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INVESTIGATION OF NUCLEON ELECTROMAGNETIC FORM FACTORS IN THE UNPHYSICAL REGION BY MEANS OF THE $\bar{N}N \to \pi l^+ l^-$ REACTIONS

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Исследование нуклонных электромагнитных формфакторов в нефизической области при помощи процессов $\bar{N}N \to \pi l^+ l^-$

Проводится теоретическое исследование процессов аннигиляции нуклон-антинуклонной пары в пион и лептонную пару. В данной работе выведена общая структура дифференциальной вероятности аннигиляции медленных антинуклонов на нуклонах, т.е. процесса $\bar{N}N \to \pi l^+ l^-$, вычислена структура электромагнитного тока процесса $\bar{p}p \to \pi^0\gamma^*$ в S-состоянии и показаны общие свойства соответствующих формфакторов.Далее, используя приближение древесных диаграмм, эти формфакторы вычислены явно. Для процесса $\bar{p}p \to \pi^0 l^+ l^-$ показано, что он полностью описывается магнитным формфактором протона в нефизической области. И последнее, показан спектр эффективных масс и интегральные коэффициенты внутренней конверсии процессов $\bar{p}p \to \pi^0 l^+ l^-$ и $\bar{p}n \to \pi^- l^+ l^-$.

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Investigation of Nucleon Electromagnetic Form Factors in the Unphysical Region by Means of the $\bar{N}N \to \pi l^+ l^-$ Reactions

A theoretical investigation $\bar{N}N \to \pi l^+ l^-$ of processes is carried out. First, the general structure of the differential probability of annihilation of very slow antinucleons on nucleons at rest into pion and lepton pairs is derived, then the structure of the electromagnetic current of $\bar{N}N \to \pi\gamma^*$ transition in the case of S-state annihilation is restored and general properties of the corresponding form factors are demonstrated. Next, by using the three-diagram approximation of the amplitude, those form factors are calculated explicitly and for the special process $(\bar{p}p) \to \pi^0\gamma^* \to \pi^0 l^+ l^-$ they are shown to be completely described by the magnetic form factor of the proton in the unphysical region. Finally, the effective mass spectra of lepton pairs and the integral coefficients of internal conversion for the $\bar{p}p \to \pi^0 l^+ l^-$ and $\bar{p}n \to \pi^- l^+ l^-$ processes are predicted.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

This paper is devoted to a theoretical investigation of $\bar{N}N \to \pi l^+ l^-$ processes, where N is a nucleon and $l^\pm$ means a lepton. As compared with the reactions $e^+e^- \to N\bar{N}$ to be a source of the information on nucleon electromagnetic (EM) form factors (FF's) in the time-like region, the reactions $\bar{N}N \to \pi l^+ l^-$ are next in order from the point of view of relevance. Since the latter are crossing reactions, on the one hand, to electroproduction [1] $e^- N \to e^- N\pi$ processes (transfer momenta are in the space-like region) and, on the other hand, to an inverse electroproduction [2] $\pi^- p \to n l^+ l^-$ process (transfer momenta are in the time-like region), by means of the $\bar{N}N \to \pi l^+ l^-$ process one can solve experimentally similar problems of electrodynamics of hadrons as in the case of $e^- N \to e^- N\pi$ and $\pi N \to N l^+ l^-$ processes.

First of all it is an investigation of the pion and nucleon electromagnetic structure in those time-like momentum transfer squared values that cannot be reached by the $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \iff \bar{N}N$ processes. If in the $e^+e^- \to \bar{N}N$ processes the nucleon EM FF's can be measured only for $k^2 \geq 4M^2$ (M is the nucleon mass and k is the four-momentum of the virtual photon), the processes $\bar{N}N \to \pi l^+ l^-$ give in principle information on the pion and nucleon EM structure starting from $k^2 \geq 4m_l^2$ (m_l is the lepton mass). The unphysical region $0 \leq k^2 \leq 4M^2$ for $e^+e^- \iff \bar{N}N$ processes is of essential interest due to the fact that just here the nucleon EM FF's are complicated in structure determined [3] by various vector mesons. As a result, the behaviour of FF's in the time-like region above the $N\bar{N}$ threshold and in the space-like region is shaped by this region.

The processes $\bar{N}N \to \pi l^+ l^-$ are tempting also from another point of view. Due to the three-body final state in the latter reaction even in annihilation of slow antinucleons with nucleons at rest one can vary values of the momentum transfer squared by varying energy of the detected pion. Therefore these processes are favorably differing from the binary reactions $e^+e^- \to \bar{N}N$, where values of the momentum transfer squared are unambiguously fixed by the energy of colliding leptons.

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FF's of different hadrons, and as a result, determine relative phases of those FF's.

We note that in the $\bar{N}N \to \pi l^+ l^-$ process for $k^2 \leq (2M - m_\pi)^2$ there are no FF's of free nucleons investigated. Really, here a vertex $NN^*\gamma^*$ with one virtual nucleon ($p^2 \neq M$) appears, and therefore, the process $\bar{N}N^* \to l^+ l^-$ is examined. It is clear that with increasing distance of k^2 from the threshold $k^2 = 4M^2$ of the reaction $\bar{N}N \to l^+ l^-$, also the virtuality of a nucleon increases. More precisely, to every value of k^2 from the interval $0 \leq k^2 \leq (2M - m_\pi)^2$ (the lepton mass is neglected) a value of p^2 corresponds from the interval $-(M^2 + m_\pi^2) \leq p^2 \leq (M - m_\pi)^2$ taking thus values of both the space-like and time-like region. A similar problem of off-mass-shell effects for nucleon EM FF's arises not only in the reaction $\bar{N}N \to \pi l^+ l^-$ but also in other processes like $e^- N \to e^- N\pi$ [4], the electrodisintegration on a deuteron $e^- D \to e^- np$ [5] and others [6]. In this paper we shall not discuss off-mass-shell effects for EM FF's of hadrons.

When studying lepton-hadron interaction processes, one has always to distinguish between the analysis of kinematical aspects and dynamical ones [7].

A generalized relativistic kinematics first contains an analysis of all consequences of one-photon exchange mechanism, then the EM current conservation of hadrons and also the results of application of various symmetries to EM interactions of hadrons. In this way, one can specify the spin structure of the EM current of hadrons, parametrizing the latter in terms of a number of amplitudes, and finally, one can express structure functions by means of those amplitudes and find out the differential cross-section dependence (and various polarization effects) on the kinematical variables that are not dynamic in nature.

A similar analysis has to precede the consideration of dynamical aspects of the problem.

In the first place, however, we clarify the general structure of the differential probability of annihilation of slow antiprotons on protons at rest into pion and lepton pairs. Afterwards, by using a simple pole model, we calculate the threshold amplitude of the process $\bar{p}p \to \pi^0\gamma^*$. Then the k^2-dependence of the process $\bar{p}p \to \pi^0 l^+ l^-$ in the afore-mentioned model is completely determined by the proton magnetic FF in the unphysical region. This k^2-dependence is predicted by using two unitary and analytic VMD models of $G_M^p(k^2)$ differing from each other by the sequence of incorporation of

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the correct FF analytic properties and the method of incorporation of the asymptotic behaviour as predicted by QCD for baryons.

2 Structure of differential probability

Usually, at a capture of slow particles by other particles at rest one can consider, instead of the cross-section, just the decay probability of the latter system.

We start with a standard expression for the probability of annihilation of very slow antinucleons on nucleons at rest into pion and lepton pairs (see Fig.1)

$$d\Gamma = (2\pi)^4 \int \frac{\overline{|\mathcal{M}|^2}}{4M} \delta(P - q - k_1 - k_2) \frac{d^3q}{(2\pi)^3 2E_\pi} \frac{d^3k_1}{(2\pi)^3 2E_+} \frac{d^3k_2}{(2\pi)^3 2E_-}. \qquad (1)$$

where P is the total momentum of an $\bar{N}N$ pair, the components of which also for very slow antinucleons are, roughly speaking, determined only by the mass as follows $P = (2M, 0)$. The quantities E_π, E_- (E_+) are energies of the pion, electron (positron) and $\vec{q}$, $\vec{k_1}$, ($\vec{k_2}$) are their three-momenta.

The matrix element $\mathcal{M}$ of the ($\bar{N}N \to \pi l^+l^-$) decay in the one-photon exchange approximation takes the form

$$\mathcal{M} = \frac{e^2}{k^2} \ell_\mu J_\mu. \qquad (2)$$

where $\ell = \bar{u}(k_1)\gamma u(k_2)$ and J_μ is the EM current of an $\bar{N}N \to \pi\gamma^*$ transition (γ^* is a virtual photon).

By means of (1) and (2) one gets

$$d\Gamma = \frac{\alpha^2}{16\pi^2} \int \frac{L_{\mu\nu} W_{\mu\nu}}{(k^2)^2 2M} dE_+ dE_- d\Omega. \qquad (3)$$

$$L_{\mu\nu} = \overline{\ell_\mu \ell_\nu^*}, \quad W_{\mu\nu} = \overline{J_\mu J_\nu^*}.$$

where the bar in the definition of $L_{\mu\nu}$ means summation over polarization states of leptons and the bar in the definition of $W_{\mu\nu}$ means averaging over polarization states of a nucleon-antinucleon pair. The quantity $d\Omega$ means the space-angle element of an outgoing electron (or positron), to be defined according to some physical reference frame. The latter can be specified if one of the initial nucleons is polarized (with a vector polarization $\vec{P}$) and if the 3-momentum of the e^+e^- pair (or the pion) is known, the latter forming, with $\vec{P}$, a plane.

If the initial particles $\bar{N}$ and N are unpolarized, then a contraction $L_{\mu\nu}W_{\mu\nu}$ does not depend on angles of $d\Omega$ and therefore in (3) one can integrate over $d\Omega$ explicitly, obtaining finally 4π.

As a consequence of the conservation of the J_μ, $k\cdot J=0$, the product $L_{\mu\nu}W_{\mu\nu}$ can be rewritten into the following form

$$L_{\mu\nu}W_{\mu\nu}=(L_{xx}+L_{yy})W_{xx}+\frac{(k^2)^2}{k_0^4}L_{zz}W_{zz}$$

by using the coordinate system, in which z-axis is parallel to the 3-momentum $\vec{k}$ of the virtual photon (k_0 is the energy of γ^*).

Now, taking into account the P-invariance of EM interactions of hadrons, the tensor W_{ij} (the space-part of $W_{\mu\nu}$) can be written in the following general form

$$W_{ij}=(\delta_{ij}-\hat{k}_i\hat{k}_j)W_1(k^2)+\hat{k}_i\hat{k}_jW_2(k^2),\quad \hat{\vec{k}}=\vec{k}/|\vec{k}| \tag{4}$$

where $W_{1,2}(k^2)$ are real structure functions (SF). SF $W_1(k^2)$ describes the creation of γ^* with the transversal polarization and the creation of γ^* with the longitudinal polarization is characterized by $W_2(k^2)$.

Production of unpolarized leptons is determined by the tensor

$$L_{\mu\nu}=4k_{1\mu}k_{2\nu}+4k_{1\nu}k_{2\mu}-2g_{\mu\nu}k^2,$$

from where

$$\begin{aligned}L_{xx}+L_{yy}&=2\frac{k^2}{\vec{k}^2}[\vec{k}^2+(E_+-E_-)^2]+8m^2,\\ L_{zz}&=2\frac{k_0^2}{\vec{k}^2}[\vec{k}^2-(E_+-E_-)^2],\end{aligned} \tag{5}$$

and m means the lepton mass.

In terms of (4) and (5), the differential probability takes the form

$$d\Gamma=\frac{\alpha^2}{32\pi}W_1(k^2)\frac{dxdy}{xM}[1+\frac{4m^2}{k^2}+R_L\frac{k^2}{k_0^2}+y^2\frac{M^2}{\vec{k}^2}(1-R_L\frac{k^2}{k_0^2})], \tag{6}$$

$$x=k^2/k^2_{max},\quad y=\frac{E_--E_+}{M},\quad 0\le x\le 1,\quad k^2_{max}=(2M-m_\pi)^2,$$

$$R_L(k^2)=W_2(k^2)/W_1(k^2),$$

$$\vec{k}^2=k_0^2-k^2=(\frac{4M^2+k^2-m_\pi^2}{4M})^2-k^2=\frac{k^2_{max}-k^2}{16M^2}[(2M+m_\pi)^2-k^2].$$

As a result, the differential probability $d^2\Gamma/dxdy$ that characterizes the Dalitz distribution for the nucleon-antinucleon annihilation at rest to a lepton pair and the pion is symmetric with respect to the change $E_+ \iff E_-$. This has to hold always in C-invariant theories.

Moreover, if the dependence of the probability $d^2\Gamma/dxdy$ on variable x is dynamical in origin, i.e. it is determined by the k^2- dependence of SF's $W_{1,2}(k^2)$, then the dependence of $d^2\Gamma/dxdy$ on variable y is pure kinematical. Therefore, by investigating the y-dependence of the differential probability of $\bar{N}N\to\pi l^+l^-$ process, one can determine the quantity R_L, the ratio of probabilities for production of longitudinal and transversal virtual photons in the $\bar{N}N\to\pi\gamma^*$ process. The fact that the probability $d^2\Gamma/dxdy$ at a fixed value of k^2 is quadratic in y,

$$d^2\Gamma/dxdy=a(k^2)+y^2b(k^2),$$

is a consequence of the one-photon exchange mechanism applied to the reaction under consideration.

Consequently, the manifestation of that y^2- dependence has the same physical reason like the $\cot^2\frac{\vartheta_e}{2}$ dependence of the differential cross-section of $e^-N\to e^-N$ process (ϑ_e is the electron scattering angle in laboratory system) or the $\cos^2\vartheta$ dependence of the differential cross-section of $e^+e^-\to h+X$ process (h is a detected hadron, X is a nonregistered bunch of created particles and ϑ is a detected hadron angle related to the three-momentum of the electron in the c.m. system of e^+e^- collisions), and that physical reason is the one-photon exchange mechanism of all afore-mentioned processes.

For a real photon $k^2=0$, therefore the quantity $W_1(0)$ determines the probability of the real-photon creation in $\bar{N}N\to\pi\gamma$ process as follows

$$\Gamma_\gamma=(2\pi)^4e^2\frac{W_{xx}+W_{yy}}{4M}\int\delta(P-q-k)\frac{d^3k}{(2\pi)^32\omega}\frac{d^3q}{(2\pi)^32E_\pi}=\alpha W_1(0)\frac{4M^2-m_\pi^2}{8M^3}.$$

Using this expression for Γ_γ one can obtain the Dalitz distribution for $\bar{N}N\to\pi l^+l^-$ process [8]

$$\frac{d\Gamma_{l^+l^-}}{\Gamma_\gamma}=\frac{\alpha}{4\pi}R_T(k^2)\frac{dxdy}{x\sqrt{k^2_{max}}}\frac{M^2}{2M+m_\pi}[1+\frac{4m^2}{k^2}+R_L\frac{k^2}{k_0^2}+y^2\frac{M^2}{\vec{k}^2}(1-R_L\frac{k^2}{k_0^2})], \tag{7}$$

where

$$R_T(k^2)=W_1(k^2)/W_1(0).$$

So, by investigating the energy distribution of leptons in the $\bar{N}N \to \pi l^+ l^-$ process on the Dalitz plane one can determine two very important quantities of the electrodynamics of hadrons, $R_T(k^2)$ and $R_L(k^2)$, which at the same time determine the energy spectrum of pions in $\bar{N}N \to \pi l^+ l^-$, i.e. the result of integration over the lepton pair

$$d\Gamma_{l^+l^-} = \frac{\alpha^2}{8\pi^3} l_{\mu\nu} W_{\mu\nu} \frac{|\vec{k}| dE_\pi}{(k^2)^2 2M} d\Omega \tag{8}$$

$$l_{\mu\nu} = \int L_{\mu\nu} \frac{d^3k_1}{2E_-} \frac{d^3k_2}{2E_+} \delta(k - k_1 - k_2) = (-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}) \frac{2\pi}{3}(k^2 + 2m^2)\sqrt{1 - \frac{4m^2}{k^2}}. \tag{9}$$

As a result, the k^2 - distribution of $\bar{N}N \to \pi l^+ l^-$ process is obtained in the form

$$\frac{d\Gamma_{l^+l^-}}{dx} / \Gamma_\gamma = \frac{\alpha}{3\pi} R_T(k^2)[I_0(k^2) + R_L(k^2) I_1(k^2)], \tag{10}$$

where

$$I_0 = \frac{1}{x}(1 + 2\frac{m^2}{k^2})\sqrt{1 - \frac{4m^2}{k^2}} \cdot \frac{|\vec{k}|}{\omega},$$

$$I_1 = \frac{1}{2x}(1 + 2\frac{m^2}{k^2})\frac{k^2}{k_0^2}\sqrt{1 - \frac{4m^2}{k^2}} \cdot \frac{|\vec{k}|}{\omega}$$

and

$$\omega = M(1 - \frac{m_\pi^2}{4M^2}).$$

To find the coefficient of internal conversion for $\bar{N}N \to \pi l^+ l^-$, which is equal to the ratio of the total probabilities of $\bar{N}N \to \pi l^+ l^-$ and $\bar{N}N \to \pi\gamma$ processes, one has to know the k^2 dependence of $R_T(k^2)$ and $R_L(k^2)$ quantities. In the next sections we calculate just $R_T(k^2)$ and $R_L(k^2)$ by using a tree-diagram approximation of the corresponding amplitude.

3 Electromagnetic current of the $\bar{N}N \to \pi\gamma^*$ transition

The structure of the EM current $\vec{J}$ of the $\bar{N}N \to \pi\gamma^*$ transition in the case of S - state annihilation can be restored generally, starting only with the P - invariance of EM interactions of hadrons. By using a two-component spinor formalism for describing $\bar{N}$ and N, one can write for $\vec{J}$ the expression

$$\vec{J} = \varphi_1^+ [i\vec{\sigma} \times \hat{\vec{k}} f_t(k^2) + \hat{\vec{k}} f_l(k^2)]\varphi_2. \tag{11}$$

The amplitude $f_t(k^2)$ is a form factor of the virtual photon production of the dipole magnetic type in $\bar{N}N \to \pi\gamma^*$ transition, the amplitude $f_l(k^2)$ is a form factor of the production of the longitudinally polarized virtual photon of E0 -type.

Both FF's, $f_{t,l}$, are complex functions of k^2 (in the time-like region) for $k^2 > k_{th}^2$, where k_{th}^2 can be found from the unitarity condition, taking into account symmetry properties of the EM current of hadrons.

The quantities $f_t(f_l)$ describe annihilation of the $\bar{N}N$ system with the angular momentum $l = 0$ and the total spin $S = 1(0)$. The system $\bar{p}p$ ($\bar{n}n$) possesses a definite C - parity defined by the relation

$$C = (-1)^{l+S}.$$

Therefore, as a consequence of the C -parity conservation in processes $\bar{p}p \to \pi^0\gamma^*$ and $\bar{n}n \to \pi^0\gamma^*$, one obtains

$$f_l(k^2) = 0. \tag{12}$$

The latter relation has to be fulfilled for any mechanism of those reactions in the whole kinematical region of k^2.

An experimental confirmation of (12) could lead to:

- verification of C - invariance of the electromagnetic interactions of hadrons
- verification of the hypothesis that the annihilation of antinucleons at rest is realized in the S - state.

Systems ($\bar{p}n$) and ($\bar{n}p$) do not possess a definite C - parity, but they have a definite G - parity determined by the relation

$$G = (-1)^{l+S+I},$$

where I is the isotopic spin of the $\bar{N}N$ system. Taking $l = 0$, $I = 1$ for $\bar{p}n$ and $\bar{n}p$ we obtain that G-parity of the singlet state is negative, and G - parity of the triplet state is positive. Accepting that the G - parity of the pion with an arbitrary charge is negative, we find that the FF $f_l(k^2)$ corresponds to the creation of an isovector virtual photon and FF $f_t(k^2)$ corresponds to the creation of an isoscalar virtual photon. Therefore, $Im f_l(k^2) \neq 0$ for $k^2 \geq 4m_\pi^2$ and $Im f_t(k^2) \neq 0$ for $k^2 \geq 9m_\pi^2$.

Consequently, $\bar{p}n \to \pi^-\gamma^*$ and $\bar{n}p \to \pi^+\gamma^*$ are described by two nonzero FF's with definite isotopic properties.

These predictions of symmetries for processes of the creation of charged and neutral pions in $\bar{N}N \to \pi l^+ l^-$ are very peculiar for polarization effects.

4 Calculation of $f_t(k^2)$ and $f_l(k^2)$ form factors

To demonstrate the sensitivity of the differential probability of $\bar{N}N \to \pi l^+ l^-$ process to the EM structure of hadrons, we calculate the form factors $f_{t,l}(k^2)$, using the simple tree-diagram-approximation model shown in Fig.2.

To "justify" the latter model, we would like to note that the poles of amplitudes, corresponding to the exchange by the nucleon, are very close to the physical region

$$s - M^2 = u - M^2 = m_\pi^2 - 2ME_\pi \le m_\pi^2 - 2Mm_\pi,$$

where $s = (k - p_1)^2$, $u = (k - p_2)^2$. So, a minimal shift of the pole from the physical region is determined by the pion mass, namely, the mass of one of the lightest hadrons.

One has to stress that similar pole models (Born approximation) are used more or less successfully in the crossing channels to the process under consideration, i.e. in $e^-p \to e^-n\pi^+$ and $\pi^-p \to ne^+e^-$ processes. For these reactions one can show two kinematical regions of invariant variables, in which the Born approximation is commonly used.

One region corresponds to the threshold of $\gamma^* N \to N\pi$ reaction, where the effective mass of the $(N\pi)$ system is very near to the sum of pion and nucleon masses and at the same time the invariant variable k^2 can take arbitrary values of the space-like region momentum transfer. For the inverse $\pi^-p \to n\gamma^*$ process, the latter region corresponds to the capture of slow pions on protons, where values of k^2 (in the time-like region) are restricted by the condition $k^2 \le m_\pi^2 + 2Mm_\pi$.

The second kinematical region of $e^-N \to e^-N\pi$ and $\pi^-p \to ne^+e^-$ processes corresponds to large values of the invariant mass of the πN system and at the same time to small values of the invariant variable $t = (k - q)^2$. Just a small value of t is responsible for a close position of the pion pole to the physical-region border, and as a

consequence, the contribution of the corresponding diagram increasses.

Measurement of the cross-section of $e^-p \to e^-n\pi^+$ process in both these regions was used for determining of the nucleon and pion EM FF's. Especially, the high energy region was used [1] for determining the charged pion FF in the region of the space-like momenta transfer (up to very high values of $|k^2|$).

The threshold region of $e^-p \to e^-n\pi$ and $\pi^-p \to ne^+e^-$ reactions has aroused a special interest in the verification of threshold theorem predictions (of the Kroll-Ruderman [9] type), and also in the verification of model predictions based on current algebra [10]. This interest has grown especially after obtaining very exciting results [11] on the cross-section of the photoproduction of π^0 - meson on protons, $\gamma p \to p\pi^0$, in the threshold region, where experimental cross-sections appeared to be substantially smaller than theoretical predictions. In principle, this problem can be connected with the "spin-crisis" problem [12] in clarifying the asymmetry in scattering of longitudinally polarized muons on polarized protons.

When returning to the $\bar{p}n \to \pi\gamma^*$ process, the EM current J_μ, corresponding to the diagrams in Fig.2, takes the form

$$\begin{aligned} J_\mu \; = \; & \sqrt{2}g_\pi \bar{u}(-p_1)[\gamma_5 \frac{\hat{p}_2 - \hat{k} + M}{(p_2 - k)^2 - M^2}(F_{1n}\gamma_\mu + \frac{F_{2n}}{2M}\sigma_{\mu\nu}k_\nu) + \\ & + \; (F_{1\bar{p}}\gamma_\mu + \frac{F_{2\bar{p}}}{2M}\sigma_{\mu\nu}k_\nu)\frac{\hat{p}_1 - \hat{k} - M}{(p_1 - k)^2 - M^2}\gamma_5 + F_\pi\gamma_5\frac{(2q + k)}{(k + q)^2 - m_\pi^2}]u(p_2), \end{aligned} \tag{13}$$

where g_π is the πNN coupling constant (for a pseudoscalar variant), F_π is the charged pion EM FF and $F_{1,2n}$ $(F_{1,2\bar{p}})$ are the neutron (antiproton) EM FF's. As a consequence of C - invariance, the relation

$$F_{1,2\bar{p}} = -F_{1,2p}$$

is valid.

For arbitrary FF's F_{1p}, F_{1n} and F_π the current (13) is not conserved

$$k \cdot J = \sqrt{2}g_\pi \bar{u}(-p_1)\gamma_5 u(p_2)(F_\pi - F_{1n} + F_{1p}) \neq 0.$$

The conservation of the EM current is ensured only when the following relation

$$F_\pi = F_{1n} - F_{1p}$$

holds valid among FF's, which, however, is not realistic at any value of k^2.

One can achieve the conservation of the current by means of a procedure which is commonly used in the investigation of inelastic scattering of electrons on nucleons and nuclei. We mean the transformation

$$J_\mu \to J'_\mu = J_\mu - \frac{k_\mu}{k^2} k \cdot J,$$

as a result of which one obtains the conservation of the current J'_μ. However, in the latter transformation, one requires the divergence of the current J_μ. An attractive property of the Born approximation for $\bar{N}N \to \pi\gamma$ process is the possibility to calculate just this current divergence $k \cdot J$.

As a consequence, for the threshold amplitudes $f_{t,l}(k^2)$ of $\bar{N}N \to \pi^\pm\gamma^*$ process one can obtain the expressions

$$f_t^{(\pi^\pm)}(k^2) = -4\sqrt{2} g_\pi G_M^{(s)}(k^2) \frac{|\vec{k}|M}{s - M^2}, \tag{14}$$

$$f_l(k^2) = 4\sqrt{2} g_\pi \frac{|\vec{k}|M}{s - M^2} \left(F_1^{(v)} + \frac{k_0}{2M} F_2^{(v)}\right) - 2\sqrt{2} g_\pi F_\pi(k^2) \frac{|\vec{k}|M}{t - m_\pi^2} - 2\sqrt{2} g_\pi M \frac{|\vec{k}|}{k^2}\left(F_\pi + 2F_1^{(v)}\right), \tag{15}$$

where

$$G_M^{(s)} = \frac{1}{2}(G_M^p + G_M^n); \quad F_{1,2}^{(v)} = \frac{1}{2}(F_{1,2}^p - F_{1,2}^n) \text{ and } G_M^N = F_1^N + F_2^N$$

is the magnetic FF of the nucleon.

As one could expect from considerations in the previous section, $f_t(k^2)$ is determined only by the isoscalar part of the magnetic FF of the nucleon (M1 transition) and $f_l(k^2)$ is determined only by the isovector parts of the Dirac and Pauli FF's and also by the pion FF F_π that is of the isovector type too.

We would like to stress the characteristic threshold behaviour of both FF's for $|\vec{k}| \to 0$

$$f_{t,l}(k^2) \approx |\vec{k}| = \sqrt{k_{max}^2 - k^2}.$$

In case of the production of neutral pions in the $\bar{p}p \to \pi\gamma^*$ process, as it was clarified earlier, there is only one nonzero FF $f_t(k^2)$ that in the framework of the Born approximation is determined by the following relation

$$f_t^{(\pi^0)}(k^2) = -4 g_\pi G_M^{(p)}(k^2) \frac{|\vec{k}|M}{s - M^2}. \tag{16}$$

5 Numerical evaluations

To find the k^2 - dependence of the differential probability of the $\bar{N}N \to \pi l^+ l^-$ process, one has to derive relations between the structure functions $W_1(k^2)$, $W_2(k^2)$ defined by the expression (4) and FF's $f_t(k^2)$, $f_l(k^2)$ defined by the relation (11).

Starting with the general structure (4) of the space-components W_{ij} of the hadronic tensor and expressing them through a product of the EM current $\vec{J}$ parametrization (11) for the $\bar{N}N \to \pi\gamma^*$ process and its complex conjugate form, after averaging over polarization states of $\bar{N}$ and N, one gets the following relations

$$W_1(k^2) = \frac{1}{2}|f_t(k^2)|^2, \quad W_2(k^2) = \frac{1}{2}|f_l(k^2)|^2. \tag{17}$$

We note that these formulae are of a general character and they have to be valid for any mechanism of $\bar{N}N \to \pi\gamma^*$ processes.

In the framework of the Born approximation for the $\bar{p}p \to \pi^0\gamma^*$ process, the effective mass spectrum of produced lepton pairs in the $\bar{p}p \to \pi^0 l^+ l^-$ process will be determined by the following expression

$$\frac{d\Gamma(\bar{p}p \to \pi^0 l^+ l^-)/dx}{d\Gamma(\bar{p}p \to \gamma\pi^0)} \equiv C^{(\pi^0)}(k^2) = \tag{18}$$

$$= \frac{\alpha}{3\pi} \left|\frac{G_M^p(k^2)}{G_M^p(0)}\right|^2 \left(1 - \frac{k^2}{4M^2 - m_\pi^2}\right)^{-2} \frac{(2M - m_\pi)^2}{k^2} \sqrt{1 - \frac{4m^2}{k^2}} \left(1 + \frac{2m^2}{k^2}\right) \cdot \left[\left(1 - \frac{k^2}{(2M - m_\pi)^2}\right)\left(1 - \frac{k^2}{(2M + m_\pi)^2}\right)\right]^{3/2}.$$

As it has to be in the general case, the expression (18) turns out to be zero on the borders of physical region, i.e. at $x = 1$ (the maximal value of k^2 at $k^2 = (2M - m_\pi)^2$) and at

$x = x_0 = \frac{4m^2}{(2M - m_\pi)^2}$ (the minimal value of k^2 at $k^2 = 4m^2$).

Therefore, the integral coefficient of the internal conversion $c_{tot} = \Gamma(\bar{p}p \to \pi^0 l^+ l^-)/\Gamma(\bar{p}p \to \pi^0\gamma)$ is determined by the formula

$$c_{tot}(\bar{p}p \to \pi^0 l^+ l^-) =$$
$$= \frac{\alpha}{3\pi} \int_{x_0}^1 dx \left(1 - \frac{k^2}{4M^2 - m_\pi^2}\right)^{-2} \frac{(2M - m_\pi)^2}{k^2} \sqrt{1 - \frac{4m^2}{k^2}} \cdot \tag{19}$$
$$\cdot \left(1 + \frac{2m^2}{k^2}\right) \left[\left(1 - \frac{k^2}{(2M - m_\pi)^2}\right)\left(1 - \frac{k^2}{(2M + m_\pi)^2}\right)\right]^{3/2} \left|\frac{G_M^p(k^2)}{G_M^p(0)}\right|^2.$$

As a consequence of the characteristic Coulomb singularity $\frac{1}{x}$, the integral coefficient c_{tot} will take its value mainly from the region $x \approx x_0$, from where one gets the well-known estimation [8]

$$c_{tot}(l^+ l^-) \approx \frac{\alpha}{3\pi} \ln(2M/m) \tag{20}$$

that does not depend on the EM structure of nucleons.

It follows from (20) that the coefficient of the internal conversion for the $\bar{p}p \to \pi^0 \mu^+ \mu^-$ process with production of a muon pairs will be smaller than the coefficient of the internal conversion for the $\bar{p}p \to \pi^0 e^+ e^-$ process with production of electron-positron pairs and

$$c_{tot}(\mu^+ \mu^-)/c_{tot}(e^+ e^-) \approx (\ln m_\mu/m_e)^{-1}. \tag{21}$$

An accurate integration in (19) confirms the previous rough estimations, as it will be shown further.

The effective mass spectrum of $l^+ l^-$ produced in the $\bar{p}p \to \pi^0 l^+ l^-$ process is calculated by means of (18) and by using two unitary and analytic VMD models [13,14] of nucleon EM FF's which differ from each other by the sequence of incorporation of the correct FF analytic properties and the method of incorporation of the asymptotic behaviour as predicted by QCD (up to logarithmic corrections) for baryons. The results are presented in Fig.3 and Fig.4, respectively.

The corresponding values of the integral coefficients of internal conversion are

$$c_{tot}(\bar{p}p \to \pi^0 \mu^+ \mu^-) = \begin{cases} OM: & 0.0723 \\ NM: & 0.0627 \end{cases}$$

and

$$c_{tot}(\bar{p}p \to \pi^0 e^+ e^-) = \begin{cases} OM: & 0.0813 \\ NM: & 0.0717 \end{cases}$$

where (OM) means the old model [13] and (NM) means the new model [14] of the nucleon EM FF's.

The effective mass spectrum of lepton pairs produced in the $\bar{p}n \to \pi^- l^+ l^-$ process contains, besides the information on the isoscalar magnetic FF of nucleons, the information on the isovector Dirac and Pauli nucleon FF's and the pion FF as well, as it follows from the relation

$$\frac{d\Gamma(\bar{p}n \to \pi^- l^+ l^-)/dx}{\Gamma(\bar{p}n \to \pi^- \gamma)} \equiv C^{(\pi^-)}(k^2) = \tag{22}$$
$$= \frac{\alpha}{3\pi} \left|\frac{G_M^{(s)}(k^2)}{G_M^{(s)}(0)}\right|^2 \left(1 - \frac{k^2}{4M^2 - m_\pi^2}\right)^{-2} \frac{(2M - m_\pi)^2}{k^2} \sqrt{1 - \frac{4m^2}{k^2}} \cdot$$
$$\cdot \left(1 + \frac{2m^2}{k^2}\right) \left[1 + \frac{k^2}{2k_0^2} R_L\right] \left[\left(1 - \frac{k^2}{(2M - m_\pi)^2}\right)\left(1 - \frac{k^2}{(2M + m_\pi)^2}\right)\right]^{3/2}$$

where

$$R_L(k^2) = \left|F_1^{(v)} + \frac{k_0}{2M} F_2^{(v)} - 2F_\pi \frac{s - M^2}{t - m_\pi^2} - 2\frac{s - M^2}{k^2}(F_\pi + 2F_1^{(v)})\right|^2 / |G_M^{(s)}(k^2)|^2.$$

and

$$k_0 = \frac{4M^2 + k^2 - m_\pi^2}{4M}.$$

Since $F_\pi(0) = -1$ (there is a negatively charged pion), $2F_1^{(v)}(0) = 1$, then the $\frac{1}{k^2}$ singularity in the ratio R_L will be cancelled. The behaviour of $R_L(k^2)$ (the $F_{\pi^-}(k^2)$ is taken from [15]) is presented in Fig.5. The effective mass-spectrum $C^{(\pi^-)}(k^2)$ is graphically presented in Fig.6 and Fig.7. The corresponding integral coefficients of internal conversion are

$$c_{tot}(\bar{p}n \to \pi^- \mu^+ \mu^-) = \begin{cases} OM: & 1.3625 \\ NM: & 0.8032 \end{cases}$$

and

$$c_{tot}(\bar{p}n \to \pi^- e^+ e^-) = \begin{cases} OM: & 1.3799 \\ NM: & 0.8149 \end{cases}$$

The quantity R_L is sensitive to the relative phases of all three isovector FF's $F_1^{(v)}$, $F_2^{(v)}$ and F_π. Therefore, its experimental determination as a function of k^2 appears to be very important in reconstructing the EM structure of hadrons in the region of time-like momenta. However, to separate contributions of $R_T(k^2)$ and $R_L(k^2)$, one has to investigate the y - dependence of the differential probability $d^2\Gamma/dxdy$ of $\bar{p}n \to \pi^- l^+ l^-$ process at a fixed value of k^2.

It is extremely useful to rewrite the quantity $R_L(k^2)$ into the following form

$$\begin{aligned} R_L(k^2) &= \Big| G_E^{(v)} + \frac{1}{2}(1 - \frac{k^2+m_\pi^2}{4M^2})F_2^{(v)} + F_\pi(1 - \frac{k^2}{4M^2-m_\pi^2}) + \\ &+ \frac{4M^2 - m_\pi^2 - k^2}{k^2}(F_\pi + 2F_1^{(v)})\Big|^2 / |G_M^{(s)}(k^2)|^2 = \\ &= \Big| G_E^{(v)} + \frac{1}{2}(1 - \frac{k^2+m_\pi^2}{4M^2}) \cdot \\ &\cdot \left[F_2^{(v)} + 2F_\pi \frac{4M^2}{4M^2 - m_\pi^2} + 8\frac{M^2}{k^2}(F_\pi + 2F_1^{(v)}) \right] \Big|^2 / |G_M^{(s)}(k^2)|^2. \end{aligned} \tag{23}$$

Since $F_2^{(v)}$=1.90, there is a strong compensation of $F_2^{(v)}$ and F_π contributions in $R_L(k^2)$ for relatively small values of k^2.

6 Conclusions and summary

We have theoretically investigated the annihilation of slow antinucleons on nucleons at rest into pion and lepton pairs with the aim of possible experimental determination of the nucleon EM FF behaviour in the unphysical region, i.e. in the region of complicated behaviour of FF's caused by the resonance formation. Since, in a process of that sort, the capture of slow particles on other particles at rest is essential and therefore, instead of the cross-section, the decay probability of a bound system is considered, we have first clarified the structure of the differential probability of $\bar{N}N \to \pi l^+ l^-$ process. It is shown that the latter generally depends on two quantities, $R_L(k^2)$ and $R_T(k^2)$, to be expressed through two corresponding structure functions. For the calculation of the k^2-dependence of $R_L(k^2)$ and $R_T(k^2)$, we have used the tree-diagram-approximation model for the $\bar{N}N \to \pi l^+ l^-$ process, in the framework of which it is clearly demonstrated that the process $\bar{p}p \to \pi^0 l^+ l^-$ is completely described by means of the magnetic FF of the proton. At the same time, the description of the $\bar{p}n \to \pi^- l^+ l^-$ process, besides the isoscalar magnetic FF of the nucleon, contains also the isovector nucleon and pion EM FF's.

By using two unitary and analytic VMD models of the nucleon EM FF's and the most accomplished up to now pion EM FF model, we have finally predicted the effective

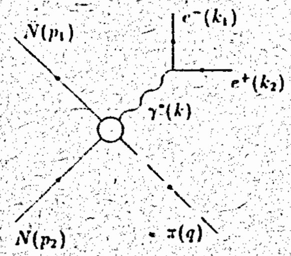

Fig 1.: Diagrammatic representation of the amplitude of annihilation of slow antinucleons on nucleons at rest into a pion and a lepton pair.

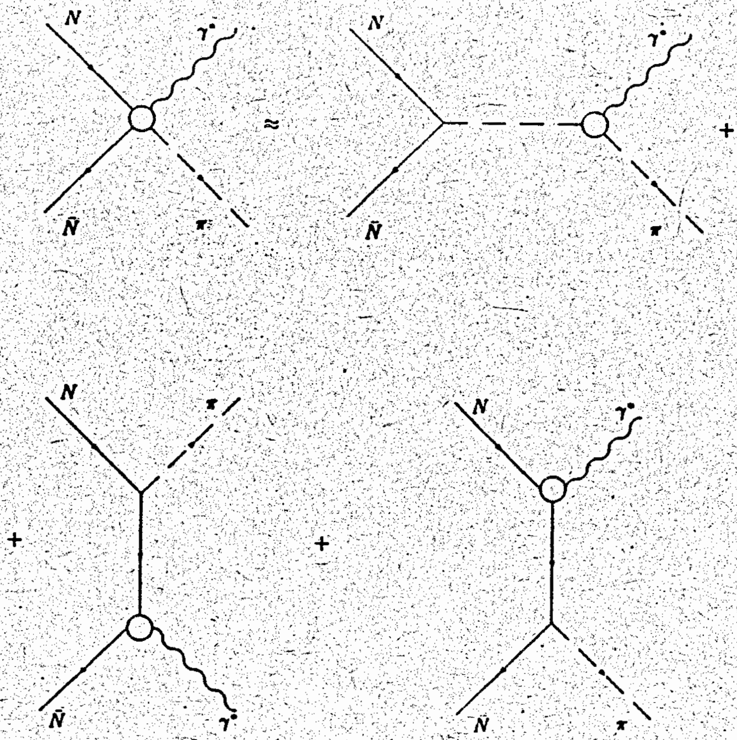

Fig 2.: Tree-diagram-approximation of the amplitude of the $\bar{N}N \to \gamma^* \pi$ process.

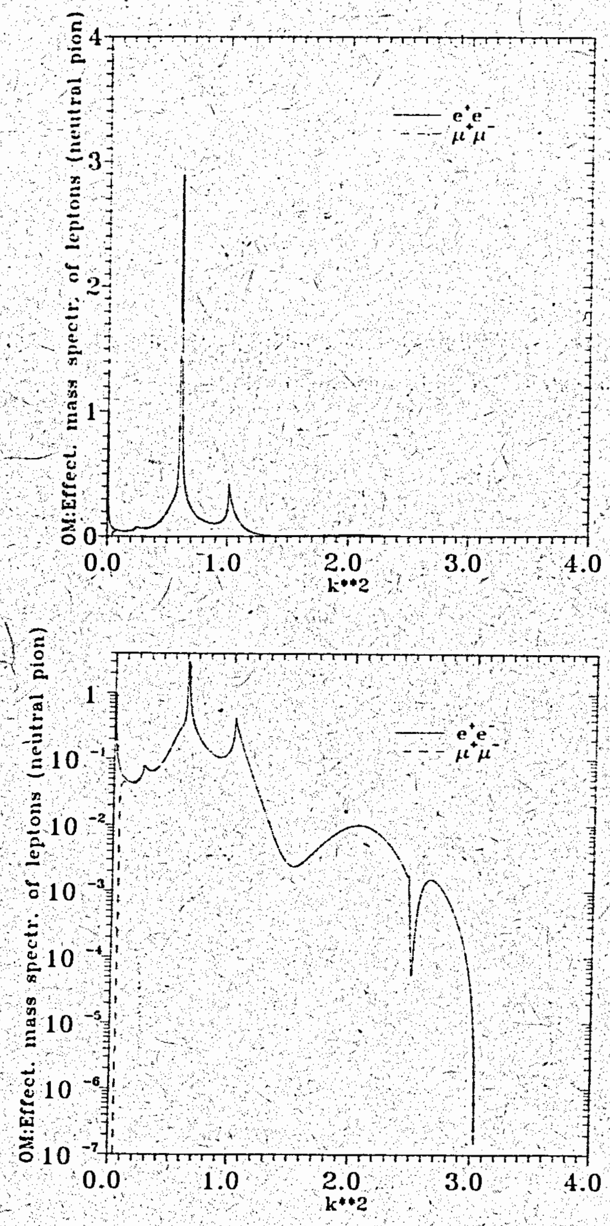

Fig 3.: The effective mass spectrum of lepton pairs produced in the $\bar{p}p \rightarrow \pi^0 l^+ l^-$ process and calculated (a)on the linear scale; b)on the logarithmic scale) by means of the old model (OM) [13] of the electromagnetic structure of nucleons.

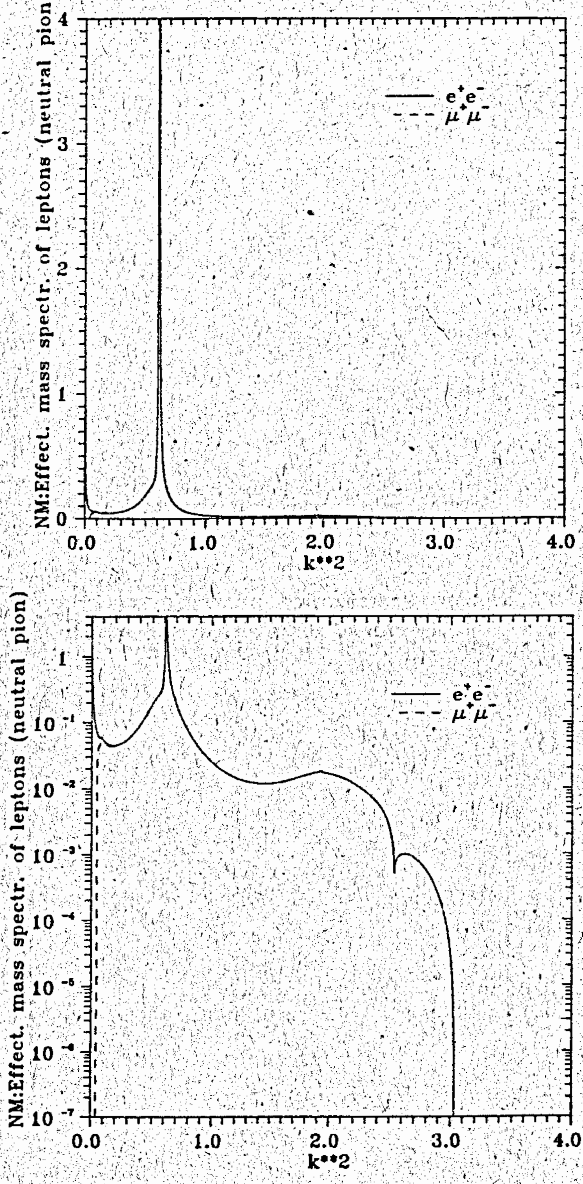

Fig 4.: The same as in Fig.3, but the new model (NM) [14] of the electromagnetic structure of nucleons was used in the calculations.

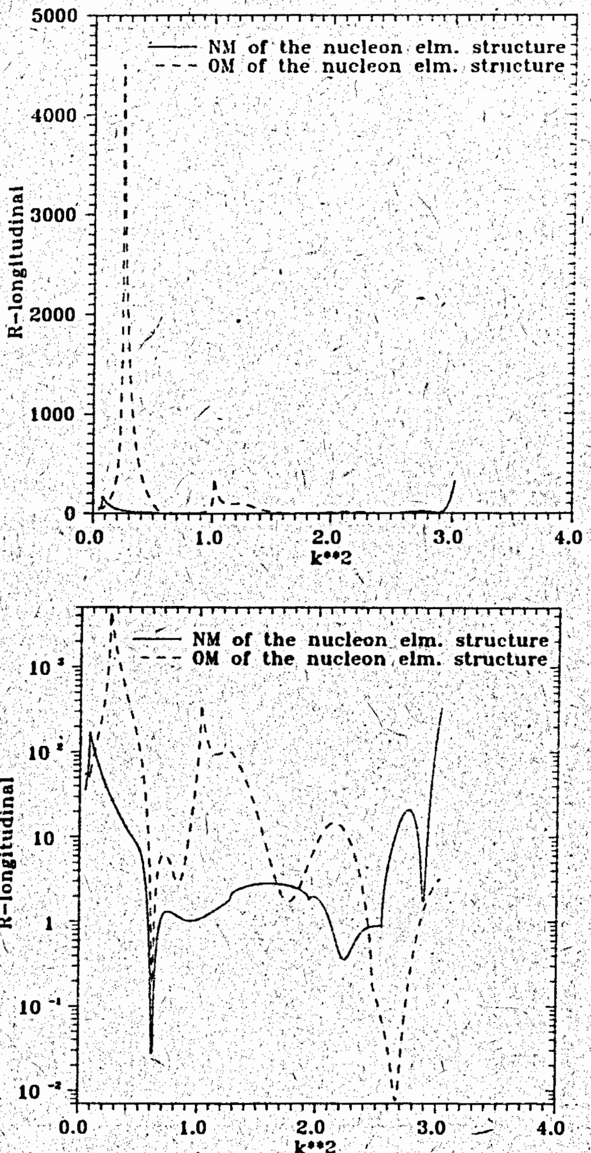

Fig 5.: Behaviour of the ratio of probabilities for production of longitudinal and transversal virtual photons in the $\bar{p}n \to \pi^- \gamma^*$ process.

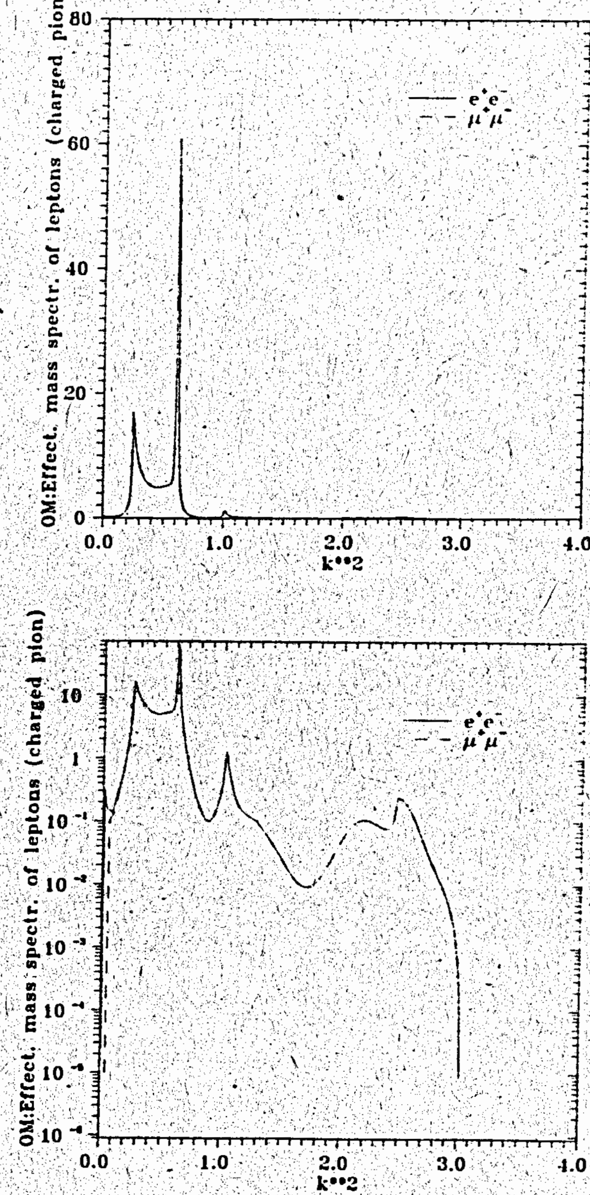

Fig 6.: The effective mass spectrum of lepton pairs produced in the $\bar{p}n \to \pi^- l^+ l^-$ process and calculated (a)on the linear scale; b)on the logarithmic scale) by means of the old model (OM) [13] of the electromagnetic structure of nucleons.

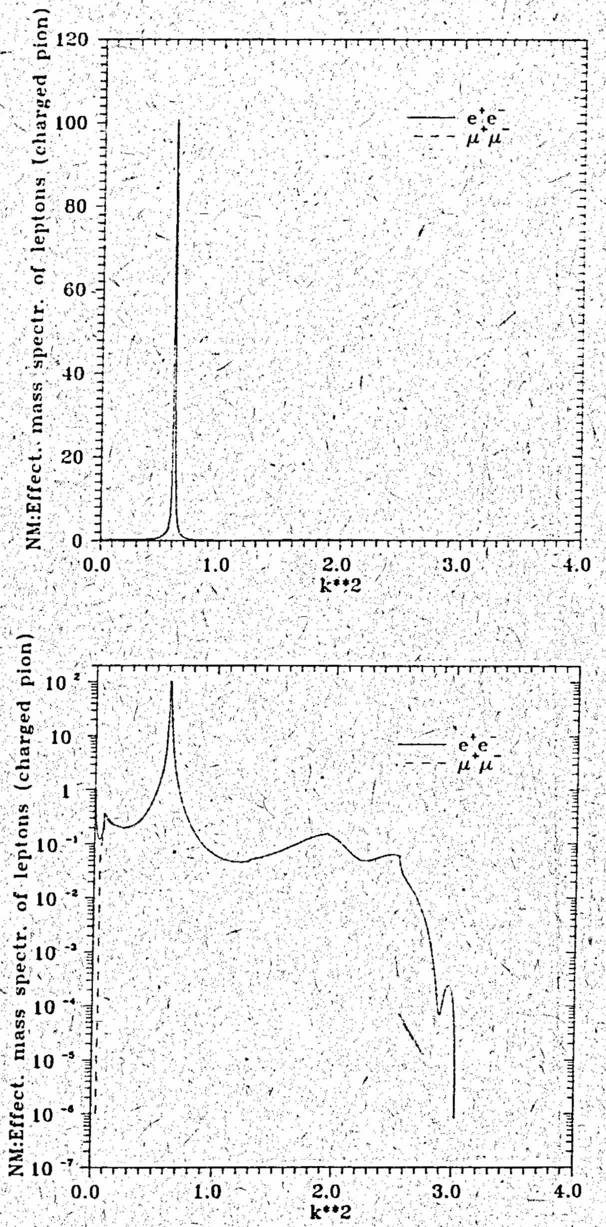

Fig 7.: The same as in Fig.6, but the new model (NM) [14] of the electromagnetic structure of nucleons was used in the calculations.

mass spectra of the lepton pairs and the integral coefficients of internal conversion for $\bar{p}p \to \pi^0 l^+ l^-$ and $\bar{p}n \to \pi^- l^+ l^-$ processes.

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