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AN IMPULSE APPROXIMATION FOR STRUCTURE FUNCTION F_2^D IN THE BETHE—SALPETER FORMALISM

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Буров В.В., Молочков А.В. Е2-95-339 Импульсное приближение для структурной функции дейтрона F_2^D в формализме Бете—Солнитера

Предложен последовательный коварнантный подход к исследованию ядерных эффектов в глубоко неупругом рассеянии электронов на дейтроне. Подход основан на формализме Бете—Солнитера. В импульсном приближении получена конволюционная формула для структурной функции дейтрона F_2^D . Проанализированы приближения с кинематикой для нуклонной амплитуды на и вне массовой поверхности.

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An Impulse Approximation for Structure Function F_2^D in the Bethe—Salpeter Formalism

A consistent covariant approach to investigate nuclear effects in the deep inelastic scattering of electrons off deuteron in the Bethe—Salpeter formalism is proposed. In the impulse approximation the convolution formula for the deuteron structure function F_2^D is derived. The approximations with the on-mass-shell kinematics and off-mass-shell one for the nucleon amplitude are analyzed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR:

1 Introduction.

Recently the interest to the Bethe-Salpeter formalism (BSF) [1] has grown up as to a covariant way to calculate various processes on light nuclei. First of all it serves for investigation, in framework of quantum field theory, of properties of a nucleus such as its mass, electromagnetic form-factors and other dynamical characteristics [3, 4, 5]. On the other hand these are expressed in terms of existing solutions of the Bethe-Salpeter equation (BSE) in the bound state region. The solutions of the BSE have been found in an effective model both with the separable form the NN-interaction [5] and a realistic meson-exchange potential [4, 6]. So we hope that the BSF points towards investigation of the relativistic nuclear structure in hard processes on light nuclei.

In present talk we propose a relativistic covariant consistent approach based on the BSF. The aim is to investigate relativistic nuclear effects in deep inelastic scattering on deuteron. As long as we have to deal with a nuclear model in which it is impossible to regard subnuclear dynamics we target our calculations at separation of the effects from the subnucleon scale in terms of on-mass-shell amplitudes of nuclear constituents.

2 Basic formalism.

It is well known that the cross section for deep inelastic scattering is proportional to the lepton and hadron tensors in one photon approximation:

$$\sigma \propto L_{\mu\nu}(p,q)W^{\mu\nu}(P,q).$$

All information about target and its nuclear properties is concentrated in the hadron tensor, which is proportional to the sum of matrix elements of transition from γ^*D state to all possible physical states. Via optical theorem we



Figure 1: Diagram schematically representing deep inelastic process.

can obtain $W_{\mu\nu}$ by means of a simpler object — the amplitude for forward Compton scattering on deuteron:

$$W_{\mu\nu}(P,q) = \frac{1}{2\pi} \text{Im} \, T_{\mu\nu}(P,q).$$
(1)

Starting from the definition for $T_{\mu\nu}(P,q)$ as T-product of electromagnetic currents averaged over the deuteron states:

$$T_{\mu\nu}(P,q) = i \int d^4x e^{iqx} \langle D | \mathrm{T} \left(J_{\mu} \left(x \right) J_{\nu} \left(0 \right) \right) | D \rangle, \tag{2}$$

we calculate this matrix element in the BSF. This yields an expression for Compton amplitude in terms of deuteron states and electromagnetic twonucleon vertex [3]:

$$T_{\mu\nu}(P,q) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \overline{\Gamma}(P,k_1) S^{(2)}(P,k_1) \Lambda_{\mu\nu}(q,P,k_1,k_2) S^{(2)}(P,k_2) \Gamma(P,k_2).$$
(3)

$$\int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \overline{\Gamma}(P,k_1) S^{(2)}(P,k_1) \Lambda_{\mu\nu}(q,P,k_1,k_2) S^{(2)}(P,k_2) \Gamma(P,k_2).$$
(3)

$$\int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \overline{\Gamma}(P,k_1) S^{(2)}(P,k_1) \Lambda_{\mu\nu}(q,P,k_1,k_2) S^{(2)}(P,k_2) \Gamma(P,k_2).$$
(3)

Here $S^{(2)}(P,k)$ is the direct product of the exact nucleon propagators:

$$S^{(2)}_{\alpha\beta\delta\gamma}(P,k) = S_{\alpha\delta}(\frac{P}{2}+k)S_{\beta\gamma}(\frac{P}{2}-k),$$

where k is the relative momentum of nucleons inside deuteron. The letters $\alpha, \beta, \delta, \gamma$ denote the spinor indexes.

Two new objects appear in expression (3). It is deuteron vertex function $\Gamma(P,k)$, and Mandelstam vertex $\Lambda_{\mu\nu}(q, P, k_1, k_2)$. The first one describes the deuteron state in terms of virtual nucleon states:

$$\Gamma(P,k) = S^{(2)^{-1}}(P,k) \int d^4x d^4X e^{ikx} e^{iPX} \langle 0 | T(\psi(X-x/2)\psi(X+x/2)) | D \rangle$$

and satisfies homogeneous BSE [1]:

$$\Gamma^{S}(P,k) = -\int \frac{d^{4}k'}{(2\pi)^{4}} \overline{G}_{4}(P,k,k') S^{(2)}(P,k') \Gamma^{S}(P,k').$$
(4)

The kernel of equation, $\overline{G}_4(P,k,k')$, is irreducible by Bethe-Salpeter [1, 2] truncated part of the exact twonucleon Green function. All existing solutions for this equation have been found in one common approximation, namely the ladder form of the kernel, which corresponds to picking only g^2 - terms in its expansion in perturbation theory series. Thus we perform all our calculation in this approximation as well.

The solution of eq. (4) is defined up to a constant. Inasmuch as BS-vertex function doesn't have probabilistic interpretation, we have to normalize the solution on some physical grounds. One way is to use the covariant normalization of the deuteron electromagnetic current at zero momentum transfer:

$$\langle D|J_{\mu}(0)|D\rangle = 2iP_{\mu}.$$
(5)

Hence one can obtain the following normalization condition for BS vertex function in the ladder approximation:

$$\int \frac{d^4k}{(2\pi)^4} \overline{\Gamma}_{\alpha\beta}(P,k) [S(\frac{P}{2}+k)\gamma_{\mu}S(\frac{P}{2}+k)]_{\alpha\gamma} S_{\beta\delta}(\frac{P}{2}-k)\Gamma_{\gamma\delta}(P,k) = -2iP_{\mu}.$$
(6)

This condition along with (4) gives inclusive description of BS-vertex function. Here we do not intend to discuss aspects of solving the BSE, since a good account may be found in [4, 6].

The other new object in (3) is the Mandelstam vertex $\Lambda_{\mu\nu}(q, P, k_1, k_2)$. It is defined by interaction of the virtual photon with system of two nucleons:

$$G_{6\mu\nu}(q,P,k',k) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} G_4(P,k',k_1) \Lambda_{\mu\nu}(q,P,k_1,k_2) G_4(P,k_2,k).$$

Here $G_{6\mu\nu}$ is the exact two nucleon green function with insertion of the electromagnetic operator:

$$G_{6\mu\nu}(q,P,k',k) = i \int d^{4}x d^{4}y d^{4}y' d^{4}Y d^{4}Y' e^{-i\hat{k}y+i\hat{k}'y'} e^{-iqx} e^{-iP(Y-Y')}$$

$$< 0|T(\bar{\psi}(Y+\frac{y}{2})\bar{\psi}(Y-\frac{y'}{2})J_{\mu}(x)J_{\nu}(0)\psi(Y'+\frac{y'}{2})\psi(Y'\cdot\frac{i}{2}\frac{y'}{2}))]0 >$$

$$(8)$$

The solution of equation (7) can be found in terms of expansion in perturbation theory. Doing so in (7) we obtain the following relation for g^n -term:

$$G_{6\mu\nu}^{(n)}(P,q;k,k') = \sum_{n_1+n_2+n_3=n} \int \frac{d^4k_2}{(2\pi)^4} \frac{d^4k_1}{(2\pi)^4} G_4^{(n_1)}(P,k,k_1) \Lambda_{\mu\nu}^{(n_2)}(P,q,k_1,k_2) G_4^{(n_3)}(P,k_2,k').$$
(9)

Choosing only zero order term we immediately obtain the corresponding term for $\Lambda_{\mu\nu}$:

$$\Lambda^{(0)}_{\mu\nu}(P,q,k,k') = S^{-1}(\frac{P}{2}+k)S^{-1}(\frac{P}{2}-k) \left[G^{(0)a}_{6\ \mu\nu}(P,q,k)(2\pi)^{4}\delta^{4}(k-k') + G^{(0)b}_{6\ \mu\nu}(P,q,k)(2\pi)^{4}\delta^{4}(k-k'-q)\right]S^{-1}(\frac{P}{2}-k')S^{-1}(\frac{P}{2}+k').$$
(10)

Two summonds are zero order of Mandelstam vertex corresponding to interaction with single isolated nucleon (a) and with two nucleons simultaneously (b), respectively. The term of order g^2 for $G_{6\mu\nu}$ depends on $\Lambda^{(0)}_{\mu\nu}$ contribution and on g^2 part of $\Lambda_{\mu\nu}$, what gives following expression for the latter:

$$\begin{split} \Lambda^{(2)}_{\mu\nu}(P,q,k,k') &= \\ S^{-1}(\frac{P}{2}+k)S^{-1}(\frac{P}{2}-k)G^{(2)}_{6\ \mu\nu}(P,q,k,k')S^{-1}(\frac{P}{2}-k')S^{-1}(\frac{P}{2}+k') \qquad (11) \\ S^{-1}(\frac{P}{2}+k)S^{-1}(\frac{P}{2}-k)\int \frac{d^4k''}{(2\pi)^4}G^{(2)}_4(P,k,k'')\Lambda^{(0)}_{\mu\nu}(P,q,k'',k')S^{-1}(\frac{P}{2}-k')S^{-1}(\frac{P}{2}+k') \end{split}$$

$$-S^{-1}(\frac{P}{2}-k)S^{-1}(\frac{P}{2}+k)\int \frac{d^4k''}{(2\pi)^4}\Lambda^{(0)}_{\mu\nu}(P,q,k,k'')G^{(2)}_4(P,k'',k')S^{-1}(\frac{P}{2}+k)S^{-1}(\frac{P}{2}-k).$$

When combining (10) and (11) we observe that $\Lambda_{\mu\nu}$ is equal to the truncated irreducible part of $G_{6\mu\nu}$.

Inasmuch as we work in the ladder approximation, it is enough, for consistency, to take only first two terms for $\Lambda_{\mu\nu}$. Substitution of them in (7) gives us Compton amplitude on deuteron in terms of the BS-vertex function and perturbation expansion of the exact twonucleon Green function with insertion:

$$T^{D}_{\mu\nu}(P,q) = -\int \frac{d^{4}k}{(2\pi)^{4}} \overline{\Gamma}(P,k) \ G_{6\mu\nu}^{(0)}(P,k,q)\Gamma(P,k) + \int \frac{d^{4}k}{(2\pi)^{4}} \frac{d^{4}k'}{(2\pi)^{4}} \overline{\Gamma}(P,k) G_{6\mu\nu}^{(2)}(P,k,k',q)\Gamma(P,k').$$

The obtained result allows us to calculate Compton amplitude on deuteron with the help of Feynman diagram technique. Indeed, calculation of all objects in a unique field theoretical model for deuteron allows us to use Feynman diagram technique to define both $G_{6\mu\nu}$ and G_4 . Enlarging this technique with a new element corresponding to BS-vertex function and introducing a rule for elimination of reducible graphs, we obtain Feynman like technique for calculation of amplitudes on deuteron in framework of the BSF. An example of such diagrams in the isoscalar meson nucleon theory is presented on fig.2. Elimination of reducible diagrams stems from fact that Mandelstam vertex is the irreducible part of $G_{6\mu\nu}$. The reducible graph can be defined as a graph which is equal to one of smaller order in g^2 . An example of reducible diagrams is shown on fig.3.



tude on deuteron in approximation up to g^2 .

of the reducible diagrams.

In general case we can highlight two groups of diagrams, and therefore corresponding amplitudes turn to a naive convolution and nonconvolution parts. On fig.2 the naive convolution approximation exhausts itself by a) and c) diagrams. Diagram a) is a representative of the relativistic impulse approximation which is nucleon part of the convolution approximation, diagram c) is mesonic exchange current contribution which is non-nucleon part. The nonconvolution correction consists of pure nucleon and mixed terms. The nucleon terms are presented by b), d), e) diagrams. Here we can suppose that d) and e) correspond to interaction corrections to the impulse approximation in particular, and can be cast into the convolution form in the Bjorken limit as it was shown in nonrelativistic consideration [7]. Role of b) and f) terms and its possible behavior are not so clear and under investigation now. The nonconvolution non-nucleon part comes in only at next order.

3 The deutéron structure function F_2^D in the impulse approximation.

Now we consider the deuteron structure function F_2^D in the impulse approximation. In view of obtained results in the previous section we can write Compton amplitude on deuteron as follows:

$$T^{D}_{\mu\nu}(P,q) = -\frac{i}{3} \sum_{S} \int \frac{d^{4}k}{(2\pi)^{4}} \overline{\Gamma}^{S}_{\alpha\beta}(P,k) \left[\frac{(\frac{P}{2} + k + m)}{(\frac{P}{2} + k)^{2} - m^{2}} \Gamma_{\mu}(q) \frac{(\frac{P}{2} + k + q)}{(\frac{P}{2} + k + q)^{2} - m^{2}} \right]_{\alpha\gamma} \frac{(\frac{P}{2} - k + m)_{\beta\delta}}{(\frac{P}{2} - k)^{2} - m^{2}} \int_{\gamma\delta} (P,k).$$
(12)-

The representation of Feynman propagator for the nucleon through the complete set of solutions for Dirac equation

$$\frac{k+m)_{\alpha\beta}}{k^2-m^2} = \frac{m}{E} \sum_{s} \frac{u_{\alpha}^s(\mathbf{k})\overline{u}_{\beta}^s(\mathbf{k})}{(k_0-E)} - \frac{m}{E} \sum_{s} \frac{v_{\alpha}^s(-\mathbf{k})\overline{v}_{\beta}^s(-\mathbf{k})}{(k_0+E)},$$
$$E = \sqrt{k^2+m^2}$$

gives us the following form of Compton amplitude which connects it with the corresponding elementary amplitudes:

Inasmuch as the highlighted amplitude has azimuthal symmetry, it is proportional to $\delta^{ss'}$. It is also symmetrical under exchange $s \to -s$ in the unpolarized case. Thus we can rewrite (13) in terms of the nucleon amplitude for the unpolarized scattering:

$$\frac{1}{6} \sum_{s,s'} \widetilde{T}_{\mu\nu}^{Ns,s'}(\frac{P}{2}+k,q) f^{s,s';S}(P,k) = T_{\mu\nu}^{N}(\frac{P}{2}+k,q) f(P,k), \qquad (14)$$

$$f(P,k) = \frac{1}{3} \sum_{s,S} f^{ss;S}(P,k).$$

That leads us to an analog of the convolution formula for Compton amplitude and via optical theorem we can obtain the same formula for the hadron tensor:

$$W^{D}_{\mu\nu}(P,q) = \int \frac{d^{4}k}{(2\pi)^{4}} W^{N}_{\mu\nu}(\frac{P}{2}+k,q) f^{N}(P,k) + \int \frac{d^{4}k}{(2\pi)^{4}} W^{\overline{N}}_{\mu\nu}(\frac{P}{2}+k,q) f^{\overline{N}}(P,k),$$
(15)

where the distribution functions are expressed via BS-vertex function:

$$f^{N}(P,k) = -\frac{1}{3} \sum_{S} \overline{\Gamma}^{S}_{\alpha\beta}(P,k) S^{+}_{\alpha\gamma}(\frac{P}{2}+k) S_{\beta\delta}(\frac{P}{2}-k) \Gamma^{S}_{\gamma\delta}(P,k) \frac{1}{2E((\frac{P}{2}+k)_{0}-E)},$$

$$f^{\overline{N}}(P,k) = -\frac{1}{3} \sum_{S} \overline{\Gamma}^{S}_{\alpha\beta}(P,k) S^{-}_{\alpha\gamma}(\frac{P}{2}+k) S_{\beta\delta}(\frac{P}{2}-k) \Gamma^{S}_{\gamma\delta}(P,k) \frac{1}{2E((\frac{P}{2}+k)_{0}+E)}.$$

Obviously, here appears separated contribution of nucleon states with the positive and negative energy. The corresponding elementary amplitude has the form, as it is shown above, similar to the on-mass-shell nucleon and antinucleon amplitudes but with dependence on time component of the nucleons relative momentum.

As the first raw approximation we can make the most common substitution — to insert on-mass-shell amplitude for the nucleon and try to obtain deuteron structure function F_2^D in Bjorken limit. To be exact we'll write our expressions in the rest frame of the deuteron, and with orientation of the virtual photon along z-direction:

$$P_D = (\dot{M}_D, \mathbf{0}),$$

 $q \approx (q_0, 0, 0, q_0).$ (16)

In case of on-mass-shell amplitudes we can have representation of the deuteron and nucleon hadron tensors through scalar structure functions F_1 and F_2 :

$$W_{\mu\nu}(P,q) = \left(-g_{\mu\nu} + \frac{q^2}{P \cdot q}\right)F_1(x) + \frac{1!}{Mq_0}\left(P_{\mu} - \frac{P \cdot q}{q^2}q_{\mu}\right)\left(P_{\nu} - \frac{P \cdot q}{q^2}q_{\nu}\right)F_2(x).$$
 (17)

Projecting the hadron tensor on $g_{\mu\nu}$ we get F_2 :

$$\lim_{Q^2 \to \infty} g_{\mu\nu} W^{\mu\nu}(P,q) = -\frac{1}{x} F_2(x)$$

If we introduce an auxiliary variable $y = \frac{x_D}{x_N}$, then the convolution formula for the deuteron structure function can be obtained as follows:

$$F_{2}^{D}(x_{D}) = \int_{x_{D}}^{1} dy F_{2}^{N}\left(\frac{x_{D}}{y}\right) \int \frac{d^{4}k}{(2\pi)^{4}} \left(f^{N}(M_{D},k) + f^{\overline{N}}(M_{D},k)\right) y \,\delta\left(y - \frac{E - k_{3}}{M_{D}}\right).$$
(18)

This formula has clear physical sense — it is the structure function of the physical nucleon in a certain state multiplied with probability of this state. Thus the distribution function must be normalized as a number of nucleons inside deuteron, i.e. baryon number:

$$\int_{0}^{1} f^{N}(y) dy = 2.$$
 (19)

If we rewrite the normalization condition for BS-vertex function with the help of trivial transformations we can obtain the following condition for f^N in system (16):

$$\int \frac{d^4k'}{(2\pi)^4} (f^N(M_D,k) + f^{\overline{N}}(M_D,k)) \frac{E-k_3}{M_D} = 2.$$

It is clear that it corresponds to baryon sum rule.

Fig.4. displays numerical results for the ratio $\frac{F_2^D}{F_2^N}$ in the effective model with the separable interaction [5]. Curve 1 corresponds to the relativistic impulse approximation. You can see that it doesn't have EMC-like behavior. It means the obtained result accounts for only effects of Fermi motion.

How can we take into account off-mass-shell effects? It was shown in [7] that in the nonrelativistic case the impulse approximation does not include any off-mass-shell effects.



Figure 4: Ratio of the deuteron and isoscalar nucleon structure functions. Curve 1 corresponds to calculation in the impulse approximation with on-mass-shell kinematics for the elementary amplitude, curve 2 corresponds to the off-mass-shell one.

They were ensured by interaction corrections. Thus our result, in the approximation with on-mass-shell amplitude, is in qualitative agreement with the nonrelativistic one. And we can take into account off-mass-shell effects by means of interaction corrections. These corrections are connected with g^2 -order of the nucleon part of nonconvolution terms. However in the relativistic case effects coming from off-mass-shell kinematics also contribute, what appears as dependence of elementary amplitude on time component of the relative momentum of nucleons. If we suppose that we can handle with such an amplitude as on-mass-shell one and neglect off-mass-shell corrections in relation (17) [8], we obtain the convolution formula for the structure function in the form similar to the obtained above, but with definition of y depending on time component of nucleon relative moment. In this case we have EMC-like behavior (curve 2 at fig.4.), with maximum deviation at medium x is about 2%. However changing definition of y-leads to changing of the normalization condition for f^N and then to violation of baryon sum rule. It shows up numerically as deviation of ratio at x = 1 from 1. This deviation is about 1%.

4 Conclusion.

In present talk we have propose the covariant approach to calculate nuclear effects in deep inelastic scattering on deuteron. This approach is based on the Bethe-Salpeter formalism. It is shown that in framework of this approach Compton amplitude on deuteron can be expressed in terms of solutions of Bethe-Salpeter equation and expansion in perturbation theory series of twonucleon Green function with insertion of electromagnetic operators.

The convolution formula for the deuteron structure function F_2^D is derived. It is shown that the impulse approximation for the structure function F_2^D with on-mass-shell kinematics for nucleon amplitude does not describe EMC-like behavior of the ratio $R = \frac{F_2^D}{F_2^N}$. Application of off-mass-shell kinematics gives similar behavior though it leads to violation of the baryon sum rule.

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References

- [1] E.E. Salpeter and H.A. Bethe, Phys. Rev. 76 769 (1949)
- [2] D. Lurie, Particles and fields, Interscience, New York, 1968
- [3] S. Mandelstam, Proc. R. Soc. London A 233, 248 (1955)
- [4] Zuilhof M.J. and Tjon J.A., Phys. Rev., C22, (1980) p.2369-2382.
- [5] Rupp G. and Tjon J.A., Phys. Rev., C41, (1990) p.471-483.
- [6] Umnikov A Yu. and Khanna F.C., Phys. Rev., C49, (1994), p.2311-2329.
 A.Yu. Umnikov, L.P. Kaptari, K.Yu. Kazakov and F.C. Khanna, Phys. Lett. B334 (1994) p.163-168.
 - [7] L.P. Kaptari, K.Yu. Kazakov, A.Yu. Umnikov, Phys. Lett. B293 (1992) p.219-225.
 - [8] W. Melnitchouk, A.W. Schreiber and A.W. Thomas, Phys. Rev. D49 (1994) 1199. A.Yu. Umnikov, L.P. Kaptari, and F. Khanna, Preprint Alberta-Thy-38-94, Univ. Alberta, 1994.

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