the known property of the Coulomb phase in the scattering of a charged particle in an external field. Not finally, that in both cases the ladder sets of loop corrections do not contribute to asymmetries.

We present here the differential cross–sections in the Born approximation. In the pair production it reads

$$\begin{aligned} d\sigma_0^{pair} &= \frac{2\alpha^3(x^2 + (1-x)^2)}{\pi^2\,q^2\,q_1^2\,q_2^2}\,dx\,d^2q_1\,d^2q_2, \\ q &= q_1 + q_2 \equiv \vec{q}_{bot} = \vec{q}_{1\perp} + \vec{q}_{2\perp}, \end{aligned} \tag{29}$$

where x and $(1-x)$ are the energy fractions of the pair components, q_1, q_2, q are the transverse to the initial photon momentum direction momenta components of the electron, the positron from the created pair and of the target respectively.

The analogous expression for the bremsstrahlung case:

$$d\sigma_0^{brem} = \frac{2\alpha^3(1+y^2)(1-y)}{\pi^2\,q^2\,k_1^2\,(k_1 - q(1-y))^2}\,dy\,d^2q\,d^2k_1, \tag{30}$$

where y and $\beta_1 = 1-y$ are the energy fractions of the scattered electron and the photon respectively, k_1 and q are the two dimensional euclidean vectors —

the transverse to the initial electron beam direction momentum components of the photon and target respectively.

We present here also the expressions for the invariant masses squared of the produced pair for the first process

$$S^{pair} = \frac{(xq_2 - (1-x)q_1)^2}{x(1-x)}, \qquad q_1^2 \sim q_2^2 \gg m^2, \tag{31}$$

and of the scattered electron in the second case

$$S^{brem} = \frac{(q(1-y)-k_1)^2}{y(1-y)}, \qquad k_1^2 \sim q^2 \gg m^2. \tag{32}$$

4 Final State Interactions

Consider the diagrams of the fig. 1c type, which describe interactions between the pair components. We need to take into account only diagrams with non-zero imaginary part. Using the Cutkosky rules one find the set of diagrams with non-zero s-channel imaginary parts:

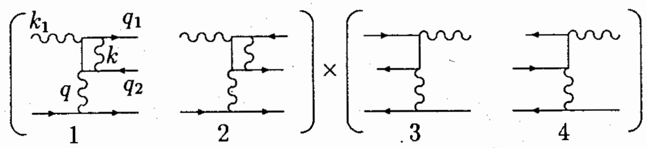

Fig. 6. The interference of the final state interactions diagram amplitudes with the Bethe-Heitler ones.

Using fig. 6 we can write the needed traces (due to the condition $q_1^2 \gg m^2$ we put $m = 0$ in the calculations of traces):

$$S_{23} = \frac{1}{4}\mathrm{Sp}\{(-\hat{q}_2)\hat{p}(-\hat{q}_2+\hat{q})\hat{e}\hat{q}_1(-\hat{q}_2-\hat{k})\hat{e}^*(\hat{q}_1-\hat{q}-\hat{k})\hat{p}(\hat{q}_1-\hat{k})\},$$
$$S_{24} = \frac{1}{4}\mathrm{Sp}\{(-\hat{q}_2)\hat{e}(\hat{q}_1-\hat{q})\hat{p}\hat{q}_1(-\hat{q}_2-\hat{k})\hat{e}^*(\hat{q}_1-\hat{q}-\hat{k})\hat{p}(\hat{q}_1-\hat{k})\}.$$

here we used representation (6) for $g^{\mu\nu}$. The remaining traces $S13$ and S_{14} could be obtained by the substitutions $q_1 \leftrightarrow -q_2$ and $q \to -q$. The correspondent denominators are transforming too. So, we have

$$2\Re\mathrm{e} M^0 M^{(1)\dagger} = -\frac{2^{11}\pi^2\alpha^4}{(q^2)^2}(1 + P(q_1 \leftrightarrow -q_2,\ q \leftrightarrow -q)) \times \int \frac{\mathrm{d}^4k}{i\pi^2(0)(1)(2)(q)}\left\{\frac{S_{23}}{(q-q_2)^2-m^2} + \frac{S_{24}}{(q_1-q)^2-m^2}\right\} \tag{33}$$

where

$$(1) \equiv (q_1-k)^2 - m^2, \qquad (2) \equiv (q_2+k)^2 - m^2,$$
$$(q) \equiv (q_1-q-k)^2 - m^2, \qquad (0) \equiv k^2, \tag{34}$$

P is the rearrangement operator. We use the following definition for the photon polarization vector and the polarization density matrix:

$$e_\mu = (e_0, e_z, e_x, e_y) = (0, 0, e_{tr}), \qquad (ae)(be^*) = i\xi_2\epsilon_{3ij}a_ib_j, \tag{35}$$

where ξ_2 is the degree of the photon circular polarization.

Performing the integration over the loop momentum (the relevant integrals are given in the Appendix) and using the expression for the cross-section in the Born approximation (29), we obtain for the analyzing power (see eq. (1)) defined as

$$A^{pair} = \frac{1}{\xi_2 \sin\phi}\frac{\mathrm{d}\sigma_{pol}^{\gamma e\to e\bar{e}e}}{\mathrm{d}\sigma_{nonpol}^{\gamma e\to e\bar{e}e}} \tag{36}$$

the following expression:

$$A^{pair} = \frac{\alpha z}{2(x^2+y^2)}\Big[\frac{2x}{z^2}(y - \frac{xc}{z})\ln\frac{x}{d} - 2y(x-2cyz)\ln\frac{z^2y}{d} + \frac{xyz}{(x+yz^2)^2}\Big[\frac{x^2}{z^3} - y^2z^3 + zy^2 - \frac{x^2}{z} + (\frac{1}{z} + 2c + z)(y^2z^2 - \frac{x^2}{z^2})\Big]\Big], \tag{37}$$
$$c = \cos\phi, \quad x = 1-y, \quad z = \frac{|\vec{q}_1|}{|\vec{q}_2|}, \quad d = x^2 + y^2z^2 - 2xyzc,$$

Note the property

$$A^{pair}(z,x,y) = -A^{pair}(z^{-1},y,x), \quad (38)$$

which comes from the symmetry in respect to the transposition of the pair components. In the limit $|\vec{q}_1| \sim |\vec{q}_2| \sim |\vec{q}| = \sqrt{q_1^2 + q_2^2 + 2q_1q_2c} \gg m$ the expression for A^{pair} does not depend on m. The value of A^{pair} slightly depends on ϕ, x and y; the dependence on z for some different fixed values of x and c is illustrated in Table 1.

Table 1. Values of A^{pair} (eq. (37)) in % for different fixed values of x and $c = \cos\phi$ as a function of z.

z	A^{pair}, % ($c = 0.5$)			A^{pair}, % ($x = 0.5$)		
	x=0.2	x=0.5	x=0.8	c=0.0	c=0.3	c=0.6
1.0	-.074	.00	.074	.00	.00	.00
1.5	-.14	-.024	.10	.23	.12	-.11
2.0	-.26	-.12	.17	.42	.16	-.28
2.5	-.43	-.30	.25	.59	.14	-.53
3.0	-.64	-.54	.33	.75	.06	-.84
3.5	-.88	-.85	.39	.90	-.06	-1.23
4.0	-1.15	-1.21	.43	1.04	-.22	-1.68

In the bremsstrahlung case only the diagrams fig. 2c have non-zero s-channel imaginary parts which leads to one-spin correlations in our energy and angle ranges.

The corresponding contribution to the matrix element has the form

$$M^{(1)} = \frac{(4\pi\alpha)^{5/2}(-2i\pi^2)}{(2\pi)^4\vec{q}^{\,2}s}\bar{u}(p')p_1u(p)\cdot\bar{u}(p_2)O^{(1)}u(p_1), \quad (39)$$

where

$$O^{(1)} = \int\frac{d^4k}{i\pi^2}\Bigg\{\frac{\gamma_\eta(\hat{p}_2-\hat{k})\hat{e}(\hat{p}_2+\hat{k}_1-\hat{k})\hat{p}(\hat{p}_1-\hat{k})\gamma_\eta}{(0)(1)(2)(q)} + \frac{\hat{e}(\hat{p}_2+\hat{k}_1)}{2p_2k_1}\left[\frac{\gamma_\eta(\hat{p}_2+\hat{k}_1-\hat{k})\hat{p}(\hat{p}_1-\hat{k})\gamma_\eta}{(0)(1)(q)} + \frac{\gamma_\eta(\hat{p}_2+\hat{k}_1-\hat{k})\gamma_\eta}{(0)(q)}\right] + \frac{\gamma_\eta(\hat{p}_2-\hat{k})\hat{e}(\hat{p}_2+\hat{k}_1-\hat{k})\gamma_\eta}{(0)(2)(q)}\cdot\frac{(\hat{p}_2+\hat{k}_1)\hat{p}}{2p_2k_1}\Bigg\}, \quad (40)$$

$$(0) \equiv (k^2-\lambda^2), \quad (1) \equiv (p_1-k)^2-m^2, \quad (2) \equiv (p_2-k)^2-m^2,$$
$$(q) \equiv (p_2-q-k)^2-m^2.$$

The asymmetry arises from the interference of the imaginary part of $M^{(1)}$ with M^{born} (eq. (24)). We use the following expression for the spin density matrix of the initial electron:

$$u(p_1)\bar{u}(p_1) = \hat{p}_1(1-\gamma_5\xi), \quad (41)$$

where ξ is the degree of its longitudinal polarization. Further we consider the terms from the whole interference

$$\Re e(M^{born})^*M^{(1)} \quad (42)$$

which contain the quantity

$$I = \xi\varepsilon_{\mu\nu\lambda\sigma}p_1^\mu p_2^\nu p^\lambda q^\sigma = \xi s[\vec{p}_2\times\vec{k}_1]_z = \xi|\vec{p}_2||\vec{k}_1|\sin\phi, \quad (43)$$

where ϕ is the azimuthal angle between the transverse components of the photon momentum and the one of the scattered electron, $0 < \phi < \pi$.

Table 2. Values of A^{brem} (eq. (44)) in % for different fixed values of y and $c = \cos\phi$ as a function of z.

z	A^{brem}, % ($c = 0.5$)			A^{brem}, % ($y = 0.5$)		
	y=0.2	y=0.5	y=0.8	c=0.0	c=0.3	c=0.6
1.0	-.28	.004	.036	.036	.005	.007
1.5	-.85	-.068	.027	.021	-.062	-.059
2.0	-1.7	-.17	.013	.012	-.15	-.16
2.5	-2.9	-.31	.000	.007	-.26	-.30
3.0	-4.4	-.49	-.012	.004	-.39	-.48
3.5	-6.3	-.71	-.022	.003	-.56	-.72
4.0	-8.4	-.97	-.032	.002	-.75	-1.0

Using the integrals form the Appendix we obtain

$$
\begin{aligned}
A^{brem} &= \frac{1}{\xi_2 \sin\phi} \frac{d\sigma_{pol}^{\bar{e}e\to\bar{e}e\gamma}}{d\sigma_{nonpol}^{\bar{e}e\to\bar{e}e\gamma}} \\
&= \frac{\alpha z(1-y)}{2(1+y)^2}\{-4(1-y)zc\ln(\frac{z^2}{y(1+2zc+z^2)}) + \frac{y}{b^2}(y+2b^2 \qquad (44)\\
&+(2-3y)b-(1+2zc+z^2)(y(1-y)+2(1-y)b))\},\\
b &= y+z^2(1-y), \quad z=\frac{|\vec{p}_2|}{|\vec{k}_1|}, \quad c=\cos\phi,
\end{aligned}
$$

where $y = 1-x$ is the energy fraction of the scattered electron. Again, the analyzing power A^{brem} does not depend on the fermion mass in the kinematical region $|\vec{p}_2| \sim |\vec{k}_1| \sim |\vec{q}| \gg m$. For some fixed values of c and y it is given in Table 2 as a function of z.

5 Conclusions

The formulae given above for the pair production case could be generalized for the case of QCD. For the two jets production process by a circularly polarized photon on a hadron the additional factor $-\alpha_S/(2N\alpha) \sim -7$, $(N=3, \alpha_S \sim 0.3)$ appears in the right part of eq. (37) for A^{pair}. In the case of the two jets production process by a circularly polarized photon on a positron the reinforcement coefficient would be even larger: $2(N^2-1)\alpha_S/\alpha$, in this case our estimation is only qualitative (it is needed to include loop corrections to the non–polarized cross section).

We do not consider here the case of a heavy pair production where the deviation from the eikonal-phase multiplier could appear, which would cause the asymmetry.

The analyzing powers of the considered processes as well as the cross sections do not decrease with increasing of the energy of the initial particles. The suggested processes and the angle ranges are convenient for the modern high energy experiments. That is an advantage in comparison with the known methods [1,2] of the polarization measurement. We underline also that in our case the target implied to be unpolarized.

Acknowledgement

We are indebted for useful discussions and criticism A. Potylitsin, E. Vinokurov, Yu. Peresun'ko and S. Gevorkyan.

Appendix

In the Appendix we list the relevant one–loop integrals as for the pair production process as for the bremsstrahlung one.

Pair Production One–Loop Integrals

The notations for the denominators:

$$
\begin{aligned}
&(1) \equiv (q_1-k)^2-m^2, \qquad (2)\equiv(q_2+k)^2-m^2, \qquad (A.1)\\
&(q) \equiv (q_1-q-k)^2-m^2, \qquad (0)\equiv k^2.
\end{aligned}
$$

The scalar integrals:

$$ I = \Im m \int \frac{d^4k}{i\pi^3}\frac{1}{(0)(q)(1)(2)}, \qquad (A.2)$$

$$ i = \Im m \int \frac{d^4k}{i\pi^3}\frac{1}{(1)(2)(q)}, \qquad (A.3)$$

$$ z = \Im m \int \frac{d^4k}{i\pi^3}\frac{1}{(1)(2)(0)}. \qquad (A.4)$$

The vector integrals:

$$ I^\mu = \Im m \int \frac{d^4k}{i\pi^3}\frac{k^\mu}{(0)(q)(1)(2)} = I_1 q_1^\mu + I_2 q_2^\mu + I_q q^\mu, \qquad (A.5)$$

$$ i^\mu = \Im m \int \frac{d^4k}{i\pi^3}\frac{k^\mu}{(1)(2)(q)} = i_1 q_1^\mu + i_2 q_2^\mu + i_q q^\mu, \qquad (A.6)$$

$$ z^\mu = \Im m \int \frac{d^4k}{i\pi^3}\frac{k^\mu}{(1)(2)(0)} = z_1 q_1^\mu + z_2 q_2^\mu. \qquad (A.7)$$

The tensor integrals:

$$
\begin{aligned}
I^{\mu\nu} &= \Im m \int \frac{d^4k}{i\pi^3}\frac{k^\mu}{(0)(q)(1)(2)} = I_g g^{\mu\nu} + I_{11}q_1^\mu q_1^\nu + I_{22}q_2^\mu q_2^\nu + I_{qq}q^\mu q^\nu\\
&+ I_{12}\{q_1,q_2\}^{\mu\nu} + I_{1q}\{q_1 q\}^{\mu\nu} + I_{2q}\{q_2,q\}^{\mu\nu}, \qquad (A.8)\\
i^{\mu\nu} &= \Im m \int \frac{d^4k}{i\pi^3}\frac{k^\mu}{(1)(2)(q)} = i_g g^{\mu\nu} + i_{11}q_1^\mu q_1^\nu + i_{22}q_2^\mu q_2^\nu + i_{qq}q^\mu q^\nu
\end{aligned}
$$

$$+i_{12}\{q_1,q_2\}^{\mu\nu}+i_{1q}\{q_1 q\}^{\mu\nu}+i_{2q}\{q_2,q\}^{\mu\nu}, \tag{A.9}$$

where $\{\ldots\}^{\mu\nu}$ means the symmetrization: $\{ab\}^{\mu\nu}=a^\mu b^\nu+a^\nu b^\mu$.

And the last integral

$$\begin{aligned}I^{\mu\nu\rho}=\Im m\int\frac{d^4k}{i\pi^2}\frac{k^\mu k^\nu k^\rho}{(0)(1)(2)(q)}&=a_1\{q,g\}+a_2\{q_1,g\}+a_3\{q_2,g\}+a_4\{q^3\}\\ &+a_5\{q_1^3\}+a_6\{q_2^3\}+a_7\{q_1,q_2,q\}+a_8\{q_1,q_2^2\}+a_9\{q_1^2,q_2\}\\ &+a_{10}\{q_1,q^2\}+a_{11}\{q_1^2,q\}+a_{12}\{q_2^2,q\}+a_{13}\{q_2,q^2\},\end{aligned}\tag{A.10}$$

where we used the notations:

$$\begin{aligned}&\{a,b,c\}=a^\mu\{bc\}^{\nu\rho}+b^\mu\{ac\}^{\nu\rho}+c^\mu\{ab\}^{\nu\rho}, &&\{a^3\}=a^\mu a^\nu a^\rho,\\ &\{a^2,b\}=a^\mu a^\nu b^\rho+a^\mu b^\nu a^\rho+b^\mu a^\nu a^\rho, &&\{g,a\}=g^{\mu\nu}a^\rho+g^{\mu\rho}a^\nu+g^{\rho\nu}a^\mu.\end{aligned}\tag{A.11}$$

All integrals besides the last one have been evaluated in [5], but in the scattering chanel. To convert them into our (pair production) chanel we have to replace $p_2\to q_1$, $p_1\to -q_2$, $k_1\to -k_1$, $q\to q$. An imaginary part appears in $l_u\equiv\ln(-u/m^2)\to\ln(-s_1/m^2-i0)=\ln(s_1/m^2)-i\pi$. In this way after the transformation we obtain using the results of paper [5]

$$\begin{aligned}&I=\frac{1}{k_2 s_1}(l_t+\ln\frac{m}{\lambda}),\quad i=\frac{1}{a}l_s,\\ &I_1=-\frac{l_s}{2k_1a}-\frac{s_1-2k_1}{4k_1k_2s_1}l_t,\qquad I_2=-\frac{\vec q^{\,2}}{2k_1s_1a}l_s+\frac{s_1-2k_2}{4k_1k_2s_1}l_t,\\ &I_q=-\frac{l_s}{2k_1a}-\frac{l_t}{4k_1k_2},\qquad i_1=-\frac{\vec q^{\,2}}{a^2}l_s-\frac{s_1-\vec q^{\,2}}{a^2},\\ &i_2=-\frac{s_1}{a^2}l_s+\frac{s_1-\vec q^{\,2}}{a^2},\qquad i_q=\frac{s_1}{a^2}l_s-2\frac{s_1}{a^2},\\ &i_g=-\frac{s_1}{4a},\qquad z-tI=-\frac{l_s}{s_1}+\frac{2}{s_1}l_t,\qquad z_1=z_2=-\frac{1}{s_1},\\ &i_{11}=\frac{(\vec q^{\,2})^2}{a^3}l_s-\frac{3(\vec q^{\,2})^2-4s_1\vec q^{\,2}-s_1^2}{2a^3},\qquad i_{22}=\frac{s_1^2}{a^3}l_s+\frac{(\vec q^{\,2})^2+4s_1\vec q^{\,2}-3s_1^2}{2a^3},\\ &i_{12}=\frac{\vec q^{\,2}s_1}{a^3}l_s+\frac{(\vec q^{\,2})^2-4s_1\vec q^{\,2}+s_1^2}{2a^3},\qquad i_{1q}=-\frac{s_1\vec q^{\,2}}{a^3}l_s-\frac{s_1(s_1-5\vec q^{\,2})}{2a^3},\\ &i_{2q}=-\frac{s_1}{a^3}l_s+\frac{s_1(5s_1-\vec q^{\,2})}{2a^3},\qquad i_{qq}=\frac{s_1^2}{a^3}l_s-\frac{3s_1^2}{a^3},\end{aligned}\tag{A.12}$$

$$\begin{aligned}&I_g=-\frac{1}{4k_1}l_s+\frac{1}{4k_1}l_t,\\ &I_{11}=-\frac{s_1k_2-2k_1\vec q^{\,2}}{2k_1^2a^2}l_s+\frac{(s_1-2k_1)^2}{8k_1^2k_2s_1}l_t-\frac{2k_1(\vec q^{\,2})^2-2k_2s_1^2}{4k_1k_2s_1a^2},\\ &I_{22}=-\frac{\vec q^{\,2}(k_2\vec q^{\,2}-2s_1k_1)}{2k_1^2a^2s_1}l_s+\frac{(s_1-2k_2)^2}{8k_1^2k_2s_1}l_t+\frac{(k_2(\vec q^{\,2})^2-k_1s_1^2)}{2k_1k_2s_1a^2},\\ &I_{qq}=-\frac{s_1(k_2+2k_1)}{2k_1^2a^2}l_s+\frac{s_1}{8k_1^2k_2}l_t-\frac{s_1(k_1-k_2)}{2k_1k_2a^2},\\ &I_{12}=-\frac{\vec q^{\,2}(k_2+2k_1)}{2k_1^2a^2}l_s+\frac{\vec q^{\,2}}{8k_1^2k_2}l_t+\frac{\vec q^{\,2}(k_2-k_1)}{2k_1k_2a^2},\\ &I_{1q}=\frac{s_1a+2k_1\vec q^{\,2}}{4k_1^2a^2}l_s+\frac{s_1-2k_2}{8k_1^2k_2}l_t+\frac{s_1k_2+k_1\vec q^{\,2}}{2k_1k_2a^2},\\ &I_{2q}=\frac{a\vec q^{\,2}+2k_1s_1}{4k_1^2a^2}l_s-\frac{s_1-2k_2}{8k_1^2k_2}l_t+\frac{k_1s_1+k_2\vec q^{\,2}}{2k_1k_2a^2},\end{aligned}$$

where we used the conservation law $q+k_1=q_1+q_2$ and the notations

$$\begin{aligned}&l_s=\ln(\frac{s_1}{m^2}),\quad l_t=\ln(-\frac{t}{m^2}),\quad s_1=2q_1q_2,\quad a\equiv s+t,\\ &t=-2q_2k_1\equiv-2k_2,\quad -2q_1k_1=-2k_1,\quad s_1-2k_1-2k_2=-\vec q^{\,2}<0.\end{aligned}\tag{A.13}$$

Here k_1, q_1 and q_2 are the four-momenta of the initial photon, the electron and the positron from the created pair respectively. q is the four momentum transferred to the target.

To evaluate the last integral $I^{\mu\nu\rho}$ (see (A.10)) we multiply it by $g^{\mu\rho}$:

$$\begin{aligned}i^\mu=&\,6a_1q^\mu+6a_2q_1^\mu+6a_3q_2^\mu+a_4q^2q^\mu+a_7(2q_1q_2\,q^\mu+2q_2q\,q_1^\mu+2q_1q\,q_2^\mu)\\ &+2a_8q_1q_2\,q_2^\mu+2a_9q_1q_2\,q_1^\mu+a_{10}(2q_1q\,q^\mu+q^2\,q_1^\mu)+2a_{11}q_1q\,q_1^\mu\\ &+2a_{12}q_2q\,q_2^\mu+a_{13}(q^2\,q_2^\mu+2q_2q\,q^\mu).\end{aligned}\tag{A.14}$$

Multiplying by $2q_1^\rho$ and taking into account that $2q_1k=k^2$ we obtain

$$\begin{aligned}i^{\mu\nu}=&\,2a_1(q_1q\,g+\{q_1,q\})+4a_2\{q_1^2\}2a_3(\{q_1,q_2\}+q_1q_2\,g)+2a_4q_1q\{q^2\}\\ &+2a_6q_1q_2\{q_2^2\}+2a_7(q_1q_2\{q_1,q\}+q_1q\{q_1,q_2\})+2a_8q_1q_2\{q_2^2\}\\ &+2a_9q_1q_2\{q_1^2\}+2a_{10}q_1q\{q_1,q\}+2a_{11}q_1q\{q_1^2\}\\ &+2a_{12}(q_1q_2\{q_2,q\}+q_1q\{q_2^2\})+2a_{13}(q_1q\{q_2,q\}+q_1q_2\{q^2\}).\end{aligned}\tag{A.15}$$

Multiplying now by $2q_2^\rho$ and canceling $k^2=-2q_2k$ we obtain

$$
\begin{aligned}
-i^{\mu\nu} = {} & 2a_1(q_2qg + \{q_2, q\}) + 2a_2(\{q_1, q_2\} + q_1q_2g) + 4a_3\{q_2^2\} + 2a_4q_2q\{q^2\} \\
& +2a_5q_1q_2\{q_1^2\} + 2a_7(q_1q_2\{q_2, q\} + q_2q\{q_1, q_2\}) \\
& +2a_8q_1q_2\{q_2^2\} + 2a_9q_1q_2\{q_1, q_2\} + 2a_{10}(q_1q_2\{q^2\} + q_2q\{q_1, q\}) \\
& +2a_{11}(q_1q_2\{q_1, q\} + q_2q\{q_1^2\}) + 2a_{12}q_2q\{q_2^2\} + 2a_{13}q_2q\{q_2q\} .
\end{aligned} \tag{A.16}
$$

And finally multiplying eq. (A.10) by $2q$ and taking only the terms with $g^{\mu\nu}$, remembering that $2qk = (q_1 - q - k)^2 + 2k_2$ we obtain

$$
z_g + 2k_2I_g = 2a_1q^2 + 2a_2q_1q + 2a_3q_2q . \tag{A.17}
$$

So, we have 18 equations for 13 variables. We present them in Table 3.

Using the strings 1–3 of Table 3 we find a_1, a_2, a_3. Then subtracting 13 from 5 and solving it with 17, we find a_{12}, a_{13} and so on. In such a way we obtain the values for a_j:

$$
\begin{aligned}
a_1 = {} & \frac{s_1}{16k_1^2}l_s - \frac{s_1}{16k_1^2}l_t + \frac{s_1}{8k_1a}, \qquad a_2 = \frac{s_1 - 2k_2}{16k_1^2}l_s - \frac{s_1 - 2k_2}{16k_1^2}l_t + \frac{s_1}{8k_1a}, \\
a_3 = {} & -\frac{s_1 - 2k_2}{16k_1^2}l_s + \frac{s_1 - 2k_2}{16k_1^2}l_t + \frac{\vec{q}^2}{8k_1a}, \\
a_5 = {} & \frac{s_1^2(3ak_1 - 2k_2^2) - 6s_1ak_1(2k_1 + k_2) - 6k_1^2a^2}{4a^3k_1^3}l_s - \frac{(s_1 - 2k_2)^3}{16s_1k_2k_1^3}l_t \\
& + \frac{s_1^3(2k_2^2 - 3k_1^2 + 5k_2k_1) + 3s_1^2k_1a(2k_2 - 3k_1) - 9s_1a^2k_1^2 - 3k_1^2a^3}{4k_2s_1a^3k_1^2}, \\
a_6 = {} & \frac{s_1^3(4k_2^2 - 6ak_1) + 3s_1^2a^3 - 6s_1k_2a^3 + 4k_2^2a^3}{8a^3k_1^3s_1}l_s + \frac{(s_1 - 2k_2)^3}{16s_1k_2k_1^3}l_t \\
& + \frac{s_1^3(3k_1^2 - 5k_2k_1 - 2k_2^2) - 3s_1^2ak_2(2k_2 + 3k_1) - 3s_1a^2k_2(k_1 + 2k_2)}{4s_1a^3k_1^2k_2} \\
& - \frac{k_2a^3(2k_2 - k_1)}{4s_1a^3k_1^2k_2},
\end{aligned} \tag{A.18}
$$

Table 3. The set of equations for a_i. The right sides of the equations are in the first column. The other columns gives the coefficients before correspondent a_i (the blanks mean zeros).

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	
i_g	$2p_1q$		$2p_1p_2$											1
$-i_g$	$2p_2q$	$2p_1p_2$												2
$z_g - I_g$	$2q^2$	$2p_1q$	$2p_2q$											3
i_1		6					$2p_2q$		$2p_1p_2$	q^2	$2p_1q$			4
i_2			6				$2p_1q$	$2p_1p_2$				$2p_2q$	q^2	5
i_q	6			q^2			$2p_1p_2$			$2p_1q$			$2p_2q$	6
i_{11}		4							$2p_1p_2$		$2p_1q$			7
$-i_{11}$					$2p_1p_2$						$2p_2q$			8
i_{22}						$2p_1p_2$						$2p_1q$		9
$-i_{22}$			4					$2p_1p_2$				$2p_2q$		10
i_{qq}				$2p_1q$									$2p_1p_2$	11
$-i_{qq}$				$2p_2q$						$2p_1p_2$				12
i_{12}			2				$2p_1q$	$2p_1p_2$						13
$-i_{12}$		2					$2p_2q$		$2p_1p_2$					14
i_{1q}	2						$2p_1p_2$			$2p_1q$				15
$-i_{1q}$										$2p_2q$	$2p_1p_2$			16
i_{2q}												$2p_1p_2$	$2p_1q$	17
$-i_{2q}$	2						$2p_1p_2$						$2p_2q$	18

$$
\begin{aligned}
a_7 = {} & \frac{2s_1^2(2k_2^2 - 3ak_1) + 2s_1a(2k_2^2 - 3ak_1) - k_1a^3}{8a^3k_1^3}l_s + \frac{s_1(s_1 + a) + 2k_1k_2}{16k_1^3k_2}l_t \\
& + \frac{s_1^2(3k_1^2 - 2k_2^2 - 5k_1k_2) + 4s_1a(3k_1^2 - 2k_2^2 - 5k_1k_2) - a^2k_1k_2}{4a^3k_1^2k_2},
\end{aligned}
$$

$$a_8 = \frac{-2s_1^2(2k_2^2-3ak_1)-4s_1a(4k_1^2+5k_1k_2+2k_2^2)-a^4}{8k_1^3a^3}l_s$$
$$+\frac{(s_1-2k_2)\vec{q}^{\,2}}{16k_1^3k_2}l_t+\frac{s_1^2(2k_2^2+5k_1k_2-3k_1^2)}{4a^3k_1^2k_2}$$
$$+\frac{s_1a(4k_2^2+8k_1k_2-3k_1^2)+a^2k_2(3k_1+2k_2)}{4a^3k_1^2k_2},$$
$$a_9 = \frac{s_1^2(2k_2^2-3ak_1)+2s_1a(5k_1^2+4k_1k_2+k_2^2)+2a^2k_1(2k_1+k_2)}{4a^3k_1^3}l_s$$
$$-\frac{\vec{q}^{\,2}(s_1-2k_1)}{16k_1^3k_2}l_t+\frac{s_1^2(3k_1^2-2k_2^2-5k_1k_2)}{4a^3k_1^2k_2}$$
$$+\frac{s_1a(6k_1^2-7k_1k_2-2k_2^2)-2ak_1(3k_1^2+k_1k_2-2k_2^2)}{4a^3k_1^2k_2},$$
$$a_{10} = -\frac{s_1^2(2k_2^2-3ak_1)+2s_1ak_1(2k_1+k_2)}{4a^3k_1^3}l_s-\frac{s_1(s_1-2k_1)}{16k_1^3k_2}l_t$$
$$+\frac{s_1^2(2k_2^2+5k_1k_2-3k_1^2)+s_1ak_1(2k_2-3k_1)}{4a^3k_1^2k_2},$$
$$a_{11} = \frac{s_1^2(3ak_1-2k_2^2)-4s_1ak_1(2k_1+k_2)-2a^2k_1^2}{4a^3k_1^3}l_s-\frac{(s_1-2k_1)^2}{16k_1^3k_2}l_t$$
$$+\frac{s_1^2(2k_2^2+5k_1k_2-3k_1^2)-2s_1ak_1(3k_1-2k_2)-3a^2k_1^2}{4a^3k_1^2k_2},$$
$$a_{12} = -\frac{s_1^2(2k_2^2-3ak_1)+s_1a^3-a^3k_2}{4a^3k_1^3}l_s-\frac{(s_1-2k_2)^2}{16k_1^3k_2}l_t$$
$$+\frac{s_1^2(2k_2^2+5k_1k_2-3k_1^2)+2s_1ak_2(3k_1+2k_2)+a^2k_2(k_1+2k_2)}{4a^3k_1^2k_2},$$
$$a_{13} = \frac{s_1^2(4k_2^2-6ak_1)+a^3s_1}{8a^3k_1^3}l_s+\frac{s_1(s_1-2k_2)}{16k_1^3k_2}l_t$$
$$+\frac{s_1^2(3k_1^2-5k_1k_2-2k_2^2)-s_1ak_2(3k_1+2k_2)}{4a^3k_1^2k_2}.$$

Bremsstrahlung One–Loop Integrals

We will use the conservation law $p_2 - q = p_1 - k_1$ and the definitions

$$\begin{aligned}&(0)\equiv(k^2-\lambda^2),\quad (1)\equiv(p_1-k)^2-m^2,\quad (2)\equiv(p_2-k)^2-m^2,\\ &(q)\equiv(p_2-q-k)^2-m^2,\quad s=2p_2k_1,\quad t=-2p_1k_1,\quad u=-2p_1p_2,\\ &b\equiv s+u,\quad q^2=-\vec{q}^{\,2}=s+t+u,\end{aligned} \tag{A.19}$$

$$l_s=\ln(-s/m^2),\quad l_q=\ln(-q^2/m^2),\quad l_u=\ln(-u/m^2).$$

$$\chi = \Im m\int\frac{d^4k}{i\pi^3}\frac{1}{(0)(q)}, \tag{A.20}$$
$$\eta = \Im m\int\frac{d^4k}{i\pi^3}\frac{1}{(0)(1)(q)}, \tag{A.21}$$
$$j = \Im m\int\frac{d^4k}{i\pi^3}\frac{1}{(0)(2)(q)}, \tag{A.22}$$
$$I = \Im m\int\frac{d^4k}{i\pi^3}\frac{1}{(0)(1)(2)(q)}, \tag{A.23}$$
$$\chi^\mu = \Im m\int\frac{d^4k}{i\pi^3}\frac{k^\mu}{(0)(q)} = \chi_2(p_2-q)^\mu, \tag{A.24}$$
$$\eta^\mu = \Im m\int\frac{d^4k}{i\pi^3}\frac{k^\mu}{(0)(1)(q)} = \eta_1p_1^\mu+\eta_2(p_2-q)^\mu, \tag{A.25}$$
$$j^\mu = \Im m\int\frac{d^4k}{i\pi^3}\frac{k^\mu}{(0)(2)(q)} = j_2p_2^\mu+j_qq^\mu, \tag{A.26}$$
$$I^\mu = \Im m\int\frac{d^4}{i\pi^3}\frac{k^\mu}{(0)(1)(2)(q)} = I_1p_1^\mu+I_2p_2\mu+I_qq^\mu, \tag{A.27}$$
$$\eta^{\mu\nu} = \Im m\int\frac{d^4k}{i\pi^3}\frac{k^\mu k^\nu}{(0)(1)(q)} = \eta_gg+\eta_11\{p_1^2\}+\eta_{22}\{(p_2-q)^2\} \tag{A.28}$$
$$-\eta\{p_1,p_2-q\},$$
$$j^{\mu\nu} = \Im m\int\frac{d^4k}{i\pi^3}\frac{k^\mu k^\nu}{(0)(2)(q)} = j_gg+j_{22}\{p_2^2\}+j_{qq}\{q^2\}+j_{2q}\{p_2,q\}, \tag{A.29}$$
$$I^{\mu\nu} = \Im m\int\frac{d^4k}{i\pi^3}\frac{k^\mu k^\nu}{(0)(1)(2)(q)} = I_gg+I_{11}\{p_1^2\}+I_{22}\{p_2^2\} \tag{A.30}$$
$$+I_{1q}\{p_1,q\}+I_{2q}\{p_2,q\}+I_{qq}\{q^2\},$$

$$I^{\mu\nu\rho} = \Im m\int\frac{d^4k}{i\pi^3}\frac{k^\mu k^\nu k^\rho}{(0)(1)(2)(q)} = a_1\{q,g\}+a_2\{p_1,g\}+a_3\{p_2,g\} \tag{A.31}$$
$$+a_4\{q^3\}+a_5\{p_1^3\}+a_6\{p_2^3\}+a_7\{p_1,p_2,q\}+a_8\{p_1,p_2^2\}+a_9\{p_1^2,p_2\}$$
$$+a_{10}\{p_1,q^2\}+a_{11}\{p_1^2,q\}+a_{12}\{p_2^2,q\}+a_{13}\{p_2,q^2\}.$$

The values of the integrals are the following:

$$\chi=-l_t,\qquad \chi_2=-\frac{1}{2}l_t,\qquad \eta=\frac{1}{2t}l_t^2, \tag{A.32}$$
$$I=\frac{2}{ut}l_ul_t,\qquad j=\frac{1}{b}(\frac{1}{2}l_t^2-2l_tl_q),\qquad \eta_1=\frac{l_t^2}{2t}-\frac{2}{t}l_t,$$

$$\eta_2 = \frac{1}{t}l_t, \qquad j_2 = -\frac{t}{b^2}(\frac{1}{2}l_t^2 - 2l_tl_q) + l_t\frac{b-2q^2}{b^2}, \qquad j_q = \frac{1}{b}l_t,$$
$$I_1 = \frac{1}{2stu}[sl_t^2 - 2bl_tl_q + 2bl_tl_u], \qquad I_q = \frac{1}{2st}[2l_tl_u - 2l_tl_q],$$
$$I_2 = \frac{1}{2stu}[\frac{st}{b}l_t^2 - 2\frac{st-uq^2}{b}l_tl_q - 2(t+u)l_tl_u], \qquad \eta_q = -\frac{1}{4}l_t,$$
$$\eta_{11} = \frac{1}{2t}l_t^2 - \frac{3}{t}l_t, \qquad \eta_{22} = \frac{1}{2t}l_t, \qquad \eta_{1q} = -\frac{1}{2t}l_t,$$
$$j_q = \frac{t}{4b}l_t, \qquad j_{22} = \frac{t^2}{b^3}(\frac{1}{2}l_t^2 - 2l_tl_q) + \frac{l_t}{b}(\frac{3t^2}{b^2} + \frac{t}{b} - \frac{1}{2}),$$
$$j_{qq} = -\frac{1}{2b}l_t, \qquad j_{2q} = \frac{b-t}{2b^2}l_t.$$

she coefficients for $I^{\mu\nu}$ (eq. (A.30)) can be expressed in terms of I_q, $\eta_{1,2}$, $j_{1,2}$:

$$I_g = \frac{1}{2}tI_q, \qquad I_{11} = \frac{1}{su}[(u+s)^2I_q + (u+s)\eta_2 + s\eta_1], \tag{A.33}$$
$$I_{22} = \frac{1}{stu}[t(u+t)^2I_q + q^2(u+t)j_q + stj_2], \qquad I_{12} = \frac{1}{su}[-uq^2I_q - (u+t)\eta_2],$$
$$I_{1q} = \frac{1}{s}[(u+s)I_q + \eta_2], \qquad I_{2q} = \frac{1}{ts}[-t(u+t)I_q - q^2j_q],$$
$$I_{qq} = \frac{1}{s}[uI_q + \eta_2].$$

Here we list the coefficients a_i of the expression for $I^{\mu\nu\rho}$ (eq. (A.31)):

$$a_1 = \frac{1}{4s}[tuI_q + l_t], \qquad a_2 = \frac{1}{4s}[tbI_q + l_t], \tag{A.34}$$
$$a_3 = \frac{1}{4s}[-t(u+t)I_q - \frac{q^2}{b}l_t], \qquad a_4 = \frac{1}{2stu}[\frac{2u^3t}{s}I_q + \frac{(2u-s)u}{s}l_t],$$
$$a_5 = \frac{1}{u}[ba_{11} + \eta_{11}], \qquad a_6 = \frac{1}{u}[j_{22} - (t+u)a_{12}],$$
$$a_7 = -\frac{ts+2q^2u}{2s^2}I_q + \frac{s-2q^2}{2s^2t}l_t, \qquad a_8 = \frac{1}{u}[2a_3 - (t+u)a_7],$$
$$a_9 = \frac{1}{u}[4a_2 - (t+u)a_{11}], \qquad a_{10} = \frac{1}{2stu}[\frac{2u^2tb}{s}I_q + \frac{(2u+s)u}{s}l_t],$$
$$a_{11} = \frac{b^2}{s^2}I_q + \frac{3s+2u}{2s^2t}l_t,$$
$$a_{12} = \frac{(u+t)^2}{s^2}I_q + \frac{-s^3 + 2u^2(u+t) + 3s(u+t)^2 + 2s^2t + 2ut^2 + 2tu^2}{2b^2s^2t}l_t,$$
$$a_{13} = \frac{1}{2stu}[-\frac{2u^2t(u+t)}{s}I_q + \frac{(s^2-(t+u)(s+2u))u}{bs}l_t].$$

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Received by Publishing Department
on February 3, 1995.

Арбузов А.Б. и др. E2-95-44

Односпиновые асимметрии в процессах рождения пар
и тормозного излучения

Рассмотрены асимметрии в процессах взаимодействия циркулярно-поляризованного фотона и продольно-поляризованного электрона с заряженной мишенью при высоких энергиях. Представлены подробности вычислений. Показано, что асимметрии пропорциональны степени поляризации начальной частицы, не падают с ростом энергии и могут достигать нескольких процентов для чистых КЭД процессов или десятков процентов в случае рождения кварк-антикварковой пары. Поперечные к направлению начального пучка компоненты выходящих частиц считаются большими по сравнению с массой электрона (кварка).

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова и Лаборатории ядерных проблем ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1995

Arbuzov A.B. et al. E2-95-44

One-Spin Asymmetries in Pair Production
and Bremsstrahlung Processes

Asymmetries in the interaction processes at high energies of circularly polarized photons or longitudinally polarized electrons with charged targets are considered. The derivations are shown in detail. The asymmetries are shown to be proportional to the degree of polarization of the initial particle, they do not decrease with increasing of the energy and could reach several percents for pure QED processes or tenth percents in the case of a quark-antiquark jets production. Transverse to the beam direction components of the outgoing particle momenta are assumed to be large compared with the electron (quark) mass.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics and Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1995