ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-95-341

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ON CONTINUOUS MASS-DEPENDENT ANALYSIS OF DIS DATA

Submitted to EPS HEP 95, Brussels, July 1995

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1995

Ширков Д.В. E2-95-341

О непрерывном, зависящем от масс анализе данных по глубоконеупругому рассеянию

Рассмотрена проблема учёта тяжёлых кварков в глубоконеупругом рассеянии. Известно, что безмассовые схемы вычитания, например $\overline{MS}$-схема, не являются физически адекватными в случае, если экспериментальные данные сосредоточены вблизи порогов тяжёлых (b,c) кварков. С другой стороны, существует техническая проблема непрерывности моментов структурных функций M_n.

Предлагается простая модификация стандартного безмассового приближения, которая приводит к более реалистическому непрерывному выражению для $M_n(Q)$ и может быть использована для практического двухпетлевого анализа данных по глубоконеупругому рассеянию.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1995

Shirkov D.V. E2-95-341

On Continuous Mass-Dependent Analysis of DIS Data

The heavy quark mass issue in deep inelastic scattering is considered. The point is that massless subtraction schemes like, e.g., $\overline{MS}$ are not physically adequate to the case of data concentrated close to heavy (b,c) quark thresholds. On the other hand, there exists some technical problem with continuity of structure function moments M_n.

We propose a simple modification of the standard massless approach that yields a more realistic continuous expression for $M_n(Q)$ and can be used in practice for two-loop DIS data analysis.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1995

1 Introduction

"Perturbative QCD" in practice is a product of combined use of genuine perturbative calculations and of a renorm-group (RG) technique. The popular RG resummation algorithm employs beta-function $\beta(\alpha_s)$, anomalous dimensions $\gamma(\alpha_s)$ and effective coupling $\bar{\alpha}_s(Q)$ calculated within some massless renormalization scheme, *e.g.*, the $\overline{\mathrm{MS}}$ scheme. This practice seems rather natural for short-distance dominating phenomena. However, in nowadays QCD it meets some specific trouble related to the fact that masses of heavy quarks (HQ) are *parametrically* large as compared to the characteristic confinement scale of an order of few hundred MeV.

In an analysis of deep inelastic hadron-lepton scattering DIS data, one commonly uses a massless version of the $\overline{\mathrm{MS}}$ subtraction scheme. Here, formally, the transition from the Q region with some value of flavour number $f-1$ to the next f-region (i.e., conjuction of the $\bar{\alpha}_s(Q)$ behavior in two different regions or "transition across the M_f threshold") can be performed with the help of the so-called "matching relations" for $\bar{\alpha}_s(Q)$ (see, *e.g.*, refs. [1]). They yield the $\bar{\alpha}_s(Q)$ continuity condition on (every) HQ mass M_f:

$$\bar{\alpha}_s(Q=M_f; f-1) = \bar{\alpha}_s(Q=M_f; f) \tag{1}$$

and provide an accurate $\bar{\alpha}_s$ (Q) decription for the Q value *not too close to the threshold region.*

To analyze DIS data, one can use the RG-summed expression for evolution of the structure function moment $M_n(Q)$. However, there is no yet a simple recipe in the massless $\overline{\mathrm{MS}}$ scheme for providing the continuity for anomalous dimensions and, hence, for the $M_n(Q)$ evolution.

The purpose of this paper is to demonstrate that there exists a rather simple modification of the massless $\overline{\mathrm{MS}}$ scheme which allows one to accurately account for heavy mass thresholds in analysing nonsinglet structure function moments in DIS.

As it has recently been shown [2] by direct summation of HQ polarization one-loop contributions to a nonsinglet light-quark structure function of DIS diagrams, the "physical" mass-dependent coupling $\bar{\alpha}_s(Q,M)$, that bears information about "Euclidean-reflected" heavy thresholds, naturally emerges in evolution equations for parton distributions in hard processes. It was also demonstrated that the same expression $\bar{\alpha}_s(Q,M)$ could be obtained by an explicit solution of massive RG equations.

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Recent estimates demonstrate a significant role of mass effects in α_s evolution if the evolution range involves HQ thresholds. In the paper [3], a quantitative discussion of the $\bar{\alpha}_s(Q,M)$ behaviour in the threshold region has been presented. It was shown for the $\bar{\alpha}_s(Q,M)$ evolution that the smooth mass dependence in the threshold vicinity could lead to an effective shift of the absolute $\bar{\alpha}_s$ values of an order of 10^{-3}. The corresponding corrections to the $\bar{\alpha}_s(M_z)$ value could reach few per cent, that is not smaller than the three-loop contribution [4].

In the next section, we give smooth analytic expressions for mass dependent $\bar{\alpha}_s(Q,M)$ and $M_n(Q,M)$ evolution based upon "massive" RG formalism devised by Bogoliubov and the present author[5], [6] in the mid-fifties. We skip all theoretical and technical details referring the reader to our previous publications[7], [8] and present below just the summary of two-loop results.

Then (Section 3) we discuss a special heavy-mass-dependent, "hybrid", renormalization scheme that naturally arises[2] in the DIS evolution equation and obeys the decoupling property. Section 4 contains some useful approximations that simplify the final two-loop expression for evolution of distribution function moments.

We shall use the following notation. First, parallel with the usual QCD coupling α_s we employ also the couplant $a = \alpha_s/4\pi$. Second, our subscripts and supersripts in brackets just correspond to the number of loops, so that, e.g., $\beta_{(k)} = \beta_{k-1}$. Then, instead of structure function moments M_n we consider only their "evolution part" $\mathcal{M}_n$ (*i.e.*, distribution function moments)

$$M_n(Q,M) = C_n(a, a(Q,M))\mathcal{M}_n(Q,M)$$

neglecting corrections to C_n.

2 Mass-dependent RG solutions

The moment's massless two-loop evolution is described by the expression

$$\frac{\mathcal{M}_n^{(2)}(Q)}{\mathcal{M}_n(\mu)} = \left(\frac{a}{\bar{a}^f(Q)}\right)^{d_n} \exp\left\{\left[a - \bar{a}^f(Q)\right] f_n\right\} \quad (2)$$

with n u m e r i c a l coefficients $d_n = \gamma_0(n)/\beta_0, \beta_0(f) = 11 - (2/3)f$, $f_n = [\beta_0\gamma_1(n) - \beta_1\gamma_0(n)]/\beta_0^2$, ... and fixed value of flavour number f.

In the mass-dependent case, one should use[7], instead of Eq.(2), a bit more complicated expression of the same structure

$$\mathcal{M}_n^{(2)}(Q,M) = \mathcal{M}_n(\mu)\left(\frac{a}{\bar{a}(Q,M)}\right)^{D_n(Q)} \exp\left\{[a - \bar{a}(Q,M)]\, F_n(Q)\right\} \quad (3)$$

with f u n c t i o n a l $Q-$ and mass-dependent coefficients D_n, F_n

$$D_n(Q,M,\mu) = \frac{\gamma_0(n)l}{A_{(1)}(Q,M,\mu)}\, ; \quad l = \ln\frac{Q^2}{\mu^2}\, , \quad (4)$$

$$F_n(Q,M,\mu) = \frac{A_{(1)}(Q)\Gamma_n^{(2)}(Q) - A_{(2)}(Q)\gamma_0(n)l}{[A_{(1)}(Q)]^2} \quad (5)$$

and a two-loop "massive" couplant $\bar{a}$ taken in the RG-summed form

$$\frac{1}{\bar{a}^{(2)}(Q,M;a)} = \frac{1}{a} + A_{(1)} + \frac{A_{(2)}}{A_{(1)}}\ln[1 + aA_{(1)}]. \quad (6)$$

Here $\Gamma_n^{(1)}(= \gamma_0(n)l$ for a non-singlet case) and HQ-mass dependent $A_{(\ell)}, \Gamma_n^{(\ell=2)}$ are the perturbation expansion coefficients:

$$\bar{a}(Q,M)_{pert} = a - a^2 A_{(1)}(Q,M,\mu) + +a^3\left\{\left[A_{(1)}(Q)\right]^2 - A_{(2)}(Q)\right\}\, , \quad (7)$$

$$\left.\frac{\mathcal{M}_n(Q)}{\mathcal{M}_n(Q_0)}\right|_{pert} = 1 + a\Gamma_n^{(1)} + a^2\left[\Gamma_n^{(2)}(Q) + \frac{\Gamma_n^{(1)}}{2}\left(\Gamma_n^{(1)} - A_{(1)}(Q)\right)\right] \quad (8)$$

satisfying the normalization conditions

$$A_{(\ell)}(Q = \mu) = \Gamma_n^{(\ell)}(Q = \mu) = 0$$

i.e., taken in "the MOM-like form"[2].

These massive RG solutions (3),(6) possess several remarkable properties:

– They are "built up of perturbative bricks", *i.e.*, loop-expansion coefficients $A_{(l)}$, $\Gamma_n^{(l)}$ [taken just in the form they appear in the perturbative input] and "contain no memory" about intermediate RG entities such as β- and γ-functions;

– In the massless "pure log" case $[A_{(k)} = \beta_{k-1}l, \Gamma^{(k)} = \gamma_{k-1}l]$ they precisely correspond to the well-known massless expressions;

– Being used in QCD, they smoothly interpolate across the HQ threshold between massless expressions with different flavour numbers f.

[2] Just this form of the $\overline{\text{MS}}$ scheme is commonly used by QCD practitioners in data analysing.

3 Hybrid scheme

In our physical situation it is adequate to present expansion coefficients as a sum of the massless part (with $f = 3$) and HQ-mass dependent contributions, *e.g.*,

$$A^H_{(k)}(Q) = \beta_{k-1}(3)l - \Delta_{(k)}(Q); \tag{9}$$
$$\Delta_{(k)}(Q) = \Delta\beta_{k-1} \sum_h I_0^{(k)}\left(Q^2/M_h^2\right)$$

with $\Delta\beta_0 = 2/3, \Delta\beta_1 = 38/3$ and summation over HQ's: $h = 4, 5$.

Here $I_0^{(k)}$, the k-loop HQ contribution, is subtracted at $Q^2 = 0$. This defines a specific, *hybrid* scheme (marked by "H") that is adequate to the case when the bulk of data lies close to the HQ thresholds (*i.e.*, $Q_0 \simeq M_{4,5}$). At $Q, \mu \ll M_{4,5}$ it smoothly approaches the massless three-flavour MOM-type scheme, *i.e.*, satisfies the *decoupling* condition. However, in the opposite, UV massless, limit it does not correspond to any well known massless scheme. Here, it is appropriate to introduce an effective coupling $\bar{a}_H(Q)$ with the expansion coefficients

$$a_{(k)}(Q) = A^H_{(k)}(Q) + \Delta_{(k)}(\mu) \; .$$

with simple UV limit $a_{(k)} \to \beta_{(k)}(5)l$. After RG summation $\bar{a}^{(2)}_H(Q)$ has the form (6) and enters into Eq. (3) that can be confronted with experimental data. However, to relate the parameter $\bar{a}^{(2)}_H(Q_0 \sim M_h) \equiv a_H(Q_0)$ extracted from these data with, *e.g.*, a widely used parameter $a_{\overline{\rm MS}}$ of the $\overline{\rm MS}$ scheme, the corresponding recalculation should be performed.

We give here the transition relation between $a_H(\mu)$ ($\mu \simeq M_h$) and $\bar{a}_{\overline{\rm MS}}(Q)$ ($Q \gg M_h$) written down in a MOM-like form $\bar{a}_5(l)$ with $f = 5$.

$$\frac{1}{\bar{a}_{\overline{\rm MS}}(Q)} = \frac{1}{\bar{a}_5(l)} = \frac{1}{a_H(\mu)} + \beta_0(5)l + \frac{\beta_1(5)}{\beta_0(5)}\ln[1 + a\beta_0(5)l] \; . \tag{10}$$

4 Useful Approximations

Analytic approximation. As it has been proposed in the paper[3], the one-loop polarization operator in the Euclidean region can be approximated with high accuracy by a simple expression

$$I_0(z) \to \tilde{I}_0(z) = \ln(1 + z/5.30)$$

that has a correct UV limit $\tilde{I}_0(z) \to \ln z - 5/3$. In combination with the pure log structure of the one-loop contribution to M_n, this yields a possibility to use one-loop coefficients with *simplified* Q^2-dependence

$$\tilde{A}_{(1)} = \beta_0(3)l - \frac{2}{3}\sum_h \ln\left(1 + Q^2/\tilde{M}_h^2\right) \; ; \quad \tilde{M}^2 = 5.30M^2 \; ; \tag{11}$$

$$\tilde{D}_n(Q) = \frac{\gamma_0(n)l}{\beta_0(3)l - \frac{2}{3}\sum_h \ln\left(1 + Q^2/\tilde{M}_h^2\right)} \; . \tag{12}$$

These expressions could be used for an evolution analysis in the one-loop approximation.

Spline approximation. For a rough estimate one can use also a simpler, "pure log", ansatz $A_{(k)}(Q) \to A^{spl}_{(k)}(l)$

$$A^{spl}_{(1)}(l) = \beta_{(1)}(3)l - \Delta\beta_{(1)}\sum_h \theta(Q^2 - (\tilde{M}_h^{(1)})^2)\{l + \lambda_h^{(1)}\}; \tag{13}$$

with

$$l = \ln(Q^2/\mu^2) \; ; \quad \lambda_h = \ln(\mu/\tilde{M}_h)^2 \; .$$

This ansatz introduces an additional error to $A_{(1)}(Q)$. However, even in the (mirror) threshold vicinity the absolute distortion is smaller than 0.05 (that corresponds to $\Delta\bar{\alpha}_s \simeq 0.01$).

The utility of this *spline-type* (in terms of the l variable) approximation, Eq. (13), is that it admits a rather simple generalization to the two-loop case "(1)" $\to$ "(2)". The point is that for actual use of our smooth mass-dependent expressions, Eqs.(3)–(6), one needs in addition to $A_{(2)}$ (that has been calculated in a single MOM scheme[9]), an explicit mass-dependent two-loop contribution $\Gamma^{(2)}_n$ to structure function moments.

As far as these terms are not known yet, we could use the massless spline-type approximation to F_n:

$$F_n^{spl}(l) = \frac{A^{spl}_{(1)}(l)\Gamma^{(2),spl}_n(l) - A^{spl}_{(2)}(l)\gamma_0(n)l}{[A^{spl}_{(1)}(l)]^2} \tag{14}$$

with explicit expressions for two-loop expansion coefficients of the same structure as in Eq. (13):

$$C_i(Q) \to C_i^{spl}(l) = c_i(3)l - \Delta c_i\sum_h \theta(Q^2 - (\tilde{M}_h^i)^2)\{l + \lambda_h^i\} \tag{15}$$

with $C_i(l) = \{A_{(2)}(l)\ ,\ \Gamma_n^{(2)}(l)\}$.

Here, a delicate issue of the $\tilde{M}^i$ definition for each two-loop contribution arises[3]. This calculation needs an asymptotic form of the mass-dependent two-loop contribution, i.e., the logarithmic and *constant* term $\ln(Q^2/M^2) - c$, $\tilde{M} = \exp(c/2)M$. As a provisional solution we propose the presumption:

for all two-loop contributions use the same one-loop polarization $\tilde{M} = 2.30M$ value as in the one-loop running $\bar{\alpha}_s$ case.

5 Final Recipe

Our main result is based upon Eq. (3) for evolution of nonsiglet structure function moments. Generally, this equation provides us with smooth, HQ mass dependent description of the $\mathcal{M}_n$ evolution on the two-loop level. Its r.h.s. consists of two factors. The structure of the first one just coincides with the one-loop expression. We shall refer to it as the "one-loop factor". The second, "two-loop", factor is absent in the one-loop case.

However, for the time being we have no possibility to use Eq. (3) as it is due to the absence of two-loop massive calculations. As a temporary remedy, for practical calculation we propose to use some intermediate (between one- and two-loop) variant for the r.h.s. of Eq. (3) –

– Take the first "1-loop factor" in the two-loop approximation with a continuous smooth mass-dependent $A_{(1)}(Q,M)$, $D_n(Q,M)$, Eqs. (6), (4) and (9).

– For the second-loop contributions $A_{(2)}, \Gamma_{(2)}, F_n$ use spline massless approximation described in Eqs. (13), (14) :

$$\frac{\mathcal{M}_n^{(2)}(Q)}{\mathcal{M}_n(\mu)} = \left(\frac{a}{\bar{a}(Q,M)}\right)^{D_n(Q,M)} \exp\left\{[a - \bar{a}(Q,M)]\, F_n^{spl}(l)\right\} . \qquad (16)$$

Acknowledgements

The author is indebted to Drs. Sergei Mikhailov and Alexander Sidorov for valuable discussions. This investigation has been supported in part by INTAS grant No.93-1180.

[3]For $A_{(2)}$ in a MOM scheme (see ref. [3]) $\tilde{M}_{(l=2)} \simeq 4.40M$.

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Received by Publishing Department
on October 26, 1995.