95-428

СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-95-428

V.N.Strel'tsov

RELATIVISTIC LENGTH EXPANSION

1995

Introduction. In relativity theory an interval takes the place of the previous «pre-relativistic» invariant — distance (length). Therefore, for example, one should say more correctly about the interval of a rod instead of its length. In the rest system of the rod, i.e., in essence in the non-relativistic limit, their values coincide that ensures the succession of corresponding theories and the necessary uniqueness of the interval. Taking into account interval Lorentz invariance in a moving system leads only to the «radar definition» of the moving rod length [1]. The consequence of this definition is the increase (but not the contraction) of longitudinal sizes of bodies in motion (see, e.g., [2]). We draw the readers' attention to the recently published book [3], where both existing approaches are considered in detail, and preference is given to «the hypothesis of length expansion». Also «the logical contradiction in the process of deriving the length contraction» is stressed. Note, besides, the recent remark [4] that Lorentz contraction is not a real physical phenomenon, and the statement about the importance of «the retarded length and volume» as the basis of «relativity theory (in contrast to the special relativity theory)»*.

Additional considerations of this problem are presented below.

The relativistic interval is a four-dimensional quantity defined by two point events and an analog of three-dimensional distance between two points. Or as one says, the metric of Minkowski's (4-dimensional) space is defined by the interval squared

$$-s^2 = (\Delta x)^2 - c^2 \Delta t^2 \qquad (1)$$

depending on the coordinate difference of these events. The interval is the main invariant of relativity theory, and so it is also named the fundamental invariant. By definition the invariant is a quantity which does not change when transiting from one inertial reference system to another one. Since this transition is related to changing motion velocity, then interval invariance must mean its independence of velocity, i.e., constancy (see, e.g., [6,7]). The material representatives of the space-like interval are scales (rods) and clocks for the time-like one.

Interval uniqueness. One of the main demands that the definition of physical notion (quantity) has to satisfy is its uniqueness**. However, at the present time we have two mutual exclusive representation of the behaviour of the longitudinal

---

*In this connection see also [5].

**In general, the demand of uniqueness of the physical notion definition is in essence a necessary condition of its fitness.

sizes of moving bodies. This position is a consequence of the violation of the indicated demand when spreading the notion of length to fast motions.

On the other hand, the interval also uniquely allows, for example, rods to be classified as it was made previously by the «non-relativistic invariant» length.

The rod length. In analytic geometry the length of a rod is defined by the first term in the right side of eq.(1). In this case it is evident that the values of projections (items) are always smaller than the very length (its sum). The length coincides with the «maximum projection» only for a rod oriented along one of the coordinate axes. One can say that, for example, on the plane this situation takes place if the rotation angle $\varphi = 0$.

The rod interval. In the relativistic case the values of the «space projection» are always larger than the value of the very interval because of the negative sign in the expression for interval (its pseudo-Euclideanness). Therefore by analogy with the previous reasoning, the angle $\psi = 0$ of Lorentzian turn has to correspond to the «minimum (space) projection» now. Remind that $\psi = \beta$, where βc is the motion velocity. Whence it follows that the «minimum projection» is simply defined by the length of a resting rod. In other words, an immovable scale (of length l^*) measures a space-like interval [8]

$$s = l^*. \tag{2}$$

It should be noted that, strictly speaking, the very representation of the dependence of the moving rod length on velocity leads with necessity to the previous result. Indeed, in accordance with the Lorentz invariance demand, only a constant (independent of velocity) quantity can define the rod interval. But this one is solely the length of a resting rod. In so doing, the equality $\Delta t^* = 0$ ensures the interval uniqueness.

On the other hand, the space part of the interval squared in a moving system or the first term in the expression

$$s^2 = \frac{(l^*)^2}{1-\beta^2} - \frac{\beta^2 (l^*)^2}{1-\beta^2} \tag{3}$$

present the moving rod length squared. Whence we have the «elongation formula»

$$l = l^*(1-\beta^2)^{-1/2} \tag{4}$$

for the length of a moving rod.

Non-invariance of «contracted interval». At the same time according to the traditional (Einstein's) definition, the interval is simply equal to the contracted length

$$s_c = l^*(1-\beta^2)^{1/2} \tag{5}$$

because of the simultaneity condition of end-marks. Thus, the «contracted interval» s_c depends evidently on velocity, and this means that the traditional definition does not satisfy the Lorentz invariance demand (or the relativity principle [2]). On the other hand, the consequence of the concept of covariant (radar) length [2] is just eq.(2), i.e., the «radar definition» satisfies this demand.

Let us touch now upon the fundamental physical consequence of relativity theory that, however, in fact is ignored up to now when defining the notion of sizes of moving objects.

Lorentz covariance. By definition mathematical quantities presenting the covariant physical notion in different reference systems are related by the Lorentz transformation. In general, covariant operations are such operations that have sense independently of the reference system. The emission and absorption of a light signal can serve as an example. In relativity theory a physical notion is described by a set of point events that can in particular reduce to a pair of events. The coordinate differences of these events are defined by the interval (4-vector) of the physical notion. We want to emphasize here that according to the considered definition, a covariant quantity is given in all reference systems by a set of the same events.

Whence it follows immediately that simultaneous events cannot be used when defining physical notions in view of simultaneity relativity. As Einstein said himself [9]: «Four-dimensional continuum does not disintegrate objectively into sections among which the sections containing all simultaneous events would be». Thus, we have here very strong argument against the traditional definition leading to the known contraction of moving bodies.

Conclusion. The interval Lorentz invariance means its independence of motion velocity, i.e., constancy. Therefore, for example, the space-like interval is defined by the length of a resting rod. In a moving reference system the «space part» of the interval is always larger than the very one because of its pseudo-Euclideanness. And this means that longitudinal sizes of bodies expand (but not contract) in motion. The account of the interval uniqueness demand also leads to the same result. What is more, Einstein's condition of end-mark simultaneity enters into a contradictions with the Lorentz covariance.

The author thanks M.S.Khvastunov for discussions.

REFERENCES

1. Strel'tsov V.N. — JINR P2-95-35, Dubna, 1995.
2. Idem — Found Phys., 1976, 6, p.293.
3. Kwak Kyeong-Do.— Length Expansion. The Inst. of Asian Studies, Seoul, 1995.

4. Jefimenko O.D. — Galilean Electrodynamics, 1995, 6, p.23.
5. Idem — Am. J. Phys., 1995, 63, p.267 & 454.
6. Strel'tsov V.N., Khvastunov M.S. — Izv. vuzov. Fizika, 1995, 38, p.125.
7. Strel'tsov V.N. — JINR D2-94-446, Dubna, 1994.
8. Mandel'shtam L.I. — Lectures on Optics, Relativity Theory and Quantum Mechanics. M.: Nauka, 1972, p.252.
9. Einstein A. — On the Special and General Theory of Relativity. London, 15th ed., 1952, Appendix V.

Received by Publishing Department
on October 16, 1995.