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VIRTUAL AND SOFT REAL PAIR PRODUCTION IN LARGE-ANGLE BHABHA SCATTERING

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1 Introduction

Due to the high development of the modern experimental technique higher order radiative corrections (RC) are of great interest. This paper presents the analytical calculations of a certain class of RC in the next–to–leading approximation to the large–angle Bhabha scattering (LABS) process.

The Bhabha scattering process is used at electron–positron colliders for luminosity measurements. Small scattering angles kinematics is used for high–energy colliders such as LEP I [1]. For the case of the colliders of moderately high energies, such as Φ, $c-\tau$ facilities, large scattering angles are more convenient [2]. The RC calculations up to the three–loop level in the leading logarithmic approximation and up to the two–loop level in the next–to–leading logarithmic approximation are required to get the theoretical accuracy better then 0.5%. For the case of small–angle Bhabha scattering (SABS) that task was fulfilled [3]. But the calculations of RC to large–angle Bhabha scattering are considerably different from the ones to SABS.

The LABS process and the first order electroweak and QED corrections to it were deeply studied in the literature (see [4] and references therein). Nevertheless the accuracy of the luminosity measurement is limited now by the accuracy of the theoretical description of the radiative corrections. Only few of $O(\alpha^2)$ RC were calculated earlier. In our recent paper [5] we started the systematic investigation of the subject, considering the hard pair production in large–angle Bhabha scattering.

This paper is devoted to the calculations of the QED $O(\alpha^2)$ RC to the large–angle Bhabha scattering connected with the production of a virtual or a soft e^+e^- pair. We work within the logarithmic accuracy and drop all the terms of the order α^2 which are not reinforced by the *large logarithm* $L = \ln(\varepsilon^2/m_e^2)$ (ε is the beam energy in the center–of–mass (CM) reference frame). In this paper we consider only the contribution of e^+e^- paris. The contributions due to muon pairs and other ones are less then the latter (they contain only the terms of the first order in L), they will be considered separately.

The general expression for the cross–section with the corrections under consideration could be presented in the form:

$$\mathrm{d}\sigma = \mathrm{d}\sigma_0\left\{1 + \left(\frac{\alpha}{\pi}\right)^2\left[\sum_{i=1}^{7}\delta_i + \delta^{\gamma}_{\mathrm{soft}} + \delta^{\gamma}_{\mathrm{hard}} + \delta^{e^+e^-}_{\mathrm{soft}}\right]\right\}, \qquad (1)$$

where $\mathrm{d}\sigma_0$ is the Born cross–section, δ_i arise from virtual corrections, $\delta^{\gamma}_{\mathrm{soft}}$ — from soft photon emission, $\delta^{\gamma}_{\mathrm{hard}}$ — from hard photon emission, and $\delta^{e^+e^-}_{\mathrm{soft}}$ — from soft pair production.

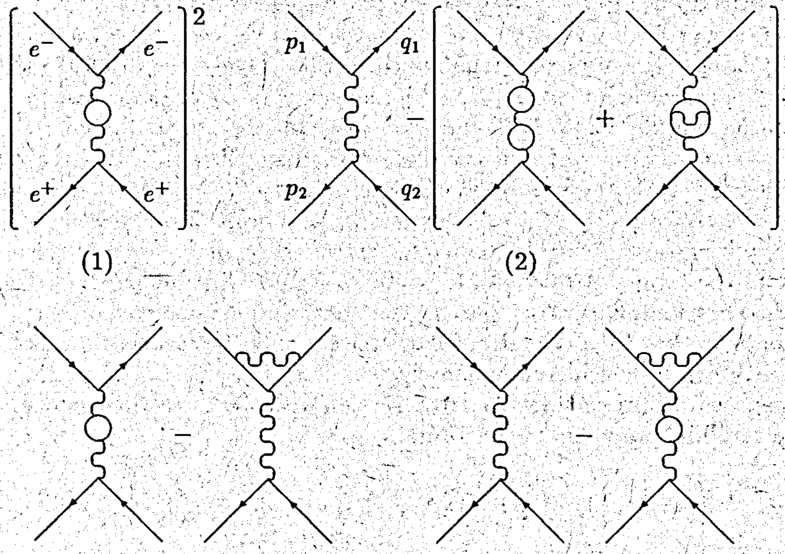
The paper is organized as follows: in the second part we consider virtual corrections; the third part is devoted to the contribution of the soft photon emission accompanied by the vacuum polarization of the virtual photon; in the fourth part we consider hard photon emission (also with vacuum polarization correction of the virtual photon propagator); in the fifth part the contribution of soft pair production is calculated; in the conclusion we sum the different contributions and discuss the results. Some useful but cumbersome expressions and integrals are given in the Appendixes.

2 Virtual Corrections

The Feynman diagrams describing the $O(\alpha^2)$ order RC to LABS process

$$e^-(p_1) + e^+(p_2) \to e^-(q_1) + e^+(q_2), \qquad (2)$$

which contain a vacuum polarization bubble could be divided into seven classes. In Fig. 1 one can see some representatives of the diagrams from different classes (any multiplication of diagrams has to be considered as a multiplication of a diagram by a conjugated one).



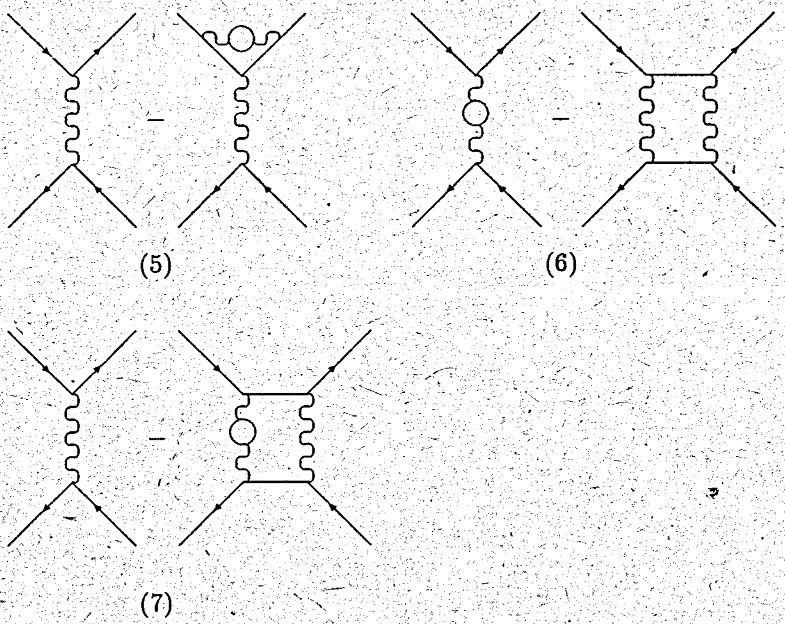


Figure 1: Representatives of Feynman diagrams for virtual pair production.

The first five contributions $\delta_{1,5}$ could be written down using the known expressions of vacuum polarization operators and vertex functions (only the Dirac form factor is relevant: the contribution of the Pauli one is proportional to m_e^2/ε^2). In the scattering channel one has for the vacuum polarization operators Π_{1t} (for a one–loop bubble) and Π_{2t} (for a two–loop bubble):

$$\Pi_{1t} = \frac{1}{3}l_t - \frac{5}{9}, \qquad \Pi_{2t} = \frac{1}{4}l_t + O(1), \qquad l_t = \ln\left(\frac{-t}{m_e^2}\right), \qquad (3)$$

and for the vertex functions F_{1t} (for a one–loop vertex correction) and F_{2t} (for the two–loop vertex correction which includes a vacuum polarization insertion (see fig. (5)):

$$\begin{aligned} F_{1t} &= (l_t - 1)(l_\lambda + 1) - \frac{1}{4}l_t - \frac{1}{4}l_t^2 + \frac{\pi^2}{12}, \qquad l_\lambda = \ln\left(\frac{\lambda}{m_e}\right), \\ F_{2t} &= -\frac{1}{36}l_t^3 + \frac{19}{72}l_t^2 - \left(\frac{\pi^2}{36} + \frac{265}{216}\right) l_t + O(1), \qquad (4) \\ t &= -2p_1q_1, \quad s = 2p_1p_2, \quad u = -2p_1q_2, \quad -t - s - m_e^2, \end{aligned}$$

λ is an auxiliary parameter — the *photon mass*. Similar expression could be written for the annihilation channel using the substitution:

$$l_t - \tilde{l}_s = l_s - i\pi, \qquad l_s = \ln\left(\frac{s}{m_e^2}\right). \tag{5}$$

After a simple algebra one obtains within the logarithmic accuracy:

$$\begin{aligned}
\sum_{i=1}^{5}\delta_i &= \left[2\left(\frac{s}{t}+\frac{t}{s}+1\right)^2\right]^{-1}\left\{\frac{s^2+u^2}{t^2}\Phi_t + \frac{t^2+u^2}{s^2}\Phi_s + \frac{2u^2}{st}\Phi_{st}\right\}, \qquad (6)\\
\Phi_t &= 3\Pi_{1t}^2 + 2\Pi_{2t} + 8\Pi_{1t}F_{1t} + 4F_{2t} = \\
&= \frac{8}{3}l_t\left(l_t - \frac{8}{3}\right)l_\lambda - \frac{7}{9}l_t^3 + \frac{9}{2}l_t^2 + l_t\left(\frac{\pi^2}{9} - \frac{311}{27}\right),\\
\Phi_s &= 3|\Pi_{1s}|^2 + 2\mathrm{Re}\,\Pi_{2s} + 8\mathrm{Re}\,\Pi_{1s}^*F_{1s} + 4\mathrm{Re}\,F_{2s} = \\
&= \frac{8}{3}l_s\left(l_s - \frac{8}{3}\right)l_\lambda - \frac{7}{9}l_s^3 + \frac{9}{2}l_s^2 + l_s\left(-\frac{2\pi^2}{9} - \frac{311}{27}\right),\\
\Phi_{st} &= \mathrm{Re}\,[\Pi_{1t}^2 + \Pi_{1s}^2 + \Pi_{1t}\Pi_{1s} + 2(\Pi_{1t} + \Pi_{1s})(F_{1t} + F_{1s}) + \Pi_{2t} + \Pi_{2s} + \\
&+ 2(F_{2t} + F_{2s})] = \left[\frac{2}{3}(l_s + l_t)^2 - \frac{32}{9}(l_s + l_t)\right]l_\lambda - \frac{2}{9}(l_t^3 + l_s^3) - \\
&- \frac{1}{6}l_tl_s(l_t + l_s) + \frac{61}{36}(l_t^2 + l_s^2) + \frac{10}{9}l_tl_s + \left(\frac{17}{36}\pi^2 - \frac{311}{54}\right)(l_t + l_s).
\end{aligned}$$

Consider now the virtual corrections of the sixth class: the ones due to the interference of the Born amplitude corrected by a vacuum polarization insertion with the box amplitude. Calculating the loop integrals over the box virtual momentum k one has to consider the scalar, vector and tensor ones. For example we present here the integrals [6] for scattering channel box diagram with uncrossed photon lines (see fig. 1(6)):

$$\begin{aligned}
b,\ J_\sigma,\ J_{\rho\sigma} &= \int\frac{\mathrm{d}^4k}{i\pi^2}\frac{1,\ k_\sigma,\ k_\rho k_\sigma}{(k^2 - \lambda^2 + i0)((q-k)^2 - \lambda^2 + i0)} - \qquad (7)\\
&- \frac{1}{((p_1+k)^2 - m_e^2 + i0)((-p_2+k)^2 - m_e^2 + i0)},\\
J_\sigma &= b_1\Delta_\sigma + b_2q_\sigma,\\
J_{\rho\sigma} &= b_3\Delta_\rho\Delta_\sigma + b_4P_\rho P_\sigma + b_5(\Delta_\rho q_\sigma + \Delta_\sigma q_\rho) + b_6q_\rho q_\sigma + b_7g_{\rho\sigma},\\
q &= q_1 - p_1 = p_2 - q_2, \qquad \Delta = \frac{1}{2}(p_2 - p_1), \qquad P = \frac{1}{2}(p_1 + p_2).
\end{aligned}$$

The explicit expressions for the coefficients $b - b_7$ are given in Appendix A.

After some algebraic work with traces one gets

$$\begin{aligned}
\delta_6 &= \left[2\left(\frac{s}{t}+\frac{t}{s}+1\right)^2\right]^{-1}\mathrm{Re}\,(1+P(s,t))\left\{\frac{\Pi_{1t}}{t}(1-P(s,u))f_1(s,t) - \right. \qquad (8)\\
&- \left.\frac{\Pi_{1s}^*}{s}(f_2(s,t) + f_3(u,t))\right\},
\end{aligned}$$

where

$$\begin{aligned}
f_1(s,t) &= -s(s^2+u^2)[b - b_1] + 2s^2t[-b_2 + b_5 + b_6] + \\
&+ \frac{s}{4}(2s^2+u^2)[-b_3 + b_4] + 2(4s^2+u^2)b_7,\\
f_2(s,t) &= u^2(s[b - b_1 + \frac{1}{4}b_3 - \frac{1}{4}b_4] - b_7),\\
f_3(u,t) &= u^2(u[-\tilde{b} + \tilde{b}_1 - \frac{1}{2}\tilde{b}_3 - \frac{1}{2}\tilde{b}_4] + 2t[-\tilde{b}_2 + \tilde{b}_5 + \tilde{b}_6] + 8\tilde{b}_7),\\
&\tilde{b},\ \tilde{b}_i = P(s,u)b,\ b_i \qquad \Pi_{1s}^* = \frac{1}{3}(l_s + i\pi) - \frac{5}{9},
\end{aligned}$$

The introduced interchange operators $P(s,t)$ and $P(s,u)$ act in the following way:

$$\begin{aligned}
P(s,t)f(s,t,u;\tilde{l}_s,l_t,l_u) &= f(t,s,u;l_t,\tilde{l}_s,l_u), \qquad (9)\\
P(s,u)f(s,t,u;\tilde{l}_s,l_t,l_u) &= f(u,t,s;l_u,l_t,\tilde{l}_s).
\end{aligned}$$

Consider now the corrections of the seventh class. In the calculations we use the substitution suggested by J. Schwinger for the photon propagator (with 4-momentum k) corrected by one-loop vacuum polarization insertion:

$$\begin{aligned}
&\frac{1}{k^2 - \lambda^2 + i0} - \frac{\alpha}{\pi}\int_0^1\frac{\mathrm{d}v\phi(v)}{1-v^2}\frac{1}{k^2 - M^2}, \qquad M^2 = \frac{4m_e^2}{1-v^2}, \qquad (10)\\
&\phi(v) = \frac{2}{3} - \frac{1}{3}(1-v^2)(2-v^2).
\end{aligned}$$

The interference of the 8 box diagrams with the Born ones gives the following contribution to the summed over spin states matrix element square:

$$\begin{aligned}
\sum_{\mathrm{spin}}|---|^2 &= \alpha^4 2^8\int_0^1\frac{\mathrm{d}v\phi(v)}{1-v^2}(1+P(s,t))\int\frac{\mathrm{d}^4k}{i\pi^2} - \qquad (11)\\
&- \frac{1}{(k^2 - \lambda^2 + i0)((k+q)^2 - M^2)}\left\{\left(\frac{S_1}{t} - \frac{A_1}{4s}\right)\frac{1}{a_1a_2} + \right.\\
&+ \left.\left(\frac{S_2}{t} - \frac{A_2}{4s}\right)\frac{1}{a_1a_3}\right\}, \qquad a_1 = (k+q_1)^2 - m_e^2 + i0,\\
a_2 &= (k-q_2)^2 - m_e^2 + i0, \qquad a_3 = (k+p_2)^2 - m_e^2 + i0,\\
S_1 &= \frac{1}{4}\mathrm{Tr}\,\gamma_\mu(\hat{q}_1 + \hat{k})\gamma_\nu\hat{p}_1\gamma_\lambda\hat{q}_1\ \frac{1}{4}\mathrm{Tr}\,\gamma_\mu(-\hat{q}_2 + \hat{k})\gamma_\nu\hat{p}_2\gamma_\lambda\hat{q}_2 = \\
&= 2(q_1+k)(-q_2+k)s^2 + 2\,\frac{1}{4}\mathrm{Tr}\,\hat{q}_1(-\hat{q}_2 + \hat{k})\hat{p}_1\hat{q}_2(\hat{q}_1 + \hat{k})\hat{p}_2,\\
S_2 &= \frac{1}{4}\mathrm{Tr}\,\gamma_\mu(\hat{q}_1 + \hat{k})\gamma_\nu\hat{p}_1\gamma_\lambda\hat{q}_1\ \frac{1}{4}\mathrm{Tr}\,\gamma_\mu(-\hat{p}_2 - \hat{k})\gamma_\nu\hat{q}_2\gamma_\lambda\hat{p}_2 = \\
&= 2(q_1+k)(-p_2-k)u^2 + 2\,\frac{1}{4}\mathrm{Tr}\,\hat{q}_1(-\hat{p}_2 - \hat{k})\hat{p}_1\hat{p}_2(\hat{q}_1 + \hat{k})\hat{q}_2,
\end{aligned}$$

$$A_1 = \frac{1}{4}\mathrm{Tr}\,\hat{q}_1\gamma_\mu(\hat{q}_1+\hat{k})\gamma_\nu\hat{p}_1\gamma_\lambda\hat{p}_2\gamma_\nu(-\hat{q}_2+\hat{k})\gamma_\mu\hat{q}_2\gamma_\lambda =$$
$$= -8\,\frac{1}{4}\mathrm{Tr}\,\hat{p}_2(\hat{q}_1+\hat{k})\hat{q}_2\hat{p}_1(-\hat{q}_2+\hat{k})\hat{q}_1\,,$$
$$A_2 = \frac{1}{4}\mathrm{Tr}\,\hat{q}_1\gamma_\mu(\hat{q}_1+\hat{k})\gamma_\nu\hat{p}_1\gamma_\lambda\hat{p}_2\gamma_\mu(-\hat{p}_2-\hat{k})\gamma_\nu\hat{q}_2\gamma_\lambda =$$
$$= -8u^2(q_1+k)(-p_2-k).$$

We used the identity:

$$\frac{1}{4}\mathrm{Tr}\,\gamma_\mu\hat{a}_1\gamma_\nu\hat{b}_1\gamma_\lambda\hat{c}_1\;\frac{1}{4}\mathrm{Tr}\,\gamma_\mu\hat{a}_2\gamma_\nu\hat{b}_2\gamma_\lambda\hat{c}_2 = 8a_1a_2 - b_1b_2 - c_1c_2 + +2\,\frac{1}{4}\mathrm{Tr}\,\hat{a}_1\hat{c}_2\hat{b}_1\hat{a}_2\hat{c}_1\hat{b}_2. \quad (12)$$

Further some scalar, vector and tensor integrals calculated within the logarithmic accuracy are necessary. We use the notations:

$$I(aba_1a_2),\, I_\rho,\, I_{\rho\sigma} = \int\limits_0^1 \frac{dv\phi(v)}{1-v^2}\int\frac{d^4k}{i\pi^2}\frac{1,\,k_\rho,\,k_\rho k_\sigma}{aba_1a_2}, \quad (13)$$

$$I(aba_2) = \int\limits_0^1\frac{dv\phi(v)}{1-v^2}\int\frac{d^4k}{i\pi^2}\frac{1}{aba_2}, \qquad I(ba_1a_2) = \int\limits_0^1\frac{dv\phi(v)}{1-v^2}\int\frac{d^4k}{i\pi^2}\frac{1}{ba_1a_2},$$

$$I_\rho = \alpha(p_{2\rho}-p_{1\rho}) + \beta q_\rho, \qquad \alpha = \frac{1}{2u}[-I(aba_1a_2) - 2I(aba_2) + I(ba_1a_2)],$$
$$\beta = \frac{1}{2tu}[(t-u)I(aba_1a_2) + sI(ba_1a_2) + 2tI(aba_2)], \quad a = k^2-\lambda^2,$$
$$b = (k+q)^2 - M^2, \quad a_1 = (k+q_1)^2 - m_e^2, \quad a_2 = (k-q_2)^2 - m_e^2,$$
$$I_{\rho\sigma} = f_0 g_{\rho\sigma} + f_1(q_{1\rho}q_{1\sigma} + q_{2\rho}q_{2\sigma}) + f_3 q_\rho q_\sigma + f_4(q_{1\rho}q_{2\sigma} + q_{2\rho}q_{1\sigma}) +$$
$$+ f_5(q_\rho(q_{2\sigma}-q_{1\sigma}) + q_\sigma(q_{2\rho}-q_{1\rho})).$$

In Appendix B we give an example of calculations and the list of scalar integrals. It appears that only two tensor coefficients, namely f_0 and f_4, are relevant. They contain only the first power of the large logarithm. Infrared parameter λ is contained only in $I(aba_1a_2)$:

$$I(aba_1a_2) = \frac{1}{3st}\mathrm{Re}\Big\{-\frac{1}{6}\tilde{l}_s^3 + \frac{1}{2}\tilde{l}_s^2 l_t + \tilde{l}_s l_t^2 - \frac{10}{3}\tilde{l}_s l_t - \frac{28}{9}l_s - - \frac{\pi^2}{6}l_t - 2\tilde{l}_s(l_t - \frac{5}{3})l_\lambda\Big\}. \quad (14)$$

3 The Soft Photon Emission

To eliminate the dependence on the photon mass λ we have to consider also the vacuum polarization corrected cross–section of the additional soft photon emission

of the energy

$$\omega < d\varepsilon, \qquad d - 1. \quad (15)$$

The correction could be obtained using the standard technique [9] of the soft radiation accounting:

$$\delta^\gamma = \Big[2\Big(\frac{s}{t}+\frac{t}{s}+1\Big)^2\Big]^{-1}\mathrm{Re}\Big[\frac{s^2+u^2}{t^2}\Pi_{1t} + \frac{t^2+u^2}{s^2}\Pi_{1s} + \quad (16)$$
$$+ \frac{u^2}{st}(\Pi_{1s}+\Pi_{1t})\Big]\Big\{4(\ln d - l_\lambda)(l_s + l_t - l_u - 1) + l_s^2 + l_t^2 -$$
$$- l_u^2 - \frac{2}{3}\pi^2 - 2\mathrm{Li}_2(\frac{1-c}{2}) + 2\mathrm{Li}_2(\frac{1+c}{2})\Big\},$$
$$c = \cos\theta, \quad \theta = \widehat{\mathbf{p}_1,\mathbf{q}_1}, \quad \mathrm{Li}_2(x) = -\int\limits_0^x\frac{dy\ln(1-y)}{y}.$$

4 The Hard Photon Emission

In order to cancel the auxiliary parameter d we have to consider also the case of hard photon (with the energy $\omega > d\varepsilon$) emission. Our method of calculations here consists in a splitting of the total kinematical region of the emitted photon into two ones: the collinear one, when the photon is emitted within a small cone in respect to one charged particle; and the semi–collinear one, when the photon moves outside of any such a cone. Then we show explicitly that the small auxiliary parameter θ_0, describing that cones, cancels in the sum of the contributions of both regions. The procedure allows us to extract explicitly the radiative corrections to the process under considerations of the orders $O(\alpha^2L^2)$ and $O(\alpha^2L)$.

4.1 The Semi–Collinear Kinematics of Hard Photon Emission

Consider at first the case, when the photon moves in respect to the directions of the charged particles (as of the initial ones as well as of the final ones) with the angles satisfying the following conditions:

$$\widehat{\mathbf{k}\mathbf{p}_{1,2}} > \theta_0, \qquad \widehat{\mathbf{k}\mathbf{q}_{1,2}} > \theta_0. \quad (17)$$

In this case the matrix element of the process is not singular and the contribution of this region in the $O(\alpha)$ order does not contain the large logarithm. In the next order in α we can just write down the contribution in the next–to–leading approximation multiplying the well known differential cross–section of a single hard photon emission [9] by the factor $2\alpha L/(3\pi)$, coming from the vacuum polarization insertion into the virtual photon:

$$
\begin{aligned}
\mathrm{d}\sigma^{\gamma}_{\text{semi-coll}} &= \frac{\alpha^3}{4s\pi^2}\frac{\alpha}{3\pi}L\frac{\mathrm{d}^3q_1\mathrm{d}^3q_2\mathrm{d}^3k}{q_1^0q_2^0k^0}W\,B\,\delta^4(p_1+p_2-q_1-q_2-k), \qquad (18)\\
W &= \frac{s}{p_1k\; p_2k}+\frac{s_1}{q_1k\; q_2k}-\frac{t}{p_1k\; q_1k}-\frac{t_1}{p_2k\; q_2k}+\frac{u}{p_1k\; q_2k}+\frac{u_1}{p_2k\; q_1k},\\
B &= \frac{ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)}{ss_1tt_1},\\
s &= 2p_1p_2, \quad t=-2p_1q_1, \quad u=-2p_1q_2, \quad p_i^2=q_i^2=0\\
s_1 &= 2q_1q_2, \quad t_1=-2p_2q_2, \quad u_1=-2p_2q_1.
\end{aligned}
$$

The contribution is to be integrated over the phase volume of the final particles, which is defined by experimental conditions, meanwhile restrictions (17) should be fulfilled.

4.2 The Collinear Kinematics of Hard Photon Emission

The contribution of the collinear kinematics of a photon emission is divided naturally into four ones in the correspondence to the cases of the photon motion in the four directions of the charged particles: 1) $\mathbf{k}-\mathbf{p}_1$, 2) $\mathbf{k}-\mathbf{p}_2$, 3) $\mathbf{k}-\mathbf{q}_1$ 4) $\mathbf{k}-\mathbf{q}_2$. So, we write the differential cross–section in the form:

$$
\frac{\mathrm{d}\sigma^{\gamma}_{\text{coll}}}{\mathrm{d}y_1\mathrm{d}c_-} = \frac{\mathrm{d}\sigma_1+\mathrm{d}\sigma_2+\mathrm{d}\sigma_3+\mathrm{d}\sigma_4}{\mathrm{d}y_1\mathrm{d}c_-}, \qquad (19)
$$

where $y_1=q_1^0/\varepsilon$ is the energy fraction of the scattered electron, $c_-=\cos\theta_-$, $\theta_-=\widehat{\mathbf{p}_1\mathbf{q}_1}$ is the electron scattering angle in the center–of–mass reference frame of the initial particles; subscripts in σ_i denote correspondent kinematical regions.

For the first region we get

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_1}{\mathrm{d}y_1\mathrm{d}c_-} &= \frac{\alpha^3y_1}{sx(1-x)(2-y_1(1-c_-))}[(1+(1-x)^2)L_0-2(1-x)] - \qquad (20)\\
&- \Big[\Pi^2_{t_1}\frac{4+(2-y_1(1-c_-))^2}{(y_1(1-c_-))^2}+-\Pi_{s_1}\frac{1}{4}[(y_1(1-c_-))^2+\\
&+(2-y_1(1-c_-))^2]--\mathrm{e}(\Pi_{t_1}\Pi_{s_1})\frac{(2-y_1(1-c_-))^2}{y_1(1-c_-)}\Big],\\
\Pi_{t_1} &= \Big(1-\frac{\alpha}{3\pi}(l_{t_1}-\frac{5}{3})\Big)^{-1}, \quad \Pi_{s_1}=\Big(1-\frac{\alpha}{3\pi}(l_{s_1}-\frac{5}{3}-i\pi)\Big)^{-1},\\
l_{t_1} &= \ln\Big(\frac{2y_1(1-c_-)(1-x)\varepsilon^2}{m_e^2}\Big), \quad l_{s_1}=\ln\Big(\frac{4\varepsilon^2(1-x)}{m_e^2}\Big),\\
x &= \frac{2(1-y_1)}{2-y_1(1-c_-)}, \quad L_0=\ln\Big(\frac{\varepsilon\theta_0}{m_e}\Big)^2-1,
\end{aligned}
$$

x is the energy fraction of the emitted photon. The energy fraction and the scattering angle of the positron in this kinematical region take the following forms:

$$
\begin{aligned}
y_2 &= \frac{q_2^0}{\varepsilon}=\frac{1+(1-y_1)^2+y_1(2-y_1)c_-}{2-y_1(1-c_-)}, \quad y_2c_+=-x-y_1c_-, \qquad (21)\\
c_+ &= \cos(\widehat{\vec{p}_1\vec{q}_2}), \quad y_2\sqrt{1-c_+^2}=y_1\sqrt{1-c_-^2}.
\end{aligned}
$$

Let us see that in the sum of the above contribution with the one of the semi–collinear region in the case when the photon is emitted close to the cone of the angle θ_0 around the initial electron beam direction the terms of the order $-L\ln\theta_0^2$ will disappear. Really, the lower limit of photon polar angle integration gives:

$$
\int\frac{\mathrm{d}^3k}{2\pi k^0}W\bigg|_{\vec{k}\|\vec{p}_1}\int_{1-\theta_0^2/2}\frac{\mathrm{d}c_-}{1-c_-}W-\mathrm{d}x\frac{2}{x}\ln\frac{1}{\theta_0^2}. \qquad (22)
$$

The second multiplier in the expression for the semi–collinear region turns out to be here:

$$
B_{\vec{k}\|\vec{p}_1}-2\Big(1-\frac{y_1(1-c_-)}{2}-\frac{2}{y_1(1-c_-)}\Big)^2\frac{1+(1-x)^2}{1-x}. \qquad (23)
$$

We need also to rearrange the phase volume:

$$
\mathrm{d}x\frac{\mathrm{d}^3q_1\mathrm{d}^3q_2}{q_1^0q_2^0}\delta^4(p_1(1-x)+p_2-q_1-q_2)-\frac{2\pi y_1\mathrm{d}y_1\mathrm{d}c_-}{2-y_1(1-c_-)}. \qquad (24)
$$

After these modifications it is easy to see that the terms $-L\ln\theta_0^2$ in the collinear and semi–collinear contributions differ only by signs.

The contribution of the second collinear region has the form

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_2}{\mathrm{d}y_1\mathrm{d}c_-} &= \frac{\alpha^3y_1}{sx(1-x)(2-y_1(1+c_-))}[(1+(1-x)^2)L_0-2(1-x)] - \qquad (25)\\
&- \Big[\Pi_t^2\frac{4+(2-y_1(1-c_-))^2}{(y_1(1-c_-))^2}+-\Pi_{s_1}\frac{1}{4}[(y_1(1-c_-))^2+\\
&+(2-y_1(1-c_-))^2]--\mathrm{e}(\Pi_t\Pi_{s_1})\frac{(2-y_1(1-c_-))^2}{y_1(1-c_-)}\Big],\\
\Pi_t &= \Big(1-\frac{\alpha}{3\pi}(l_t-\frac{5}{3})\Big)^{-1}, \quad l_t=\ln\Big(\frac{2y_1(1-c_-)\varepsilon^2}{m_e^2}\Big).
\end{aligned}
$$

We put here also the expressions for the photon and positron energy fractions and for the positron scattering angle:

$$
\begin{aligned}
1-x &= \frac{y_1(1-c_-)}{2-y_1(1+c_-)}, \quad y_2=\frac{1+(1-y_1)^2-y_1(2-y_1)c_-}{2-y_1(1+c_-)}, \qquad (26)\\
y_2c_+ &= x-y_1c_-.
\end{aligned}
$$

Again one can be convinced in the cancellation of $L\ln\theta_0^2$ in the sum with the relevant term of the semi-collinear contribution.

The contribution of the third collinear region and the parameters of the scattered positron are:

$$\frac{d\sigma_3}{dy_1 dc_-} = \frac{\alpha^3 y_1^2}{2sx(1-x)}\Big[\frac{1+(1-x)^2}{1-x}(L_0+2\ln(1-x))-2\Big] - \Big[|\Pi_t|^2\frac{4+(1+c_-)^2}{(1-c_-)^2} + |\Pi_s|^2\frac{(1-c_-)^2+(1+c_-)^2}{4} - \mathrm{Re}(\Pi_t\Pi_s^*)\frac{(1+c_-)^2}{1-c_-}\Big], \quad \Pi_s = \Big(1-\frac{\alpha}{3\pi}(l_s-\frac{5}{3}-i\pi)\Big)^{-1}, \tag{27}$$

$$l_s = \ln\Big(\frac{4\varepsilon^2}{m_e^2}\Big), \quad y_2 = 1, \quad 1-x = y_1, \quad c_+ = -c_-.$$

Finally, the in the fourth collinear region the energy fraction of the scattered electron is unity and the final particles move back to back as well as in the third region. The correspondent contribution reads

$$\frac{d\sigma_4}{dy_1 dc_-} = \int \frac{\alpha^3\delta(1-y_1)}{2sx(1-x)} y_2^2 dy_2\Big[\frac{1+(1-x)^2}{1-x}(L_0+2\ln(1-x))-2\Big] - \Big[|\Pi_t|^2\frac{4+(1+c_-)^2}{(1-c_-)^2} + |\Pi_s|^2\frac{(1-c_-)^2+(1+c_-)^2}{4} - \mathrm{Re}(\Pi_t\Pi_s^*)\frac{(1+c_-)^2}{1-c_-}\Big], \quad 1-x = y_2, \quad c_+ = -c_-. \tag{28}$$

Again one can see the cancellation of the dependence on the auxiliary parameter θ_0 in the sum of the third and the fourth collinear region contributions with the semi-collinear ones.

So, the total contribution to the LABS process differential cross-section due to a hard photon emission with vacuum polarization correction of the virtual photon propagator reads

$$\delta^\gamma_{\mathrm{hard}} = \Big(\frac{d\sigma_0}{dy_1 dc_-}\Big)^{-1}\Big[\frac{d\sigma^\gamma_{\mathrm{semi\text{-}coll}}}{dy_1 dc_-} + \frac{d\sigma^\gamma_{\mathrm{coll}}}{dy_1 dc_-}\Big]. \tag{29}$$

The auxiliary parameter d cancels in the above sum.

5 Soft Pair Production

Here we consider the process

$$e^-(p_1) + e^+(p_2) \to e^-(q_1) + e^+(q_2) + e^-(p_-) + e^+(p_+), \tag{30}$$

where $e^-(p_-) + e^+(p_+)$ is the created soft pair. It gives to the cross-section an important contribution containing cubic in the large logarithm terms. The maximum energy of the soft pair is taken as $D\varepsilon$, it is assumed to be large compared with the electron mass:

$$2m_e < D\varepsilon \ll \varepsilon. \tag{31}$$

The contributions containing L^3 will cancel with the terms due to virtual corrections and the dependence on the auxiliary parameter D will disappear in the sum with the contribution of the hard (with the energy of pair components larger then $D\varepsilon$) pair emission.

Recently the contribution of the soft pairs production was calculated in two limiting cases: for the process e^+e^- annihilation into hadrons [7] and for the case of small angle Bhabha scattering [8]. Here we carry out the calculations for arbitrary scattering angles.

Due to the smallness of the energy of the pair components the matrix element M of a hard process with the charged particles with momenta p_1, q_1, accompanied by soft pair emission, could be expressed through the matrix element of the hard subprocess M_0 without pair production in the way:

$$M = M_0\frac{4\pi\alpha}{k^2}\bar{v}(p_+)\gamma_\mu u(p_-)\,J_\mu, \quad k = p_+ + p_-, \tag{32}$$

The classic [9, 10] accompanied radiation current could be put in the form:

$$J_\mu = -\frac{p_{1\mu}}{p_1k+\frac{1}{2}k^2} + \frac{q_{1\mu}}{q_1k-\frac{1}{2}k^2} + \frac{p_{2\mu}}{p_2k-\frac{1}{2}k^2} - \frac{q_{2\mu}}{q_2k+\frac{1}{2}k^2}. \tag{33}$$

Performing the covariant integration of the summed over spin states matrix element modulus over the pair components momenta, we obtain:

$$\sum_{\mathrm{spin}} |\bar{v}(p_+)\gamma_\mu u(p_-)|^2 = 4(p_+^\mu p_-^\nu + p_+^\nu p_-^\mu - \frac{k^2}{2}g^{\mu\nu}), \tag{34}$$

$$\int\frac{d^3p_+d^3p_-}{p_+^0p_-^0}\delta^4(p_++p_--k)(p_+^\mu p_-^\nu + p_+^\nu p_-^\mu - \frac{1}{2}k^2g_{\mu\nu}) = $$

$$= \Big(-\frac{2\pi}{3}(k^2+2m_e^2)\sqrt{1-\frac{4m_e^2}{k^2}}\Big)(g_{\mu\nu} - \frac{1}{k^2}k_\mu k_\nu).$$

At first we parametrize the phase volume of the pair momentum as

$$d^4k = dk_0(\vec{k})^2 d|\vec{k}|dO_k = dk_0\,dk^2\sqrt{k_0^2-k^2}\,\pi\,dc_k. \tag{35}$$

Neglecting the invariant mass of the pair $\overline{k^2}$ compared to the energies of its components and omitting the terms of order $m_e^2/(p_1k)^2$ (this simplifications does not violate the logarithmical accuracy that we keep here) we perform the angular integration:

$$I = \int \frac{\mathrm{d}O_k}{2\pi}\frac{2p_1q_1}{2p_1k\,2q_1k} = \int_0^1 \mathrm{d}x \int_{-1}^{1} \frac{\mathrm{d}c_k\, 2p_1q_1}{4\varepsilon^2(k_0 - \vec{k}\vec{n} c_k)^2} = \quad (36)$$

$$= \frac{2p_1q_1}{2\varepsilon^2}\int_0^1 \frac{\mathrm{d}x}{k_0^2 - (\vec{k})^2(\vec{n})^2}, \qquad \vec{n} = x\frac{\vec{p}_1}{\varepsilon} + (1-x)\frac{\vec{q}_1}{\varepsilon},$$

$$(\vec{n})^2 = 1 - 4x(1-x)z^2, \quad z = \sqrt{\frac{1-c}{2}}, \quad c = \cos\theta, \quad \theta = \widehat{\mathbf{p}_1\mathbf{q}_1}.$$

Integrating over auxiliary variable x we obtain:

$$I = \frac{2J}{k_0^2(1-y)}, \qquad J = \left(1 + \frac{y}{(1-y)z^2}\right)^{-\frac{1}{2}}\Big[\ln z - \frac{1}{2}\ln y + \quad (37)$$

$$+ \ln 2 + \ln\Big(\frac{\sqrt{1-y} + \sqrt{1+y(z^{-2}-1)}}{2}\Big)\Big].$$

The result can be expressed as a ratio of the cross-sections of the processes of electron scattering in an external field with soft pair production to the Born one:

$$\frac{\mathrm{d}\sigma^{\mathrm{sp}}}{\mathrm{d}\sigma_0} = \frac{\alpha^2}{3\pi^2}\int_0^N \frac{\mathrm{d}t}{t}\int_{1/t}^{1}\frac{\mathrm{d}y}{y^2}\Big(y + \frac{1}{2t^2}\Big)\sqrt{1 - \frac{1}{yt^2}}\,(1+y(z^{-2}-1))^{-\frac{1}{2}} - \quad (38)$$

$$-\Big[2\ln z + 2\ln 2 - \ln y + 2\ln\Big(\frac{\sqrt{1-y} + \sqrt{1+y(z^{-2}-1)}}{2}\Big)\Big],$$

$$N = \left(\frac{D\varepsilon}{2m}\right)^2 - 1.$$

Integrating firstly over t we omit terms of the order N^{-2}. Then we introduce variable $x = \sigma/y$ and split the integration using the parameter η $(1 - \eta - N^{-1})$. Within the logarithmic accuracy we obtain:

$$\frac{\mathrm{d}\sigma^{\mathrm{sp}}}{\mathrm{d}\sigma_0} = \frac{\alpha^2}{3\pi^2}\Big\{\int_{1/N}^{\eta}\frac{\mathrm{d}x}{\sqrt{x(x+N^{-1}(z^{-2}-1))}}\Big(-\frac{5}{3} + 2\ln 2 - \ln x\Big)\Big[2\ln z + \quad (39)$$

$$+ \ln N + 2\ln 2 + \ln x + 2\ln\Big(\frac{\sqrt{1-(Nx)^{-1}} + \sqrt{1+(Nx)^{-1}(z^{-2}-1)}}{2}\Big)\Big] +$$

$$+ \ln N\int_{\eta}^{1}\frac{\mathrm{d}x}{x}\Big[\Big(-\frac{5}{3} - \frac{1}{3}x\Big)\sqrt{1-x} + \ln\frac{(1+\sqrt{1-x})^2}{x}\Big]\Big\}.$$

The final expression reads

$$\frac{\mathrm{d}\sigma^{\mathrm{sp}}}{\mathrm{d}\sigma_0} = \frac{\alpha^2}{6\pi^2}\Big\{\frac{1}{3}L^3 + L^2 - \Big(2\ln D - \frac{5}{3}\Big) + L\Big[4\ln^2 D - \quad (40)$$

$$-\frac{20}{3}\ln D + \frac{56}{9} - \frac{2}{3}\pi^2 + 2\mathrm{Li}_2\Big(\frac{1+c}{2}\Big)\Big] + O(1)\Big\},$$

where

$$L = \ln\frac{2\varepsilon^2(1-c)}{m_e^2} - 1, \quad z = \sqrt{\frac{1-c}{2}}. \quad (41)$$

Using general expression 40 we reproduce the results obtained earlier in the annihilation channel ($c = -1,\ z = 1$) [7]

$$\frac{\mathrm{d}\sigma^{\mathrm{sp}}}{\mathrm{d}\sigma_0}\Big|_{z=1} = \frac{\alpha^2}{6\pi^2}\Big\{\frac{1}{3}(\rho + 2\ln D)^3 - \frac{5}{3}(\rho + 2\ln D)^2 + \quad (42)$$

$$+4(\rho + 2\ln D)\Big(\frac{14}{9} - \frac{\pi^2}{6}\Big)\Big\}, \qquad \rho = \ln\Big(\frac{4\varepsilon^2}{m_e^2}\Big);$$

and in the small angle scattering channel ($c - 1$) [8]:

$$\frac{\mathrm{d}\sigma^{\mathrm{sp}}}{\mathrm{d}\sigma_0}\Big|_{z\ll 1} = \frac{\alpha^2}{6\pi^2}\Big\{\frac{1}{3}(L + 2\ln D)^3 - \frac{5}{3}(L + 2\ln D)^2 + \quad (43)$$

$$+4(L + 2\ln D)\Big(\frac{14}{9} - \frac{\pi^2}{12}\Big)\Big\}, \qquad L = \ln\Big(\frac{\varepsilon^2\theta^2}{m_e^2}\Big), \quad z - \frac{\theta}{2} - 0, \quad L - 1.$$

To obtain the total contribution of the soft pair production we have to multiply by factor 2 (to account the pair emission from the positron line) and to add the t– and u–channel contributions, which could be obtained by simple substitutions. In this way one gets:

$$\delta^{e^+e^-} = \frac{1}{3}\Big\{\frac{1}{3}l_s^3 + l_s^2\Big(2\ln D - \frac{5}{3}\Big) + l_s\Big(4\ln^2 D - \frac{20}{3}\ln D + A_s\Big) + \quad (44)$$

$$+ \frac{1}{3}l_t^3 + l_t^2\Big(2\ln D - \frac{5}{3}\Big) + l_t\Big(4\ln^2 D - \frac{20}{3}\ln D + A_t\Big) -$$

$$- \frac{1}{3}l_u^3 - l_u^2\Big(2\ln D - \frac{5}{3}\Big) - l_u\Big(4\ln^2 D - \frac{20}{3}\ln D + A_u\Big)\Big\},$$

where

$$A_s = \frac{56}{9} - \frac{2}{3}\pi^2, \qquad A_t = \frac{56}{9} - \frac{2}{3}\pi^2 + 2\mathrm{Li}_2\Big(\frac{1+c}{2}\Big), \quad (45)$$

$$A_u = \frac{56}{9} - \frac{2}{3}\pi^2 + 2\mathrm{Li}_2\Big(\frac{1-c}{2}\Big).$$

6 Conclusions

The total sum of the considered corrections does not contain parameter λ and cubic in large logarithm terms. It reads

$$
\begin{aligned}
\mathrm{d}\sigma &= \mathrm{d}\sigma_0 - 1 + \left(\frac{\alpha}{\pi}\right)^2 (\delta + \delta^{\gamma}_{\text{hard}}) -, \qquad (46)\\
\delta &= l_s\Big[\frac{8}{3} l_s \ln d - \frac{64}{9}\ln d + \frac{8}{3}\ln d \ln(\frac{s}{-u}) + \frac{4}{3}\ln^2 D + \ln D\Big(\frac{4}{3}\ln(\frac{t}{u}) + \frac{2}{3} l_s - \\
&- \frac{20}{9}\Big) + \frac{17}{6} l_s - \frac{4}{3}\mathrm{Li}_2(\frac{1-c}{2}) + \frac{4}{3}\mathrm{Li}_2(\frac{1+c}{2}) - \frac{311}{27} + \frac{1}{3}(A_s + A_t - A_u) - \\
&- \frac{4}{3}\ln d \ln(\frac{s}{-t})\Big(1 + \frac{s}{t} + \frac{t}{s}\Big)^{-2}(4\frac{s^2}{t^2} + 7\frac{s}{t} + 5\frac{t}{s} + 9) + H(c)\Big],
\end{aligned}
$$

where $H(c)$ is a function of the scattering angle, its analytical expression and the table of values in several points are presented in Appendix C. $H(c)$ as could be seen from Table 1 is not small. That convinces us in the importance of the non-leading terms. Parameters d and D will cancel in the sum with the contributions due to the emission of a hard photon $\delta^{\gamma}_{\text{hard}}$ and a hard pair [5]. It is remarkable that the qubic in large logarithm terms cancel.

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Appendix A

Here we present the quantities b_i which enter the integral over the virtual 4–momentum in box diagrams (see eq. (7)).

$$
\begin{aligned}
b &= \frac{2}{st}\tilde{l}_s(l_t - 2l_\lambda), \qquad b_1 = -\frac{2}{u}\Big(\frac{1}{s}\tilde{l}_s(l_t - l_s) + \Psi_s + \Psi_t\Big), \qquad (A.1)\\
b_2 &= \frac{1}{2}(b - b_1), \qquad b_3 = -\frac{1}{u}\Big((t-s)b_1 + 4(\Psi_t - \frac{1}{t}l_t) - \frac{4}{s}\tilde{l}_s\Big),\\
b_4 &= b_1 - \frac{4}{st}l_t, \qquad b_5 = -\frac{s}{u}b_1 + \frac{2}{u}(\Psi_t - \frac{1}{t}l_t) - \frac{2}{su}\tilde{l}_s,\\
b_6 &= \frac{s-u}{2u}b_1 + \frac{1}{2}b + \frac{t-u}{tus}\tilde{l}_s - \frac{1}{u}(\Psi_t - \frac{1}{t}l_t), \qquad b_7 = -\frac{s}{4}b_1 + \frac{1}{2}\Psi_t,\\
\Psi_t &= \frac{1}{t}\Big(\frac{2\pi^2}{3} + \frac{1}{2}l_t^2\Big), \qquad \Psi_s = \frac{1}{s}\Big(\frac{2\pi^2}{3} + \frac{1}{2}l_s^2\Big).
\end{aligned}
$$

Appendix B

We present here the integrals for the box diagrams with a vacuum polarization insertion in one photon propagator. Consider first the scalar integral with four denominators:

$$
I(aba_1a_2) = \int\limits_0^1 \frac{\mathrm{d}v\phi(v)}{1-v^2}\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{1}{aba_1a_2}, \qquad (B.1)
$$

$$
\begin{aligned}
a &= k^2 - \lambda^2 + i0, \quad b = (q+k)^2 - M^2 + i0, \quad M^2 = \frac{4m_e^2}{1-v^2},\\
a_1 &= (k+q_1)^2 - m_e^2 + i0, \quad a_2 = (k-q_2)^2 - m_e^2 + i0, \quad q_i^2 = p_i^2 = m_e^2.
\end{aligned}
$$

We join the denominators using the known Feynman trick and transform the total denominator to the form:

$$
\begin{aligned}
& z[y(xa_1 + (1-x)a_2) + b(1-y)] + a(1-z) = (k - zP_y)^2 - \qquad (B.2)\\
& -[z^2P_y^2 - z(1-y)(t-M^2) + (1-z)\lambda^2]; \quad P_y = yP_x - (1-y)q,\\
& P_x = xq_2 - (1-x)q_1, \quad q = q_1 - p_1, \quad t = q^2.
\end{aligned}
$$

Performing the integration over the loop momentum we obtain:

$$
\begin{aligned}
I(aba_1a_2) &= \int\limits_0^1 z^2\mathrm{d}z\int\limits_0^1 y\mathrm{d}y\int\limits_0^1\mathrm{d}x\int\limits_0^1\frac{\mathrm{d}v\phi(v)}{1-v^2} - \qquad (B.3)\\
&- [z^2P_y^2 - z(1-y)(t-M^2) + (1-z)\lambda^2 - i0]^{-2},\\
P_y^2 &= y^2P_x^2 + (1-y)t, \quad P_x^2 = m_e^2 - x(1-x)s.
\end{aligned}
$$

To extract the infrared singularity it is convenient to split the integration over y into two parts using the auxiliary parameter σ $((\lambda/m_e)^2 - \sigma - 1)$:

$$
I(aba_1a_2) = I_1(o < y < 1-\sigma) + I_2(1-\sigma < y < 1). \qquad (B.4)
$$

In each region the integration over z could be carried out explicitly:

$$
\begin{aligned}
I(aba_1a_2) &= \int\limits_0^1\frac{\mathrm{d}v\phi(v)}{1-v^2}\int\limits_0^1\frac{\mathrm{d}x}{P_x^2(t-M^2)}\Big\{-\ln\frac{1}{\sigma} - \int\limits_0^1\frac{\mathrm{d}y(P_x^2y - M^2)}{y^2P_x^2 + (1-y)M^2} - \\
&- \frac{1}{2}\ln\frac{\sigma^2(t-M^2)^2}{4P_x^2\lambda^2} - \int\limits_0^\infty\frac{\mathrm{d}t}{(t^2+1)^{3/2}}\ln(t + \sqrt{t^2+1})\Big\}. \qquad (B.5)
\end{aligned}
$$

Further integration could be simplified by the usage of the following relations:

$$
\int\limits_0^1\frac{\mathrm{d}v\phi(v)}{(1-v^2)(t-M^2)} = \frac{1}{3t}(l_t - \frac{5}{3}), \qquad (B.6)
$$

$$\int_0^1 \frac{dv\,\phi(v)}{(1-v^2)(t-M^2)}\ln\left(\frac{-t+M^2}{m_e^2}\right) = \frac{1}{3t}\left(l_t^2 - \frac{5}{3}l_t + \frac{\pi^2}{6}\right),$$

$$\int_0^1 \frac{dx}{P_x^2} = -\frac{2}{s}\tilde{l}_s, \qquad \int_0^1 \frac{dx}{P_x^2}\ln\frac{P_x^2}{m_e^2} = -\frac{1}{s}\left(\tilde{l}_s^2 - \frac{\pi^2}{3}\right),$$

$$\int_0^1 \frac{dx}{P_x^2}\ln^2\frac{P_x^2}{m_e^2} = -\frac{2}{3t}\left(\tilde{l}_s^3 - \frac{\pi^2}{3}\tilde{l}_s\right), \qquad \tilde{l}_s = l_s - i\pi.$$

The remaining integral over v could be calculated by splitting of the interval:

$$0 < v < 1-\sigma_1, \qquad 1-\sigma_1 < v < 1, -t\sigma_1 - m_e^2. \tag{B.7}$$

Now the integral reads

$$\int_0^1 \frac{dv\,\phi(v)}{(1-v^2)(t-M^2)}\int_0^1 dx \int_0^1 \frac{y\,dy}{y^2P_x^2+(1-y)M^2} = \tag{B.8}$$
$$= \frac{1}{3t}\int_0^{1-\sigma_1} dx \int_0^1 y\,dy[y^2P_x^2+(1-y)M^2]^{-1} +$$
$$+\frac{1}{3}\int_{1-\sigma_1}^1 \frac{dv}{t(1-v)-2m_e^2}\int_0^1 dx\int_0^1 y\,dy\frac{(1-v)}{y^2P_x^2(1-v)+2m_e^2(1-y)} = J_1 + J_2.$$

For the first integral within the logarithmic accuracy we get

$$J_1 = \frac{1}{3t}\int_0^{1-\sigma_1} dv\left(\frac{1}{1-v}+\frac{1}{1+v}-2+v^2\right)\left[-\frac{1}{2s}l_s^2+\frac{1}{s}l_s\ln\frac{M^2}{m_e^2}\right] = \tag{B.9}$$
$$= \frac{1}{3st}\left[-\frac{1}{2}l_s^2\left(-\ln\sigma_1+\ln 2-\frac{5}{3}\right)+l_s\left(\frac{1}{2}\ln^2 2+\frac{1}{2}\ln^2\sigma_1 - \right.\right.$$
$$\left.\left. - \ln 2\ln\sigma_1+\frac{\pi^2}{6}-\frac{28}{9}\right)\right].$$

The second integral after a simple integration over v turns to

$$J_2 = \frac{1}{st}\int_0^1\frac{dx}{P_x^2}\int_0^1\frac{dy}{y}\ln\left(1+\frac{\sigma_1P_x^2y^2}{2m_e^2(1-y)}\right) = \frac{1}{12t}\int_0^1\frac{dx}{P_x^2}\ln^2\left(\frac{P_x^2\sigma_1}{2m_e^2}\right) =$$
$$= -\frac{1}{6st}\Re e\left[\frac{1}{3}(\tilde{l}_s+\ln\sigma_1-\ln 2)^3-\tilde{l}_s\frac{\pi^2}{3}\right]. \tag{B.10}$$

The total result for the considered scalar integral reads

$$I(aba_1a_2) = \frac{1}{3st}\Re e\left\{-\frac{1}{6}\tilde{l}_s^3+\frac{1}{2}\tilde{l}_s^2l_t+\tilde{l}_sl_t^2-\frac{10}{3}\tilde{l}_sl_t-\frac{28}{9}\tilde{l}_s - \right.$$
$$\left. - \frac{\pi^2}{6}l_t+\tilde{l}_s\left(l_t-\frac{5}{3}\right)\ln\frac{m_e^2}{\lambda^2}+\mathcal{O}(\text{const})\right\}. \tag{B.11}$$

For the scalar integral with three denominators b, a_1 and a_2 one obtains in a similar way the following result:

$$I(ba_1a_2) = \int_0^1\frac{dv\,\phi(v)}{1-v^2}\int\frac{d^4k}{i\pi^2}\frac{1}{ba_1a_2} =$$
$$= -\int_0^1\frac{dv\,\phi(v)}{1-v^2}\int_0^1 dx\int_0^1\frac{dy\,(1-y)}{(1-y)^2P_x^2+yM^2}. \tag{B.12}$$

Again, splitting the integration over v one can simplify calculations. We give here the results for two relevant integrals with three denominators:

$$I(ba_1a_2) = \frac{1}{3s}\Re e\left\{\frac{1}{6}\tilde{l}_s^3-\frac{5}{6}\tilde{l}_s^2+\tilde{l}_s\left(\frac{28}{9}+\frac{\pi^2}{6}\right)+\mathcal{O}(\text{const})\right\}, \tag{B.13}$$
$$I(aba_1) = \frac{1}{3t}\Re e\left\{\frac{1}{3}l_t^3-\frac{5}{6}l_t^2+\frac{\pi^2}{3}l_t+\mathcal{O}(\text{const})\right\}.$$

The remaining needed integrals with three denominators could be obtained by the following substitutions:

$$I(aba_2) = I(aba_3) = I(aba_1), \qquad I(ba_1a_3) = I(ba_1a_2)(\tilde{l}_s \to l_u, s \to u). \tag{B.14}$$

Consider now a vector integral with four denominators. In the same manner as above we put it in the form

$$I_\sigma(aba_1a_2) = \int_0^1\frac{dv\,\phi(v)}{1-v^2}\int\frac{d^4k}{i\pi^2}\frac{k_\sigma}{aba_1a_2} = \alpha(p_2-p_1)^\sigma+\beta(p_1-q_1)^\sigma = \tag{B.15}$$
$$= \int_0^1\frac{dv\,\phi(v)}{1-v^2}\int_0^1 dz\int_0^1 dz\int_0^1 y\,dy\int_0^1 dx\,\frac{z(yP_x-(1-y)q)^\sigma}{[yP_y^2-(1-y)(t-M^2)]^2}.$$

Multiplying this vector equation by q_1^σ or by p_2^σ we obtain the system of linear equations:

$$\begin{cases} \alpha(t-u)-\beta t = I(aba_2)-I(ba_1a_2) \\ \alpha s+\beta t = I+I(aba_2), \end{cases} \tag{B.16}$$

where quantity I has the form

$$I = \int_0^1\frac{dv\,\phi(v)}{1-v^2}\int_0^1 dx\left\{-\frac{1}{P_x^2\ln\left(\frac{-t+M^2}{M^2}\right)}+\int_0^1\frac{y\,dy}{y^2P_x^2+(1-y)M^2}\right\}. \tag{B.17}$$

The final answer for the coefficients reads:

$$\alpha = \frac{1}{2u}[-I-2I(aba_2)+I(ba_1a_2)], \tag{B.18}$$
$$\beta = \frac{1}{2tu}[(u-t)I-sI(ba_1a_2)-2tI(aba_2)],$$
$$I = \frac{1}{3s}\Re e\left\{2\tilde{l}_s\left[\frac{1}{2}l_t^2-\frac{5}{3}l_t+\frac{28}{9}\right]-\frac{1}{6}\tilde{l}_s^3+\frac{5}{6}\tilde{l}_s^2+\tilde{l}_s\left(\frac{8}{9}\pi^2-\frac{28}{9}\right)+\mathcal{O}(\text{const})\right\}.$$

Let us, finally, consider the tensor integral

$$I_{\sigma\rho} = \int_0^1 \frac{\mathrm{d}v\phi(v)}{1-v^2}\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{k_\sigma k_\rho}{aba_1a_2} = f_0g^{\sigma\rho} + f_1(q_1^\rho q_1^\sigma + q_2^\rho q_2^\sigma) +$$
$$+ \quad f_3q^\rho q^\sigma + f_4(q_1^\rho q_2^\sigma + q_2^\rho q_1^\sigma) + f_5(q^\rho(q_2^\sigma - q_1^\sigma) + q^\sigma(q_2^\rho - q_1^\rho)). \quad \text{(B.19)}$$

Fortunately within the logarithmic accuracy one can present the square of the matrix element in the form, where only coefficients f_0 and f_4 enter. Using the procedure described above we obtain

$$f_0 = -\frac{1}{2}\int_0^1\frac{\mathrm{d}v\phi(v)}{1-v^2}\int_0^1 z\,\mathrm{d}z\int_0^1 y\,\mathrm{d}y\int_0^1\frac{\mathrm{d}x}{z(y^2P_x^2+(1-y)t)-(1-y)(t-M^2)},$$
$$f_4 = 2\frac{\partial f_0}{\partial s}. \quad \text{(B.20)}$$

One can see that both f_0 and f_4 enter have only the first power of large logarithm:

$$f_0 = -\frac{1}{6}\int_{1-\sigma}^1 \mathrm{d}v\iiint\frac{zy\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x}{(1-v)[zy^2P_x^2-(1-y)(1-z)t]+2m_e^2(1-y)} =$$
$$= \frac{l_t}{6t}M(\frac{s}{t}), \qquad f_4 = -\frac{l_t}{3t^2}M_1(\frac{s}{t}), \qquad \sigma - 1, \quad \text{(B.21)}$$

where we defined

$$M(\xi) = \int_0^1\int_0^1\int_0^1\frac{zy\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x}{zy^2x(1-x)\xi+(1-y)(1-z)}, \quad \text{(B.22)}$$
$$M_1(\xi) = \int_0^1\int_0^1\int_0^1\frac{z^2y^3x(1-x)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x}{[zy^2x(1-x)\xi+(1-y)(1-z)]^2}.$$

These quantities are convenient for numerical integration for $\xi > 0$, for $\xi < 0$ it is better to use the following expressions:

$$M(\xi) = \int_0^1 f(x)\phi(x,\xi)\,\mathrm{d}x, \qquad \xi < 0, \quad \text{(B.23)}$$
$$f(x) = (1-\frac{1}{1+4x})\ln x + \frac{2}{1+4x}\ln(1+\frac{2}{1+\sqrt{1+4x}}),$$
$$\phi(x,\xi) = \frac{\xi}{\Delta}\Big[\frac{1}{x}+\frac{2}{\Delta}\ln\Big|\frac{-\xi+\Delta}{-\xi-\Delta}\Big|\Big], \qquad \Delta = \xi^2-4\xi x > 0.$$

We did not manage to perform this integration analytically, except the limiting case:

$$M(\xi)_{|\xi|\to\infty} = \frac{1}{\xi}-\mathrm{e}\Big(\frac{1}{2}\ln^2(-\xi)-\frac{\pi^2}{6}\Big). \quad \text{(B.24)}$$

The expression for $M_1(\xi)$ has the following form:

$$M_1(\xi) = \int_0^1 f(x)\psi(x,\xi)\,\mathrm{d}x, \qquad \xi < 0, \quad \text{(B.25)}$$
$$\psi(x,\xi) = \frac{1}{\Delta^2}\Big[\frac{\xi^2}{x}+2\xi+\frac{4(\xi^2-x\xi)}{\Delta}\ln\Big|\frac{-\xi+\Delta}{-\xi-\Delta}\Big|\Big].$$

Appendix C

We present here the explicit expression for function $H(c)$ entering eq. (46):

$$H(c) = \Big(1+\frac{s}{t}+\frac{t}{s}\Big)^{-1}\Big\{\frac{u^3}{3t^3}f_{4p}+\frac{s^2u}{3t^3}f_{4up}-\frac{t^2}{3s^2}h_{0up}+\frac{t^2u}{3s^3}h_{4up}- \quad \text{(C.1)}$$
$$-\frac{s^2}{3t^2}f_{0up}-\frac{u^3}{3st^2}f_{0p}-\frac{u^3}{3s^2t}h_{0p}+\frac{u^3}{3s^3}h_{4p}+\frac{\pi^2}{6}\Big(-\frac{4t^2}{s^2}+\frac{10t}{3s}+\frac{125}{18}-$$
$$-\frac{2s^2}{t^2}+\frac{7s}{3t}\Big)+l_{st}^2\Big(-\frac{11s}{12t}+\frac{7t}{12s}+\frac{t^2}{s^2}-\frac{5}{12}\Big)+l_{su}^2\Big(-\frac{s^2}{t^2}-\frac{5s}{2t}-$$
$$-\frac{5t}{2s}-\frac{t^2}{s^2}-\frac{19}{6}\Big)+l_{st}l_{su}\Big(\frac{2s^2}{t^2}+\frac{4s}{t}+\frac{t}{s}+\frac{19}{6}\Big)+$$
$$+\; l_{st}\Big(-\frac{17s^2}{3t^2}-\frac{25s}{3t}-\frac{17t}{6s}-\frac{17}{2}\Big)+l_{su}\Big(-\frac{s}{6t}-\frac{t}{6s}\Big)\Big\}-$$
$$-\; 2\mathrm{Li}_2\Big(\frac{1-c}{2}\Big)+2\mathrm{Li}_2\Big(\frac{1+c}{2}\Big)-\frac{2\pi^2}{9},$$

where

$$f_{0p} = M(\frac{s}{t}), \quad f_{4p} = M_1(\frac{s}{t}), \quad h_{0p} = M(\frac{t}{s}), \quad h_{4p} = M_1(\frac{t}{s}), \quad \text{(C.2)}$$
$$f_{0up} = M(\frac{u}{t}), \quad f_{4up} = M_1(\frac{u}{t}), \quad h_{0up} = M(\frac{t}{u}), \quad h_{4up} = M_1(\frac{t}{u}),$$
$$\frac{t}{s} = -\frac{1-c}{2}, \quad \frac{u}{s} = -\frac{1+c}{2}, \quad \frac{t}{u} = \frac{1-c}{1+c},$$
$$l_{st} = \ln(\frac{2}{1-c}), \quad l_{su} = \ln(\frac{2}{1+c}).$$

Functions M and M_1 are given in Appendix B. For an illustration in Table 1 we give function $H(c)$ for different c values.

Table 1: $H(c)$ as a function of c.

c	-0.8	-0.6	-0.4	-0.2	0.0	0.2	0.4	0.6	0.8
$H(c)$	-13.9	-12.7	-12.2	-12.2	-12.5	-13.0	-13.7	-14.7	-17.1

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