ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-95-386

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$1/N_c$-EXPANSION OF THE QUARK CONDENSATE AT FINITE TEMPERATURE

Submitted to «Physical Review C»

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1995

Блашке Д. и др. E2-95-386

$1/N_c$-разложение кваркового конденсата при конечной температуре

Кварковые и мезонные свойства в многокварковых системах ранее изучались при конечных температурах в эффективных QCD-моделях в приближении Хартри. В этой работе рассматривается влияние мезонных поправок на кварковую собственную энергию и на кварковый пропагатор в систематическом $1/N_c$-разложении. Используя общий анализ сепарабельности для нелокального взаимодействия, выводится самосогласованное уравнение для $1/N_c$-поправок к кварковому пропагатору. Используя процедуру суммирования Мацубары, найдена замкнутая форма для $1/N_c$-поправок к кварковому конденсату при конечных температурах. Конденсат уменьшается на 10% при нулевой температуре по абсолютному значению и мезонные флуктуации стабилизируют его температурное поведение.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1995

Blaschke D. et al. E2-95-386

$1/N_c$-Expansion of the Quark Condensate at Finite Temperature

Previously the quark and meson properties in many quark systems at finite temperature have been studied within effective QCD approaches in the Hartree approximation. In the present paper we consider the influence of the mesonic correlations on the quark self energy and on the quark propagator within a systematic $1/N_c$ expansion. Using a general separable ansatz for the nonlocal interaction, we derive a selfconsistent equation for the $1/N_c$ corrections to the quark propagator. Performing the Matsubara summation the corresponding $1/N_c$ corrections to the quark condensate at finite temperature are found in closed form. For a NJL-type model, we obtain a decrease of the condensate of the order of 10% at zero temperature and a stabilization of the temperature behaviour against the medium effects due to the mesonic fluctuations.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1995

1 Introduction

QCD motivated effective theories are the most promising approaches to the low energy behaviour of QCD and the meson physics in terms of QCD degrees of freedom and symmetries. Starting from chiral quark model lagrangians a perturbative approach to the occurence of a chiral condensate below a critical temperature T_c in mean field approximation is usually considered. Simultaneously, the pseudoscalar Goldstone boson, the pion, occurs. Perturbation theory can be formulated in $1/N_c$, where N_c is the number of colors. The leading order is the Hartree approximation, results are reported in Refs.[1, 2, 3, 4]. A more general approach, where a nonlocal instantaneous interaction is applied, has been presented in Refs. [5, 6, 7, 8, 9, 10]. A still open but very important question is the influence of terms beyond the Hartree approximation, which neglects mesonic degrees of freedom which are supposed to be dominant in the low temperature limit. For the NJL model, an effective $1/N_c$ expansion which accounts for the mesonic fluctuations has been considered in [11]. However, the set of diagrams for the self energy in next to leading order considered in this reference was not complete. This has been observed in Ref. [12] where also the role of the scalar iso-vector mesons in the $1/N_c$ - approximation was discussed. It was shown that in the $1/N_c$ expansion the Schwinger Dyson equation for the quark self energy is different from the gap equation for the quark condensate and has to be solved separately. A complete collection of diagrams in $1/N_c$ was given in Ref. [13] and recently studied in the chiral limit $m_0 = 0$ at $T = 0$ by Ref.[14]. At $T = 0$, effects of the order of $10\% - 20\%$ have been obtained in these approaches, showing that mesonic fluctuations play an important role.

In the present work, we consider the influence of mesonic correlations on the quark condensate at finite temperature. It is expected that such a calculation beyond the Hartree level of description will lead to corrections to the temperature behaviour of the quark condensate since the medium allows for mesonic degrees of freedom. The relation of a generalized gap equation to the thermodynamical potential of a quark meson plasma has been considered in Ref. [15]. This paper is a first step for a consistent description of a meson gas in terms of quark degrees of freedom.

The paper is organized as follows: In Section 2 the nonlocal chiral quark model is briefly introduced, which is used in Section 3 to derive a generalized formula for the quark condensate in $\mathcal{O}(1/N_c)$ expansion. In Section 4 we involve dynamical fluctuations into the self energy and treat the scalar und pseudoscalar contributions within the pole approximation. The numerical results for a calculation within the NJL model at finite temperature are discussed in Section 5.

2 The model

Our starting point is the chiral symmetric effective Lagrangian in the quark sector of the following general form

$$\mathcal{L} = \bar{q}_1(p)(\gamma_\mu p^\mu - m_0)q_1(p) + \mathcal{L}_{\rm int}\,, \tag{1}$$

where the interaction term

$$\mathcal{L}_{\rm int} = -\frac{1}{2}\bar{q}_1(p_1)\Lambda^\Phi_{12}q_2(p_2)K(p_1,p_2,p_{1'},p_{2'})\bar{q}_{2'}(p_{2'})\Lambda^\Phi_{1'2'}q_{1'}(p_{1'}) \tag{2}$$

is given as a nonlocal generalization of the current-current type interaction. Here the matrices Λ^Φ_{12} denote the decomposition into the color (c), flavor (f) and Dirac (D) channels. In this work we restrict us to scalar and pseudoscalar channels. Therefore we choose $\Lambda^\sigma_{12} = [1_c \cdot 1_f \cdot 1_D]_{12}$ and $\Lambda^\pi_{12} = [1_c \cdot \tau_f \cdot i\gamma_5]_{12}$.

Not discussing the gluonic background of the quark interaction, we take a phenomenological instantaneous kernel. For instantaneous interactions, the kernel can be formulated in a covariant way [10]. We employ here a separable ansatz for the nonlocal 4 point interaction of the form

$$K(p_1,p_2,p_1',p_2') = -K_0 g(\frac{|\mathbf{p}_1+\mathbf{p}_2|}{2})g(\frac{|\mathbf{p}_1'+\mathbf{p}_2'|}{2})\delta_{p_1-p_2,p_1'-p_2'}. \tag{3}$$

We assume the separable form (3) of the potential. The dependence of the formfactor on the modulus of the three-momentum ($|\mathbf{p}| = \mathrm{p}$) has been discussed for different shapes, e.g. as a Gaussian shape ($g(\mathrm{p}) = \exp[-(\mathrm{p}/\Lambda_{Gauss})^2]$) or as the well-known NJL type interaction ($g(\mathrm{p}) = \Theta(1-\mathrm{p}/\Lambda_{NJL})$) , see [9]. Note that the potential does not depend on the energy and we obtain therefore the NJL- model with a three-momentum cut–off. The spectral properties of the quark model defined by the Lagrangian (1) are obtained from the single particle propagator

$$G_{12}(p_1p_2) = [G(p_1)1_c1_f]_{12}\delta_{p_1,p_2} \tag{4}$$

which is a diagonal matrix in color, flavor and momentum space. The matrix element $G(p)$ obeys the Dyson equation

$$G(p) = [G_0^{-1}(p) - \Sigma(p)]^{-1}\,, \tag{5}$$

where $G_0^{-1}(p) = \gamma_\mu p^\mu - m_0$ is the vacuum Green function, see Fig. 1. The self energy $\Sigma(p)$ is defined by an analysis of all one particle irreducible diagrams contributing to the propagator. Having the single particle propagator at our disposal, the physical quantity of interest which is straightforwardly evaluated is the quark condensate. For our separable potential we introduce the nonlocal quark condensate as

$$<\bar{q}q>= N_cN_f\sum_p g(\mathrm{p})\mathrm{Tr}\,[G(p)]\,. \tag{6}$$

where Tr stands for the trace over the Dirac space only. The finite temperature investigations are performed using the Matsubara technique [17, 18, 19], where $p_0 = i\omega_n$ with the fermionic Matsubara frequencies $\omega_n = (2n+1)\pi T$ and $\sum_p$ stands short for $T\sum_n\int\frac{d\mathbf{p}}{(2\pi)^3}$. In order to obtain estimates for the quark condensate one has to make approximations for the self energy.

The first step towards a systematic investigation of the Dyson equation (5) is the selfconsistent Hartree approximation , see Fig. 2

$$\Sigma^H[\mathrm{p};G^H] = -K_0N_cN_fg(\mathrm{p})\sum_k g(\mathrm{k})\mathrm{Tr}[G^H(k)]\,, \tag{7}$$

which defines upon insertion in (5) the propagator in Hartree approximation

$$G^H(k) = [G_0^{-1}(k) - \Sigma^H[\mathrm{k};G^H]]^{-1}\,. \tag{8}$$

The Hartree self energy (7) is a Dirac scalar and appears as a mass term in the propagator,

$$m^H(\mathrm{p}) = m_0 + g(\mathrm{p})K_0 <\bar{q}q>^H \tag{9}$$

with a momentum dependence due to the nonlocality of the interaction kernel (3). The quark condensate in Hartree approximation is, cf. Eqs. (7) and (6),

$$\begin{aligned} <\bar{q}q>^H &= N_cN_f\sum_k g(\mathrm{k})\mathrm{Tr}\Big[G^H(k)\Big]\,, \\ &= -2N_cN_f\int\frac{d^3\mathbf{k}}{(2\pi)^3}g(\mathrm{k})\frac{m^H(\mathrm{k})}{E(\mathrm{p})}[1-2f(E(\mathrm{k}))]\,, \end{aligned} \tag{10}$$

for details see e.g. Refs [1, 2, 3, 4, 9]. In this approximation, the magnitude as well as the temperature dependence of the dynamical mass generation is determined from the condensate only. Note that the restoration of the chiral symmetry at temperatures above the critical one ($T_c \sim 200$) MeV, depending on the choice of the potential, see Ref. [9]) is governed by the Fermi distribution function of quarks in the medium, $f(E(\mathrm{p})) = \{\exp[(E(\mathrm{p}))/T] + 1\}^{-1}$, where the quasiparticle dispersion relation

$$E(\mathrm{p}) = \sqrt{\mathrm{p}^2 + (m^H(\mathrm{p}))^2} \tag{11}$$

contains the momentum dependent Hartree mass (9).

It is, however, questionable whether the Hartree approximation is appropriate for the description of the nonperturbative low energy region of QCD where free quarks should be absent due to confinement. Since mesonic correlations are supposed to dominate the low energy excitation spectrum of the quark matter system one has to study their influence on the results obtained within the Hartree approximation. A systematic perturbation theory for strong interactions is however lacking. Instead, one resorts to an expansion of diagrams to orders $1/N_c$, which we will investigate in this work at finite temperatures.

3 $1/N_c$ expansion

Introducing a new coupling constant $K_0' = K_0 N_c$, the self energy in the selfconsistent Hartree approximation appears of the order O[1] as the leading term in the $1/N_c$ expansion. In order to improve this approximation, we will study next to leading order diagrams, i.e. $\mathcal{O}[1/N_c]$- contributions. Therefore we make the following ansatz for the self energy and for the quark propagator

$$\Sigma(p) = \Sigma^H[p;G] + \frac{1}{N_c}\delta\Sigma[p;G] + \mathcal{O}[1/N_c^2] , \quad (12)$$

$$G(p) = G^H(p) + \frac{1}{N_c}\delta G(p) + \mathcal{O}(1/N_c^2) , \quad (13)$$

where the corrections to the self energy depend in the general case on the full Green function $G(p)$ and on the momentum p. The corrections to the self energy $\delta\Sigma[p;G]$ are not yet specified and will be discussed in the following section. Using the $1/N_c$-approximation (12) for $\Sigma[p;G]$ in the Dyson equation (5), the $(1/N_c)$- expansion to the propagator is given as

$$G(p) = \left(G_0^{-1}(p) - \Sigma[p;G]\right)^{-1}$$
$$= \left(G_0^{-1}(p) - \Sigma^H[p;G^H] - \frac{1}{N_c}\left[\Sigma^H[p;\delta G] + \delta\Sigma[p;G^H] + \mathcal{O}(1/N_c)\right]\right)^{-1} \quad (14)$$

Expanding the $1/N_c$ contribution in the denominator and comparing with (13), we obtain a selfconsistent equation for $\delta G(p)$ in the form

$$\delta G(p) = G^H(p)\Sigma^H[p;\delta G]G^H(p) + G^H(p)\delta\Sigma[p;G^H]G^H(p) . \quad (15)$$

Note, that this consistent $1/N_c$- expansion for the quark propagator is a new result of this paper. In particular, the first term on the r.h.s. of (15) has not been considered in some of the previous approaches, see [11, 16]. In order to get a closed expression we use the fact that the functional dependence of the Hartree selfenergy on the $1/N_c$ corrections to the quark propagator δG is known from Eq. (7). After insertion of $\Sigma^H[p;\delta G]$ on the r.h.s. of Eq. (15), we obtain

$$\sum_p g(\mathrm{p})\mathbf{Tr}[\delta G(p)] = -K_0 N_c N_f \sum_p g^2(\mathrm{p})\mathbf{Tr}\left[G^H(p)G^H(p)\right]\sum_k g(\mathrm{k})\mathbf{Tr}[\delta G(k)]$$
$$+ \sum_p g(\mathrm{p})\mathbf{Tr}\left[G^H(p)\delta\Sigma[p;G^H]G^H(p)\right]$$
$$= \frac{1}{1-J^\sigma(0)}\sum_p g(\mathrm{p})\mathbf{Tr}\left[G^H(p)\delta\Sigma[p;G^H]G^H(p)\right] , \quad (16)$$

where the scalar quark loop integral $J^\sigma(0)$ is defined in the Appendix. The $1/N_c$ expansion of the quark condensate corresponding to that of the propagator (13) and the definiton of the quark condensate (6) reads

$$< \bar{q}q > = < \bar{q}q >^H + \delta < \bar{q}q > + \mathcal{O}[1/N_c^2] . \quad (17)$$

The $1/N_c$ correction to the condensate is obtained in closed form using the result (16)

$$\delta < \bar{q}q > = Z N_f \sum_p g(\mathrm{p})\mathbf{Tr}\left[G^H(p)\delta\Sigma[p;G^H]G^H(p)\right] \quad (18)$$

with a prefactor $Z = 1/(1-J^\sigma(0))$ as derived in (16) coming from the $1/N_c$ contributions to the Hartree self energy $\Sigma^H[\mathrm{p},\delta G]$, see in Fig. 3. This prefactor leads to a considerable rescaling ($Z \sim 4$) which has been discussed before in Refs. [12, 14] for the NJL model and is obtained here for the more general case of a nonlocal separable interaction.

4 Mesonic Fluctuations

Within the chiral quark model as defined in Section 2, the complete set of diagrams contributing in $\mathcal{O}[1/N_c]$ to the self energy is given in Fig. 4. The double line corresponds to the RPA - type partial resummation of the chain of bubble diagrams, where the quark - antiquark loop in Hartree approximation defines the polarization functions $J^\phi(p-k)$ in the scalar and pseudoscalar channel ($\phi = \sigma, \pi$), see Appendix A.

The $\mathcal{O}[1/N_c]$ self energy contribution is given by

$$\delta\Sigma[p;G^H] = K_0' \sum_k g^2\left(\frac{|\mathbf{p}+\mathbf{k}|}{2}\right)\left[\frac{G^H(k)}{1-J^\sigma(p-k)} - (N_f^2-1)\frac{\gamma_5 G^H(k)\gamma_5}{1-J^\pi(p-k)}\right] . \quad (19)$$

The denominators $1 - J^\phi(p-k)$ occur due to the resummation and thus strong correlations can be described. Note that the $1/N_c$ self energy is a dynamical quantity and has not yet been solved in its complexity. The most dramatic effect is the occurence of collective excitations in the quark - antiquark channel when $\mathrm{Re}J^\Phi(P = M_\Phi) = 1$ (and $\mathrm{Im}J^\Phi(P = M_\Phi) = 0$) which correspond to mesonic bound states. The full treatment of the RPA approximation which contains bound and scattering states is possible for the separable interaction and will be regarded in an additional work. In what follows we restrict us to the consideration of bound states only and use an expansion of the polarization function at the mesonic poles (pole approximation) which leads to the introduction of meson propagators and meson - quark-antiquark form factors (see Appendix A)

$$\frac{1}{1-J^\phi(P)} = \frac{1}{M_\phi^2 - P^2}\frac{g_{\phi q\bar{q}}^2(M_\phi)}{N_f K_0'} . \quad (20)$$

Using this ansatz for the self-energy and the short notation with $\phi = \sigma, \pi$, $\Gamma^\sigma = 1_D$ and $\Gamma^\pi = i\gamma_5$ we obtain

$$\delta < \bar{q}q >^\phi = \frac{g^2_{\phi qq}}{1 - J^\sigma(0)} \int \frac{d^3\mathbf{p}}{(2\pi)^3} g(\mathbf{p}) \int \frac{d^3\mathbf{k}}{(2\pi)^3} g^2\left(\frac{|\mathbf{p}+\mathbf{k}|}{2}\right) \int \frac{dp_0}{2\pi} \int \frac{dk_0}{2\pi} \frac{1}{M^2_\phi - (k-p)^2} \mathbf{Tr}\left[G^H(p)\Gamma^\phi G^H(k)\Gamma^\phi G^H(p)\right] \quad (21)$$

For the $1/N_c$ mesonic contributions (21) we obtain after performing the Dirac trace and the Matsubara summation

$$\begin{aligned}
\delta < \bar{q}q >^\phi = & \frac{2g^2_{\phi qq}}{1 - J^\sigma(0)} \int \frac{d^3\mathbf{p}}{(2\pi)^3} g(\mathbf{p}) \int \frac{d^3\mathbf{k}}{(2\pi)^3} g^2\left(\frac{|\mathbf{p}+\mathbf{k}|}{2}\right) \\
& \left\{ \frac{m^H(\mathbf{p})}{E^2(\mathbf{p})} \frac{1-2f(E(\mathbf{p}))}{E(\mathbf{p})} \left(\frac{1-2f(E(\mathbf{k}))}{E(\mathbf{k})} - \frac{1+2n(E_\phi(\mathbf{k}-\mathbf{p}))}{E_\phi(\mathbf{k}-\mathbf{p})} \right) \right. \\
& - \left(\left[\left[\frac{m^H(\mathbf{p})}{E^3(\mathbf{p})} \frac{(M^2_\phi - [m^H(\mathbf{k}) \pm m^H(\mathbf{p})]^2)(E_\phi(\mathbf{k}-\mathbf{p}) + E(\mathbf{k}))}{E_\phi(\mathbf{k}-\mathbf{p})E(\mathbf{k})(E_\phi(\mathbf{k}-\mathbf{p}) + E(\mathbf{p}) + E(\mathbf{k}))} \right. \right. \right. \\
& - \left. \frac{m^H(\mathbf{k}) \pm m^H(\mathbf{p})}{E_\phi(\mathbf{k}-\mathbf{p})E(\mathbf{k})E(\mathbf{p})} \right] \\
& \frac{[1 + n(E(\mathbf{p}) + E(\mathbf{k})) + n(E_\phi(\mathbf{k}-\mathbf{p}))]}{E_\phi(\mathbf{k}-\mathbf{p}) + E(\mathbf{p}) + E(\mathbf{k})} [1 - f(E(\mathbf{p})) - f(E(\mathbf{k}))] \\
& \left. \left. \left. + [E(\mathbf{k}) \leftrightarrow -E(\mathbf{k})] \right] + [E(\mathbf{p}) \leftrightarrow -E(\mathbf{p})] \right) \right\}
\end{aligned} \quad (22)$$

with the energies $E_\phi(\mathbf{k}-\mathbf{p}) = \sqrt{(\mathbf{k}-\mathbf{p})^2 + M^2_\phi}$ and the bosonic distribution function $n(E_\Phi) = [\exp(E_\Phi/T) - 1]^{-1}$.

In order to compare our results with previous works we will now discuss the $T = 0$ case. At zero temperature the Fermi and Bose distribution functions vanish. For this case we obtain the same results as in Refs. [12, 14]. Note that the latter work is formulated at zero temperature in a 4d notation. In the $T = 0$ limit of Eqs. (10) and (22) we obtain

$$\begin{aligned}
\delta < \bar{q}q >^\phi = & \frac{2g^2_{\phi qq}}{1 - J^\sigma(0)} \int \frac{d^3\mathbf{p}}{(2\pi)^3} g(\mathbf{p}) \int \frac{d^3\mathbf{k}}{(2\pi)^3} g^2\left(\frac{|\mathbf{p}+\mathbf{k}|}{2}\right) \left\{ \frac{m^H(\mathbf{p})}{E^3(\mathbf{p})E(\mathbf{k})E_\phi(\mathbf{k}-\mathbf{p})} \right. \\
& \left[E_\phi(\mathbf{k}-\mathbf{p}) - E(\mathbf{k}) - (E_\phi(\mathbf{k}-\mathbf{p}) + E(\mathbf{k})) \frac{M^2_\phi - [m^H(\mathbf{k}) \pm m^H(\mathbf{p})]^2}{(E_\phi(\mathbf{k}-\mathbf{p}) + E(\mathbf{p}) + E(\mathbf{k}))^2} \right. \\
& \left. \left. + \frac{E^2(\mathbf{p})[m^H(\mathbf{k}) \pm m^H(\mathbf{p})]}{m^H(\mathbf{p})(E_\phi(\mathbf{k}-\mathbf{p}) + E(\mathbf{p}) + E(\mathbf{k}))} \right] \right\} .
\end{aligned} \quad (23)$$

In the following section we present the numerical evaluation of the above discussed $1/N_c$ corrections to the quark condensate.

5 Numerical results and discussion

In section 2 we have introduced a general nonlocal interaction kernel in separable form. In order to compare the numerical results with previous approaches within the NJL model, we will restrict us in this paper to the discussion of a cut-off formfactor

$$g\left(\frac{|\mathbf{p}+\mathbf{k}|}{2}\right) = \Theta\left(1 - \frac{|\mathbf{p}+\mathbf{k}|}{2\Lambda_{NJL}}\right). \quad (24)$$

The chiral quark model with soft formfactors as, e.g. a Gaussian one, has been discussed in Refs. [8, 9].

After fixing the parameters of the model as described in Appendix A, we obtain for the quark condensate in the Hartree approximation $< \bar{u}u >^H = -(250 \text{ MeV})^3$ and for the quark mass $m^H = 300$ MeV in agreement with the well-known data of the literature [1, 2, 3, 4].

In the next step, discussed in Section 4, we have included dynamical mesonic fluctuations. Compared with the Hartree term (10) where we have to solve a one-loop integral, the next to leading order contribution is a two-loop integral which after summation over both Matsubara frequencies (k_0, p_0) reduces to three-dimensional integrals over the variables $\mathbf{k}, \mathbf{p}, z$.

At first we want to discuss the $T = 0$ limit. An open question which occurs in the NJL model is the choice of the cut-off for the second momentum integral in Eqs. (22) and (23) over $\mathbf{k}$. A very crude approximation presented in Ref.[16] is the neglection of the second integral by assuming that $\mathbf{k} = 0$. In Refs. [12, 14] the second cut-off $\bar{\Lambda}$ was discussed. Ref.[12] assumes that $\Lambda_{NJL} = \bar{\Lambda}$ and in Ref.[14] upper and lower limits are determined from a calculation of f_π in $(1/N_c)$. In the formulation which we have chosen such a problem does not exist since the integrals are regularized in the separable approach by the proper treatment of the formfactors. That leads to a cut-off Λ_{NJL} which does regularize the integral over $\mathbf{p}$ and an integral limit $\bar{\Lambda}$ which regularizes the $\mathbf{k}$- integration $\bar{\Lambda} = -pz + \sqrt{4\Lambda^2_{NJL} - p^2(1-z^2)}$ where $z = \cos\theta$, if θ denotes the angle between the momenta $\mathbf{k}$ and $\mathbf{p}$. The second cut-off $\bar{\Lambda}$ runs between $\Lambda_{NJL} < \bar{\Lambda} < 3\Lambda_{NJL}$. Note that in solving (21) one has to check the integral limits for each term separately due to different combinations of formfactors partly hidden in the momentum dependent quark mass $m^H(\mathbf{p}) = g(\mathbf{p})m^H$.

The result for such a calculation for the $T = 0$ limit is a decreased condensate $< \bar{u}u >= -(240 \text{ MeV})^3$. For comparison, a decrease of the quark mass due to the mesonic correlations in $1/N_c$ has been obtained in Ref.[12].

Let us now consider the finite temperature case. The results are plotted in Fig. 5. Paying attention to the shape of the chiral phase transition, we observe that the Hartree contribution shows a soft transition. The inclusion of mesonic fluctuations stabilizes the quark condensate against medium effects.

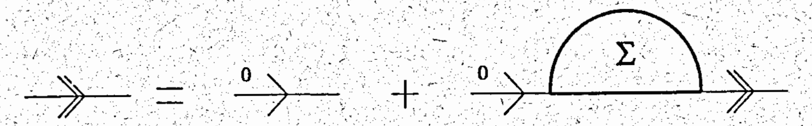

Figure 1. The Dyson equation with the full self energy.

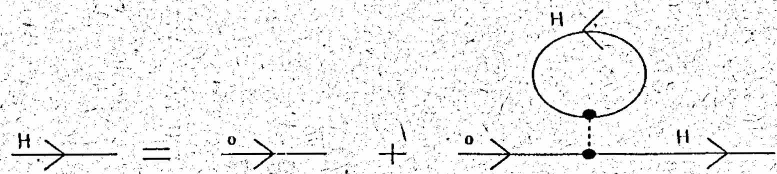

Figure 2. The Dyson equation in selfconsistent Hartree approximation.

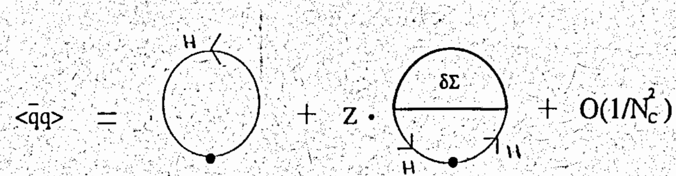

Figure 3. $1/N_c$-expansion of the quark condensate.

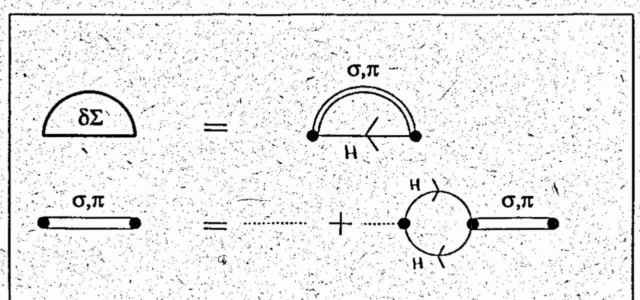

Figure 4. $1/N_c$- approximation for the self energy. The scalar- and pseudoscalar fluctuations are described as RPA- type correlations.

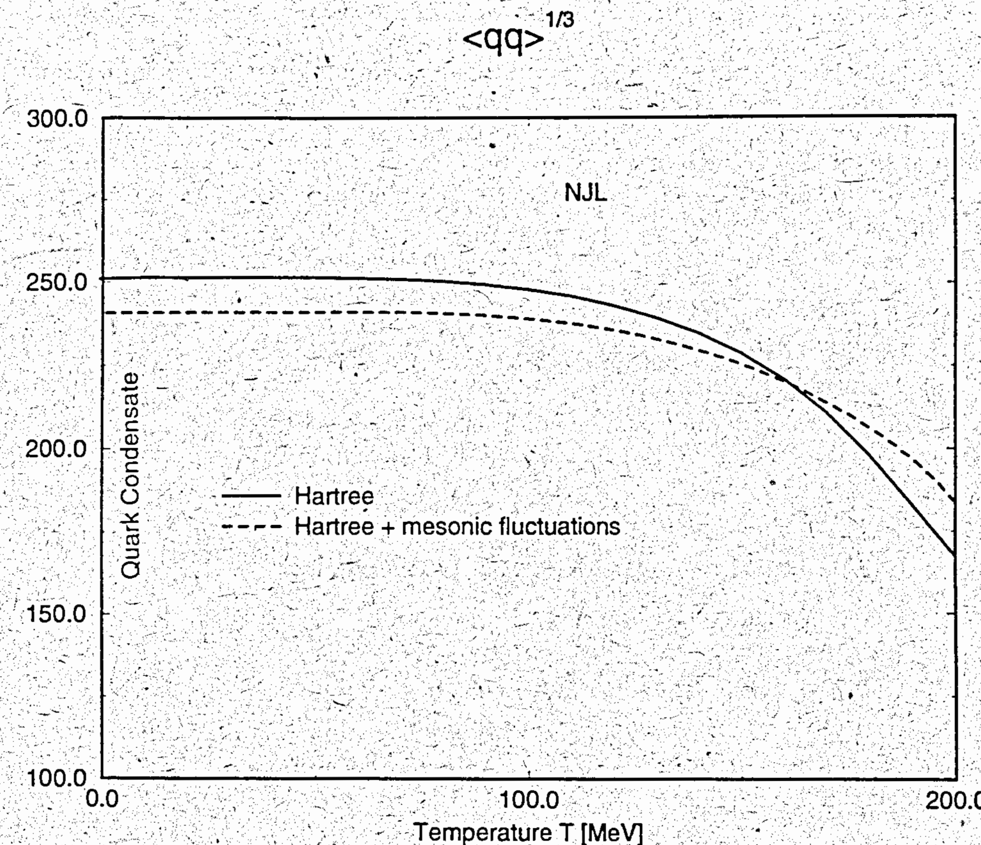

Figure 5. The quark condensate as a function of the temperature in self consistent Hartree approximation (solid line) and with inclusion of mesonic fluctuations (dotted line).

6 Conclusions

In conclusion we have obtained the following new results: (i) a consistent $1/N_c$ expansion for the self energy as well as for the propagator and a closed formula for the quark condensate in the $1/N_c$ expansion, (ii) a finite temperature result for the $1/N_c$ quark condensate in pole approximation within the Matsubara formalism, (iii) a proper treatment of the integral limits for the two loop diagrams.

The numerical evaluation for a NJL-type model shows that the mesonic fluctuations lead to a decrease of the quark condensate at $T = 0$ and to a stabilization against melting in a medium at finite temperature, compared with the Hartree approximation.

Acknowledgement

Our thanks go to the DEUTSCHE FORSCHUNGSGEMEINSCHAFT which gave a research grant to one of us (Yu. K.). G.R. and S.S. are grateful for the hospitality provided to them during a stay in Dubna. M.K.V. acknowledges the support provided by INTAS grant No. W 94-2915 and RFFI grant No. W 93-02-14411 as well as the hospitality extended to him during a stay in Rostock.

A Table of integrals and parameter fixing

The polarization operators are given as

$$J^\phi(P) = -K_0' N_f \sum_q g^2(\mathrm{q}) \mathbf{Tr}\left[\Gamma^\phi G^H(q+P/2)\Gamma^\phi G^H(q-P/2)\right] . \tag{25}$$

The temperature dependence meson masses are obtained from the solution of the Bethe-Salpeter equation

$$1 - J^\phi(P_0 = M_\phi(T), \mathbf{P} = 0) = 0 \ , \tag{26}$$

where the polarization operators $J^\phi(P_0)$ are defined as

$$J^\pi(P_0) = \frac{2K_0' N_f}{\pi^2} \int d\mathrm{q}\mathrm{q}^2 g^2(\mathrm{q}) \frac{E(\mathrm{q})}{E^2(\mathrm{q}) - (P_0/2)^2} [1 - 2f(E(\mathrm{q}))] \ , \tag{27}$$

$$J^\sigma(P_0) = \frac{2K_0' N_f}{\pi^2} \int d\mathrm{q}\mathrm{q}^2 \frac{g^2(\mathrm{q})}{E(\mathrm{q})} \frac{\mathrm{q}^2}{E^2(\mathrm{q}) - (P_0/2)^2} [1 - 2f(E(\mathrm{q}))] \ , \tag{28}$$

the quark meson coupling constants in the rest frame can be written as

$$g^{-2}_{\phi q\bar q}(M_\phi) = \frac{1}{2K_0' N_f} \frac{d}{dP^2} J^\phi(P)\Big|_{P^2 = M_\phi^2} \approx \frac{1}{2K_0' N_f} \frac{d}{2P_0 dP_0} J^\phi(P_0, 0)\Big|_{P_0 = M_\phi} \tag{29}$$

$$g^{-2}_{\pi q\bar q} = \frac{1}{2\pi^2} \int d\mathrm{q}\, \mathrm{q}^2\, g^2(\mathrm{q}) \frac{E(\mathrm{q})}{(E^2(\mathrm{q}) - M_\pi^2/4)^2} \Big[1 - 2f(E(\mathrm{q}))\Big] , \tag{30}$$

$$g^{-2}_{\sigma q\bar q} = \frac{1}{2\pi^2} \int d\mathrm{q}\, \mathrm{q}^2\, g^2(\mathrm{q}) \frac{\mathrm{q}^2}{E(\mathrm{q})} \frac{1}{(E^2(\mathrm{q}) - M_\sigma^2/4)^2} \Big[1 - 2f(E(\mathrm{q}))\Big] .$$

The pion decay constant which we use for the parameter fixing at zero temperature is calculated by

$$f_\pi = \frac{\sqrt{N_c} g_{\pi q\bar q}}{2\pi^2} \int d\mathrm{q}\, \mathrm{q}^2\, g(\mathrm{q}) \frac{m^H(\mathrm{q})}{E(\mathrm{q})(E^2(\mathrm{q}) - M_\pi^2/4)} , \tag{31}$$

The model contains three parameters: the coupling constant K_0, the current quark mass m_0 and the range of the formfactor of the potential. We fix these 3 parameters to reproduce the pion mass ($M_\pi = 140 MeV$) Eqs.(26) and (27), the pion decay constant ($f_\pi = 93 MeV$) Eqs.(30),(31) and the quarkmass on Hartree level at zero temperature. This choice of the paramters is usual for the NJL- model and can also be found in the same order of magnitude in Refs. [2, 3, 11, 12, 14, 16].

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Received by Publishing Department
on September 5, 1995.