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VECTOR MESON RESONANCE CONTRIBUTIONS TO $e^+e^- \to \pi^0\pi^0\gamma$ AND THE WIDTH OF $\rho \to \pi^0\pi^0\gamma$ DECAY

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1 Introduction

A considerable attention now is paid to the investigation of electromagnetic properties of neutral pion. Electric and magnetic polarizabilities are the subject as of theoretical investigations [1,2] as well as of experimental [3] ones. Some of results such as the total cross-section of $\gamma\gamma \to \pi^0\pi^0$ process may be explained rather well in the frame of the Chiral Perturbation theory (ChPT) [4].An essential role of vector mesons in the description of polarizabilities is understood in [1,2,5].

In this note we turn to the possibility to investigate the neutral pion electromagnetic properties in the feasible for experimental measurement [6,7] process:

$$e^+(p_+) + e^-(p_-) \longrightarrow \pi^0(p_1) + \pi^0(p_2) + \gamma(k). \qquad (1)$$

It is interesting to note that in the process one has a possibility to measure $\gamma\gamma\pi_0\pi_0$ interaction without any contamination. Really, the emission of a photon by the initial particles is strictly forbidden due to the fact that the system of two identical particles $\pi_0\pi_0$ can not be in the state with an odd orbital momentum (in the lowest order of the perturbation theory). As for pure QED process $e^+e^- \to 5\gamma$, which gives a similar to (1) final state (after decays $2\pi^0 \to 4\gamma$), it has rather small cross-section of about 10^{-35} cm^2 [8]. It can be excluded by a criterion on invariant masses of photon pairs: they are to coincide with the pion masses.

The general way, accepted now for the description of processes with photon–pion interactions, is the ChPT. We argue nevertheless that the description in the frames of the ChPT is non adequate for process (1) in the energy range $s \sim 1$ GeV^2. Really, here resonant properties of vector mesons play an essential role, whereas the dynamic character of intermediate vector meson states can not be described in finite number of terms from ChPT expansions. The vector meson dominance model is relevant here (see fig. 1).

2 Calculations

The matrix element of the process (1) reads

$$\mathcal{M}^{e\bar{e}\to\pi^0\pi^0\gamma} = \frac{ie\bar{v}\hat{\epsilon}u}{s}\,\epsilon^*_\rho(k_1)e_\sigma(k)T_{\rho\sigma}\,, \qquad (2)$$

where k_1 and k are the four–momenta of the virtual and real photons, ϵ and e — their polarization vectors, $s = 4\varepsilon^2 = k_1^2 = (p_- + p_+)^2$, $k^2 = 0$, $p_1^2 = p_2^2 = m_\pi^2$,

$k_1 = p_1 + p_2 + k$. Tensor $T_{\rho\sigma}$ [6] has the following general form:

$$T_{\rho\sigma} = 8\pi m_\pi \left(a_1 L^{(1)}_{\rho\sigma} + a_2 L^{(2)}_{\rho\sigma} + a_3 L^{(3)}_{\rho\sigma} + a_4 L^{(4)}_{\rho\sigma} + a_5 L^{(5)}_{\rho\sigma} \right), \tag{3}$$

where

$$\begin{aligned} L^{(1)}_{\rho\sigma} &= (k_1 k) g_{\rho\sigma} - k_{1\sigma} k_\rho, \\ L^{(2)}_{\rho\sigma} &= (k_1 k) Q_\rho Q_\sigma - (kQ)(k_{1\sigma} Q_\rho + k_\rho Q_\sigma) + (kQ)^2 g_{\rho\sigma}, \\ L^{(3)}_{\rho\sigma} &= (kQ)(k_1^2 g_{\rho\sigma} - k_{1\rho} k_{1\sigma}) + Q_\sigma((k_1 k) k_{1\rho} - k_1^2 k_\rho), \\ L^{(4)}_{\rho\sigma} &= k_\sigma((k_1 k) k_{1\rho} - k_1^2 k_\rho), \qquad L^{(5)}_{\rho\sigma} = k_\sigma((k_1 k) Q_{1\rho} - (k_1 Q) k_\rho). \end{aligned} \tag{4}$$

Tensor structures $L^{(i)}$ are chosen so that the correspondent coefficients a_i do not have kinematical zeros and singularities. They obey the gauge conditions:

$$T_{\rho\sigma} k_1^\rho = 0, \qquad T_{\rho\sigma} k^\sigma = 0. \tag{5}$$

Note that structures $L^{(4)}$, $L^{(5)}$ do not contribute to the cross-section. Quantities a_1, a_2 and a_3 describe some dynamic properties of neutral pion. Tensor structure $L^{(1)}$ coincides with the coefficient before the difference of electric and magnetic polarizabilities in the Compton scattering tensor on pion.

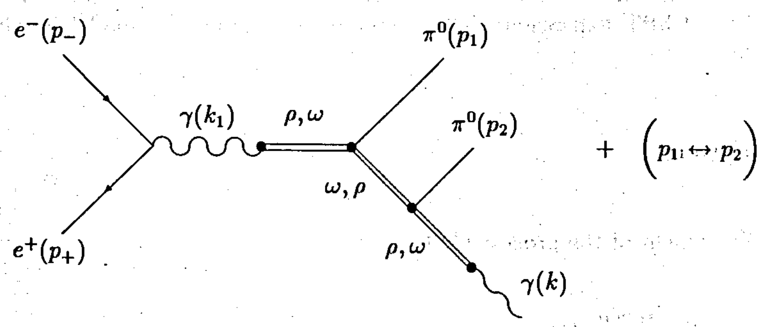

Fig. 1. Relevant Feynman diagrams in the vector meson dominance model.

Anomalous part of the effective chiral lagrangian and the terms, which describe vertexes with photons and vector mesons, reads

$$\mathcal{L} = -\frac{3}{16}\frac{g^2}{\pi^2 f_\pi}\varepsilon^{\mu\nu\alpha\beta}\pi^0 \partial_\mu \omega_\nu \partial_\alpha \rho^0_\beta + \frac{\sqrt{2}e}{g} a_\mu \left(m_\rho^2 \rho^0_\mu + \frac{1}{3} m_\omega^2 \omega_\mu \right), \tag{6}$$
$$g = \sqrt{2} g_{\rho\pi\pi} \approx 8.1, \qquad f_\pi \approx 94\text{MeV}.$$

Using the above expression and the Feynman diagram shown in fig. 1 we obtain the following tensor:

$$T^{\rho\sigma} = -\left[a_{12} p_1^\alpha p_2^\beta + a_{21} p_2^\alpha p_1^\beta \right] \varepsilon^{\nu\mu\alpha\rho} \varepsilon^{\nu\lambda\beta\sigma} k_1^\mu k^\lambda, \tag{7}$$

where

$$\begin{aligned} a_{12} &= \frac{a_\rho (1+\delta)}{s^2 (1 - x_1 + \gamma_\omega)}, \qquad \delta = \frac{1}{9}\frac{a_\omega}{a_\rho}\frac{1 - x_1 + \gamma_\omega}{1 - x_1 + \gamma_\rho}, \\ a_{21} &= a_{12}(x_1 \to x_2), \qquad \gamma_i = \frac{1}{s}(m^2 - m_i^2 + i m_i \Gamma_i), \qquad i = \rho, \omega, \\ a_i &= \frac{9 g^2_{\rho\pi\pi} \alpha m_i^2 s}{16 \pi^3 f_\pi^2 (s - m_i^2 + i m_i \Gamma_i)}, \end{aligned} \tag{8}$$

$\alpha \approx 1/137$ is the fine structure constant, $x = k^0/\varepsilon$, $x_{1,2} = p^0_{1,2}/\varepsilon$ are the energy fractions of the photon and the pions in the final state which satisfy the conditions

$$x_1 + x_2 + x = 2, \qquad \frac{m}{\varepsilon} < x_{1,2} < 1 - \frac{\delta\varepsilon}{2\varepsilon}; \tag{9}$$

m_ρ, m_ω, Γ_ρ, Γ_ω are the masses and the widths of ρ and ω mesons; m is the pion mass; $\delta\varepsilon$ is the threshold of the photon registration.

Comparing expressions (7) and (3) we find that

$$\begin{aligned} 8\pi m a_1 &= \left(\frac{1}{4}(k_1 k) + Q^2\right)(a_{12} + a_{21}) + (Qk)(a_{21} - a_{12}), \\ 8\pi m a_2 &= -(a_{12} + a_{21}), \qquad 8\pi m a_3 = \frac{1}{2}(a_{12} - a_{21}). \end{aligned} \tag{10}$$

Consider now the differential on the energy fractions of the final particles cross-section of the process. To obtain it we note that the current tensor

$$L_{\rho\xi} = \sum_{\sigma\zeta} T^{\rho\sigma} e_\sigma(k) \Big(T^{\xi\zeta} e_\zeta(k) \Big)^* \tag{11}$$

can be presented in the form

$$L_{\rho\xi} = -\frac{A}{3}\left(g_{\rho\xi} - \frac{k_1^\rho k_1^\xi}{k_1^2} \right), \qquad A = T^{\lambda\eta}(T_{\lambda\eta})^*, \tag{12}$$

which follows from the gauge invariance conditions (5). Quantity A can be presented in the form

$$\begin{aligned} A &= |a_\rho|^2 F(x_1, x_2), \\ F(x_1,x_2) &= (1 + P(x_1,x_2))\left\{ \left| \frac{1+\delta}{1 - x_1 + \gamma_\omega} \right|^2 r_1 \right. \\ &\quad \left. + \Re e\left(\left(\frac{1+\delta}{1-x_1+\gamma_\omega} \right)^* \frac{1+\delta}{1-x_2+\gamma_\omega} \right) r_2 \right\}, \\ r_1 &= \frac{1}{4}x^2(1-\beta^2)^2 + (1-\beta^2)\big[(1-x_2)^2 + 2(1-x_1)(1-x_2) \\ &\quad - x_1 x(1-x_1) - x(x_1+x_2-2x_1x_2)\big] + 2x_1x(1-x_1)(1-x) \\ &\quad + 2x_1x_2(1-x_1)(1-x_2) - 4(1-x_1)(1-x_2)(1-x), \\ r_2 &= \frac{1}{4}x^2(1-\beta^2)^2 + (1-\beta^2)\big[-x^2(1-x_1) + (1-x_1)(1-x_2) \\ &\quad + \frac{1}{2}x(x_1+x_2-x_1^2-x_2^2)\big] + x^2(1-x)^2 - 2(1-x)(1-x_1)(1-x_2) \\ &\quad - x(1-x)(x_1+x_2-x_1^2-x_2^2) + 2x_1x_2(1-x_1)(1-x_2), \end{aligned} \tag{13}$$

where $\beta^2 = 1 - (m/\varepsilon)^2$, $P(x_1,x_2)$ is the interchange operator: $P(x_1,x_2)f(x_1,x_2) = f(x_2,x_1)$. After the following rearrangement of the phase space volume of the final particles:

$$\frac{d^3p_1 d^3p_2 d^3k}{p_1^0 p_2^0 k^0}\delta^{(4)}(k_1 - p_1 - p_2 - k) = 2s\pi^2 dx dx_1 dx_2 \delta(x + x_1 + x_2 - 2), \tag{14}$$

we obtain the differential cross-section:

$$\begin{aligned} \left.\frac{d\sigma}{dx_1 dx_2}\right|_{s\sim m_\rho^2} &= \frac{3^3\alpha^3 m_\rho^4 s g_{\rho\pi\pi}^4 2^{-19}}{\pi^8 f_\pi^4[(s-m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2]} F(x_1,x_2) \\ &\approx F(x_1,x_2)\frac{\sigma_0}{(1-m_\rho^2/s)^2 + \Gamma_\rho^2/m_\rho^2}, \end{aligned} \tag{15}$$

$$\sigma_0 \approx 6\cdot 10^{-34}\text{cm}^2.$$

Function $F(x_1,x_2)$ is illustrated in Table 1 below.

Table 1. Function $F(x_1,x_2)$ (see eq. (13)) for different values of the variables.

x_1	0.6	0.7	0.7	0.8	0.8	0.9	0.9	0.9
x_2	0.5	0.4	0.6	0.5	0.7	0.4	0.6	0.8
F	1.6	1.3	0.2	0.2	0.05	0.1	0.15	0.05

Using the same technique as above we can estimate the partial width of the decay $\rho \to \pi^0\pi^0\gamma$. The differential width of the decay can be expressed as follows:

$$\frac{d\Gamma}{dx_1 dx_2} = \frac{27\alpha g_{\rho\pi\pi}}{2^{20}\pi^{10}} m_\rho \left(\frac{m_\rho}{f_\pi}\right)^4 (1 + P(x_1,x_2))F(x_1,x_2), \tag{16}$$

where function $F(x_1,x_2)$ is defined above in eq. (13). The integration of this expression over the pions energy fractions gives for the partial width and the relevant branching ratio:

$$\Gamma(\rho\to\pi^0\pi^0\gamma) = 56\text{keV}, \qquad Br(\rho\to\pi^0\pi^0\gamma) = 3.9\times 10^{-4}. \tag{17}$$

3 Conclusions

In conclusion we note that quantities a_1, a_2 and a_3 are complex–valued functions of energy $2\varepsilon = \sqrt{s}$, i.e. virtuality of the heavy photon, and of the energy fractions of the pions. The results of the comparison of a_1 with the static polarizability [5]

$$\alpha_{\pi^0} = -\frac{\alpha}{96\pi^2 m f_\pi^2} = -0.5\cdot 10^{-43}\text{cm}^3 \tag{18}$$

can be seen in Table 2, where we present function $f(x_1,x_2)$ defined by the following equation:

$$\begin{aligned} a_1 &= \frac{\alpha_{\pi^0} f(x_1,x_2)}{s/m_\rho^2 - 1 + i\Gamma_\rho/m_\rho}, \\ f(x_1,x_2) &= -\frac{27 g_{\rho\pi\pi}^2(2\gamma + x)(3x/2 - \beta^2) - (x_1 - x_2)^2}{16\pi^2(1 - x_1 + \gamma)(1 - x_2 + \gamma)}, \end{aligned} \tag{19}$$

$$\beta = 1 - \frac{4m^2}{s} \approx 1 - \frac{4m^2}{m_\rho^2}, \quad \gamma = -1 + \frac{m^2}{m_\rho^2}.$$

The total cross–section obtained by the integration of eq. (15) resonance like function of the beam energy:

$$\sigma(s) = N\sigma_0 \left[\left(1 - \frac{m_\rho^2}{s}\right)^2 + \frac{\Gamma_\rho^2}{m_\rho^2} \right]^{-1}. \tag{20}$$

In the nearest to the ρ–resonance area it reaches the magnitude

$$\sigma(s)\Big|_{s\approx m_\rho^2} \approx \left(\frac{m_\rho}{\Gamma_\rho}\right)^2 N\sigma_0(m_\rho^2) = 47\text{pb}, \tag{21}$$

$$N = \int d^2x F(x_1, x_2) = 0.26\,.$$

This quantity is large enough to be measured at colliders with the luminosity $\mathcal{L} \sim 10^{33}\text{cm}^{-2}\text{sec}^{-1}$.

The cross–section of the process calculated [9] in frames of the Chiral perturbation theory in the region $\sqrt{s} \sim 1$ GeV has too large value ($\sigma \sim 6$ nb) in comparison with the existing experimental bound [10] ($\sigma < 0.4$ nb).

Table 2. Function $f(x_1, x_2)$ (see eq. (19)) for different values of the variables.

x_1	0.6	0.7	0.7	0.8	0.8	0.9	0.9	0.9
x_2	0.5	0.4	0.6	0.5	0.7	0.4	0.6	0.8
f	10.3	12.9	3.2	4.6	−1.8	7.9	−1.0	−5.6

The consideration given above does not take into account the $f^0(980)$ meson contribution, which enters through the following mechanism: $4e\bar{e} \to \gamma^*_{} \to \gamma f^0 \to \gamma\pi^0\pi^0$. The relevant contribution to the tensor $T_\rho\sigma$ may be expressed by the replacement

$$T^{\rho\sigma} \to T^{\rho\sigma} + g_0 \frac{k_1 k\, g^{\rho\sigma} - k_1^\sigma k^\rho}{(k_1 - k)^2 - m_f^2 + i m_f \Gamma_f}, \tag{22}$$

where

$$|g_0| = 32\pi \frac{\Gamma_f}{m_f} \left[\frac{2}{3} B_2^{\gamma\gamma} B_2^{\lambda\pi} \sqrt{1 - \frac{4m^2}{m_f^2}} \right]^{1/2}. \tag{23}$$

Using the known f_0 meson parameters [11] we obtain $|g_0| \approx \sqrt{\Gamma_f/m_f}\ /20$. This quantity (taking into account the uncertainty in the value of the total f_0 meson width) is $\approx (1 \div 3) \times 10^{-2}$, which is at least one order of magnitude less than the quantity $|a_\rho| \approx \frac{1}{3}|s/m_\rho^2 - 1 + i\Gamma_\rho/m_\rho|^{-1}$. So, within the accuracy not worse then 10% we may neglect the f_0 meson contribution.

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