ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-95-385

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NON-PERTURBATIVE EXPANSION WITH MASSIVE QUARKS AND e^+e^- ANNIHILATION AT LOW ENERGIES

Submitted to «Modern Physics Letters A»

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1995

Эберт Д., Соловцов И.Л., Соловцова О.П. E2-95-385
Непертурбативное разложение с массивными кварками
и e^+e^--аннигиляция при низких энергиях

В квантовой хромодинамике с массивными кварками строится непертурбативное разложение, основанное на новом малом параметре. Вычисляется непертурбативная β-функция и обсуждается ее связь со статическим кварк-антикварковым потенциалом. Анализируется процесс e^+e^--аннигиляции в адроны при низких энергиях с помощью специального метода сглаживания резонансов.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1995

Ebert D., Solovtsov I.L., Solovtsova O.P. E2-95-385
Non-Perturbative Expansion with Massive Quarks
and e^+e^- Annihilation at Low Energies

A non-perturbative expansion in QCD with massive quarks that is based on a new small expansion parameter is constructed. We calculate a non-perturbative β function and discuss its behaviour in connection with the static quark-antiquark potential. By using the «smearing» method of Poggio, Quinn, and Weinberg, the process of e^+e^- annihilation into hadrons at low energies is analyzed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1995

The idea of constructing the so-called floating or variational series in quantum theories was suggested and applied to the anharmonic oscillator case in Ref. [1]. Within this approach, a certain variational principle is combined with a possibility of calculating corrections to the main contribution that gives a possibility to answer the question about the validity of the principal contribution and the region of applicability of the results obtained. At present, this idea finds many applications in developing various approaches, which somehow make it possible to go beyond perturbation theory. Among them, there are the Gaussian effective potential method [2-4], the optimized linear δ–expansion [5,6], and the variational perturbation theory [7-9].

In this paper, we will apply the method of variational perturbation theory for constructing a non-perturbative QCD expansion based on a new small expansion parameter. Within this method, a quantity can be approximated by a series, which is different from the perturbative expansion and which can be used to go beyond the weak–coupling regime and allows one to deal with considerably lower energies than in the case of perturbation theory. We will consider a massive renormalization scheme, in which the quark masses are renormalized so that the value of m_q is the position of the pole in the quark propagator: $S_q^{-1}(\hat{p} = m_q) = 0$. In the case of perturbation theory, the renormalization prescription of such type was considered in Ref. [10] (in this connection, see also Refs. [11-13]). In the framework of this scheme, the effective coupling constant depends on the quark masses that provides a natural way to include into consideration the threshold effects without any additional matching procedure. The non-perturbative expansion in the case of massless MS–like renormalization scheme was considered in Refs. [14,15], and some applications of this method were given in Refs. [16,17]. Within this approach, a quantity under consideration is represented in the form of a power series with a new expansion parameter a associated with the initial coupling constant $\lambda = g^2/(4\pi)^2 = \alpha_s/(4\pi)$ by the following equation

$$\lambda = \frac{1}{C}\,\frac{a^2}{(1-a)^3}\,. \tag{1}$$

It is clear that for all values of the coupling constant $\lambda \geq 0$ the expansion parameter a obeys the inequality $0 < a \leq 1$. The positive parameter C plays the role of a variational parameter, which is associated with the use of the variational or floating series. The original quantity, which is approximated by this expansion, does not depend on the auxiliary parameters C, however any finite approximation depends on it on account of the truncation of the series. Here we will fix this parameter using some further information, which comes from the potential approach to meson spectroscopy.

In the first order of our approximation, the renormalization constant $Z_\lambda(\mu',\mu)$, which describes the modification of the coupling constant $\lambda(\mu)$ when we change the scale parameter from μ to μ', has the following form

$$Z_\lambda(\mu',\mu) = 1 + \lambda_{\rm eff}\left[J(\mu'^2) - J(\mu^2)\right]. \tag{2}$$

Here

$$\lambda_{\rm eff} = \frac{1}{C}\,a^2\,(1+3\,a) \tag{3}$$

and

$$J(\mu'^2) - J(\mu^2) = 11\,\ln\frac{\mu'^2}{\mu^2} - \frac{2}{3}\sum_f\left[I\left(\frac{\mu'^2}{m_f^2}\right) - I\left(\frac{\mu^2}{m_f^2}\right)\right], \tag{4}$$

where the function $I(\mu^2/m^2)$ is the well known one-loop integral

$$I\left(\frac{\mu^2}{m^2}\right) = 6\int_0^1 dx\, x\,(1-x)\,\ln\left[1+\frac{\mu^2}{m^2}x\,(1-x)\right]. \tag{5}$$

The non-relativistic static quark-antiquark potential used in quark models is associated with the running coupling constant $\lambda(Q^2)$ as follows

$$V(r) \sim \int d\mathbf{Q}\,\exp\left(\mathrm{i}\mathbf{Q}\cdot\mathbf{r}\right)\frac{\lambda(Q^2)}{Q^2}, \tag{6}$$

If in the infrared region at small Q^2 the running coupling constant behaves as

$$\lambda(Q^2) \sim (Q^2)^{-\kappa}, \tag{7}$$

it immediately follows that at long distances the quark interaction potential has the following asymptotic behaviour

$$V(r) \sim r^{2\kappa-1}. \tag{8}$$

In the literature, for the description of the phenomenology of meson spectroscopy one uses various types of a confinement potential. In particular, from Eqs. (7) and (8) we can obtain the linear potential by setting $\kappa = 1$, the oscillator potential by taking $\kappa = 3/2$, and the potential close to logarithmic one for $\kappa \simeq 1/2$. We will consider all these possibilities admitted in meson spectroscopy. Thus, in this context, the parameter C turns out to be with some uncertainty factor about 3.

The value of κ is associated with the asymptotic behaviour of the renormalization group β-function at large λ:

$$-\frac{\beta(\lambda)}{\lambda} = -\frac{1}{\lambda}\mu^2\frac{d\,\lambda(\mu^2)}{d\,\mu^2} \simeq \kappa. \tag{9}$$

From Eq.(2), for the β-function one finds

$$-\frac{\beta(\lambda)}{\lambda} = \lambda_{\rm eff}(a)\left[11-\frac{2}{3}\sum_f F_1\left(\frac{m_f^2}{\mu^2}\right)\right], \tag{10}$$

where $F_1(m^2/\mu^2) = \mu^2/m^2\, I'(\mu^2/m^2)$.

The renormalization scale dependence of the running expansion parameter $a = a(\mu^2)$ is defined by the following equation

$$C\left[U(a)-U(a_0)\right] = 11\ln\frac{\mu^2}{\mu_0^2} - \frac{2}{3}\sum_f\left[I\left(\frac{\mu^2}{m_f^2}\right) - I\left(\frac{\mu_0{}^2}{m_f^2}\right)\right], \tag{11}$$

where μ_0 is some normalization point, $a_0 = a(\mu_0^2)$, and the function $U(a)$ has the following form

$$U(a) = \frac{1}{a^2} - \frac{3}{a} - 12\ln a + \frac{3}{4}\ln(1-a) + \frac{45}{4}\ln(1+3a). \tag{12}$$

In the limit $\mu \to 0$, the function $F_1 \to 0$, $a \to 1$, and $-\beta(\lambda)/\lambda \to \kappa_{\rm asympt} = 44/C_{\rm asympt}$. However, in an appropriate region of the momentum, we will choose the value of C slightly

less than the value of $C_{\rm asympt}$. Fig. 1 shows function (10) for three values of the parameter C as a function of $1/Q$, in order to have some analogy with distances. For $C = 27$, the value of the function $-\beta/\lambda \simeq 3/2$ at appropriate large λ and the shape of the quark-antiquark interaction at long distances is close to the oscillator potential; for $C = 39$, $-\beta/\lambda \simeq 1$ and the potential is close to linear potential; and for $C = 78$, $-\beta/\lambda \simeq 1/2$ and the corresponding potential is close to logarithmic potential. The curves in Fig. 1 are derived at the following values of the pole quark masses: $m_u = m_d = 0.33$ GeV, $m_s = 0.50$ GeV, $m_c = 1.5$ GeV, $m_b = 4.7$ GeV, and $m_t = 174$ GeV. Here we use the arguments of Ref. [18,19] that the pole quark masses are much the same as the constituent quark masses.

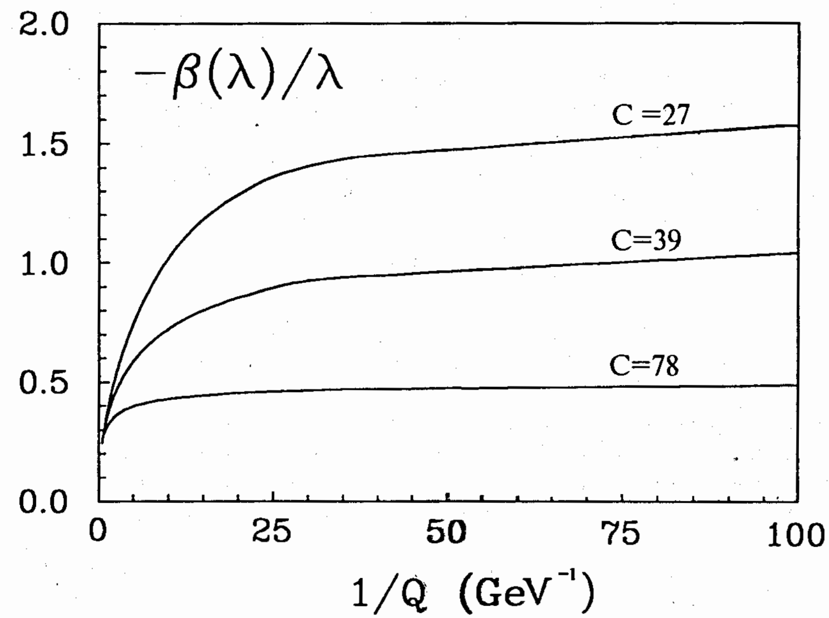

Fig. 1. The function $-\beta(\lambda)/\lambda$ versus $1/Q$ for C =27, 39, and 78.

Following Ref. [20], let us consider the smeared quantity

$$R_\Delta(s) = \frac{1}{2\mathrm{i}}\left[\Pi(s+\mathrm{i}\Delta) - \Pi(s-\mathrm{i}\Delta)\right]. \tag{13}$$

If Δ is sufficiently large, this quantity (13) permits one to avoid the problem of threshold singularities coming from the higher order diagrams. By using the dispersion relation for the hadronic vacuum polarization $\Pi(q^2)$, one finds

$$R_\Delta(s) = \frac{\Delta}{\pi}\int_0^\infty ds'\frac{R(s')}{(s'-s)^2+\Delta^2}, \tag{14}$$

where $R(s)$ is the ratio of e^+e^- annihilation into hadrons. The smeared quantity $R_\Delta(s)$ allows one to compare the experimental data with the theoretical predictions obtained in the framework of QCD [21,22,16]. Here, for the same aims, we will consider the function

$$W_\Delta(q^2) = \frac{d\,R_\Delta(q^2)}{d\,q^2} = -\frac{1}{2\mathrm{i}}\left[\frac{D(q^2+\mathrm{i}\Delta)}{q^2+\mathrm{i}\Delta} - \frac{D(q^2-\mathrm{i}\Delta)}{q^2-\mathrm{i}\Delta}\right], \tag{15}$$

where for the $D(q^2)$-function we obtain

$$\begin{aligned} D(q^2) &= -q^2\frac{d}{d\,q^2}\,\Pi(q^2) \\ &= \frac{3}{\pi}\sum_f Q_f^2\left[F_1\left(\frac{m_f^2}{-q^2}\right) + 4\,\lambda_{\rm eff}(q^2)\,F_2\left(\frac{m_f^2}{-q^2}\right)\right]. \end{aligned} \tag{16}$$

For the function F_2 we will use the result obtained on the basis of the Schwinger approximation of the imaginary part of the vacuum polarization function:

$$F_2\left(\frac{m^2}{-q^2}\right) = \frac{4\pi}{3}\,\frac{4m^2}{-q^2}\int_0^1 dv\,\frac{v^2\,(3-v^2)}{[4m^2/(-q^2)+1-v^2]^2}\,f(v) \tag{17}$$

with

$$f(v) = \frac{\pi}{2v} - \frac{3+v}{4}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right). \tag{18}$$

The effective coupling constant $\lambda_{\rm eff}$ in Eq.(16) is defined by Eq.(3) with the running expansion parameter $a(-q^2)$, which can be find as a solution of Eq. (11).

We should note that in contrast to perturbation theory, where at $q^2 = -\Lambda^2_{\rm QCD}$ the running coupling constant has an infrared singularity, which contradict the dispersion relation for the D-function, the effective coupling constant $\lambda_{\rm eff}$ is a finite function in the infrared region with freezing behaviour at small Q^2. (The discussion of the behaviour and analytic properties of the effective coupling constant in the massless case was considered in Ref. [17].)

In Fig. 2, we plotted this effective coupling constant $\alpha_s = 4\pi\lambda_{\rm eff}$ at small Q for three values of the parameter C. As the normalization point μ_0 in Eq.(11) we used the τ lepton mass $\mu_0 = M_\tau = 1.777$ GeV with $\alpha_s(M_\tau) = 0.36$. After this, all our parameters are fixed, and we do not fit any parameters by using the e^+e^- experimental data. Thus, the effective coupling constant α_s is frozen, and, at very small Q, $\alpha_s \simeq 1.8$, 1.2, and 0.6 for $C = 27$, 39, and 78, respectively. The idea that the QCD coupling constant can be frozen at low energies has been discussed within many approaches (see the discussion in Refs. [21,22]). The following fit-invariant integral characteristic

$$\int_0^1 dQ\,\frac{\alpha_s(Q^2)}{\pi} \simeq 0.2 \text{ GeV} \tag{19}$$

is the experimental result associated with a freezing of the coupling constant [21]. For integral (19), we obtain 0.22, 0.19, and 0.14 for $C = 27$, 39, and 78, respectively. Thus, the value of (19) is rather associated with the linear shape of the potential, and we will use the corresponding value of $C = 39$ in the following consideration.

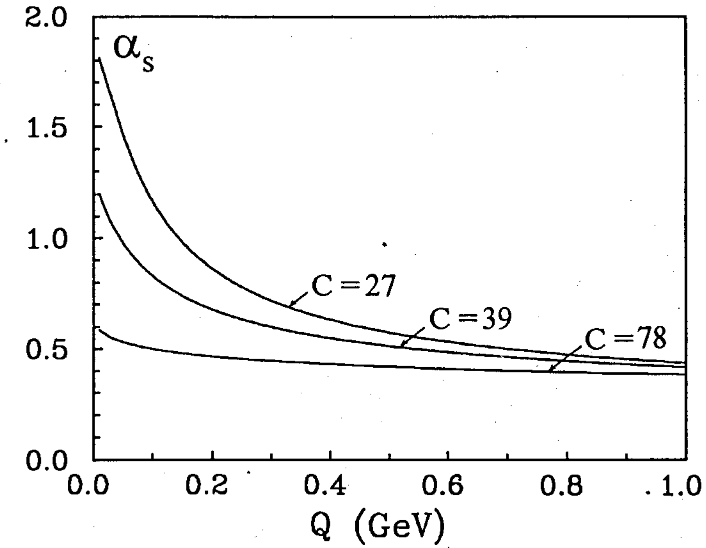

Fig. 2. The function $\alpha_s = 4\pi\lambda_{\rm eff}$ versus Q for $C = 27$, 39, and 78.

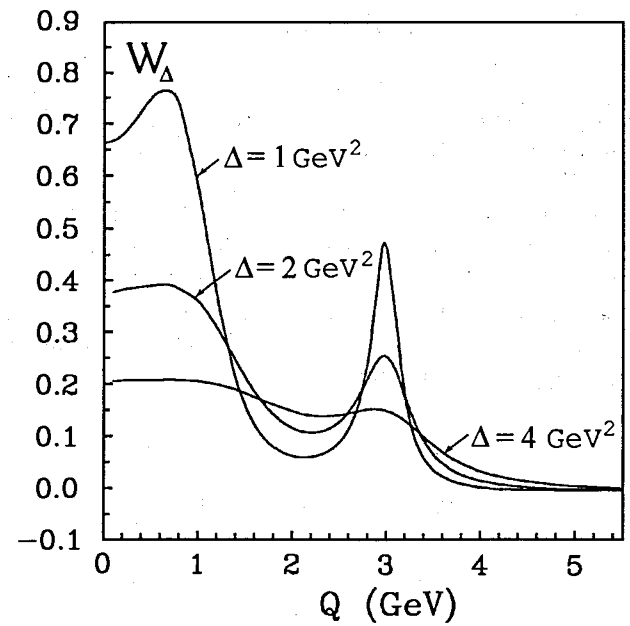

Fig. 3. The function W_Δ versus $Q = \sqrt{s}$ for $\Delta = 1$, 2, and 4 GeV^2.

Fig. 3 shows the function $W_\Delta(s)$ from Eq.(15) for $\Delta = 1$, 2, and 4 GeV^2. Two peaks in this figure are associated with the regions of the ρ-meson resonance and of the charmonium. In Fig. 4, we compare the function $W_\Delta(s)$ for $\Delta = 2$ GeV^2 (solid line) and the smeared experimental data (dashed line) from Ref. [21]. It should be emphasized that our curve obtained in the first non-trivial order of $\lambda_{\rm eff}$ are close to the theoretical predictions obtained in Ref. [21] on the basis of optimization of the third-order QCD perturbative corrections to $R_{e^+e^-}$ (e.g., see dot-dashed curve in Fig. 4).

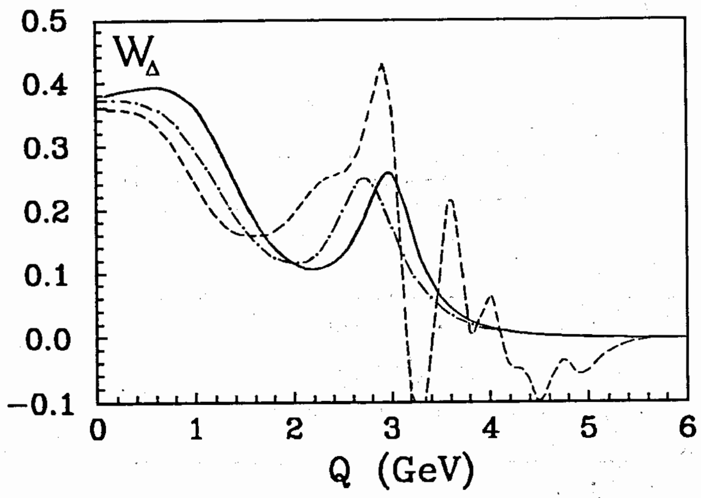

Fig. 4. The function W_Δ versus $Q = \sqrt{s}$ for $\Delta = 2$ GeV^2. The solid curve is obtained from Eq. (15), the dashed one from the smeared experimental data (taken from Ref. [21]) and the dot-dashed one from applying the optimization procedure to the third-order calculation of $R_{e^+e^-}$ (Ref. [21]).

As we can see from Fig. 4, the value of $\Delta = 2$ GeV^2 is not sufficiently large to smooth the region of charm resonances; by increasing the value of Δ up to 4 GeV^2 the experimental curve is drawn near the theoretical prediction shown in Fig. 3.

Acknowledgments. We would like to thank H.F. Jones, A.L. Kataev, D.I. Kazakov, V.E. Rochev, and A.N. Sissakian for interest in the work and useful comments.
This work was supported in part by the Heisenberg-Landau program.

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Received by Publishing Department
on September 5, 1995.