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ON THE DETERMINATION OF α_s FROM THE QCD-ANALYSIS OF THE SCALING VIOLATION IN THE FRAGMENTATION FUNCTIONS OF THE PROCESS $e^+e^- \to h + X$ STUDIED BY DELPHI, TASSO AND OTHER COLLABORATIONS

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1 Introduction

In papers [1] the idea of unification of the coupling constants of strong electromagnetic and weak interactions has been tested on the basis of new data of the DELPHI Collaboration.

The essential point for the analysis performed in [1] was the input value for the strong interaction coupling constant $\alpha_s(Q^2)$ found at $s(s \equiv Q^2) = M_z^2$.

As has been shown in papers [2]-[4], the DELPHI data on differential cross sections of the process $e^+e^- \to h+X$ allow one to enlarge essentially the region, available for a QCD-analysis of the scaling violation effects in the fragmentation functions, as compared to the region where the data were collected by MARK II, TASSO and other non-LEP collaborations and analysed for the first time in the framework of the QCD evolution equations in papers [5] (TASSO and MARKII data) and [3] (analysis of TASSO data with the evolution in Q^2 of the fitted theoretical expression into the DELPHI region).

In recent papers [6],[7], the QCD analysis of the scaling violation in the fragmentation functions has been perfomed for the first time with inclusion of new LEP data on the process $e^+e^- \to h+X$ (X are all other particles). The method of the QCD analysis used in these papers is based on the method of a direct numerical integration of the exact QCD matrix element (including the second order of perturbation theory) which are implemented in the Lund string model.

Here we shall report on the preliminary results of QCD analysis of the data of the same collaborations as in Ref. [6],[7] but our present analysis is based on another method, that was applied earlier in Ref.[3] for analysing scaling violation in TASSO data for the fragmentation functions. The main features of the program, used in our analysis, are the same as of the program used in Ref. [8] for analysing the BCDMS collabaration data on deep inelastic muon-nucleon structure functions (see for details Ref. [9]). Of course, the modifications of the anomalous dimensions (or splitting functions) for annihilation channel were taken into account.

2 QCD formulae for the fragmentation functions

The process of producing of the inclusive hadron h (of the momentum p) $e^+e^- \to h + X$ is discribed in terms of two kinematic variables: 1) the square of the momentum transferred from the leptonic (e^+e^-) block to the hadronic one $(h+X)$, i.e. $Q^2 = q^2 = (k_{e^+} + k_{e^-})^2 \equiv s(= 4E^2_{beam}$ in c.m.s.) and 2) the fraction of the beam energy E_{beam} carried by the inclusive hadron h, i.e. $z = 2pq/Q^2(= E^h/E_{beam}$ in c.m.s.).

The fragmentation functions, we are interested in, are defined through the cross section normalized to the total one, i.e.

$$\bar{D}(z, Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(z, Q^2)}{dz}, \qquad (1)$$

We shall restrict our analysis to the region of $z > 0.18$ where, as it is known from the experiment performed at PETRA energies [10], the contribution of a longitudinal part of the cross section is negligible as compared to a transversal one.

This transversal part is defined through the quark and antiquark fragmentation functions (we follow the notations and definations given at [11]) as follows (i=γ or Z)

$$\bar{D}^{ih}_T(z, Q^2) = \sum_{f=1}^{N_f} \lambda^i_f [\bar{D}^h_{q_f}(z, Q^2) + \bar{D}^h_{\bar{q}_f}(z, Q^2)], \qquad (2)$$

where $\lambda^\gamma_f = Q^2_f$; $\lambda^Z_f = v^2_f + a^2_f$, and v_f, a_f are electroweak coupling constants.

Now after these necessary remarks we proceed directly to the discussion of QCD analysis that became possible for an annihilation channel due to an application of a "cut vertex formalizm" , developed by A.H.Mueller [12].

The fragmentation functions (2) do obey the Altarelly–Parisi integro-differential evolution equations. These equations after passing to a singlet ($i \equiv f_i$, $j \equiv f_j$)

$$\bar{D}^h_\Sigma(z, Q^2) = \sum_{i=1}^{N_f} (\bar{D}^h_{q_i}(z, Q^2) + \bar{D}^h_{\bar{q}_i}(z, Q^2)) \qquad (3)$$

and a nonsinglet (NS)

$$\bar{D}^h_{\Delta q_i q_j} = \bar{D}^h_{q_i}(z, Q^2) - \bar{D}^h_{q_j}(z, Q^2) \qquad (4)$$

combinations transform into a set of a single equation for a nonsinglet fragmentation function and two coupled equations for a singlet sector.

The solutions to integro-differential Altarelli-Parisi equations have not yet been found in an explicit analytical form. They are found only for the moments

$$\langle \bar{D}^h_{q_i}(Q^2) \rangle_n = \int_0^1 dz \cdot z^{n-2} \cdot z \bar{D}^h_{q_i}(z, Q^2). \qquad (5)$$

This moments do satisfy the evolution equations

$$Q^2\frac{\partial}{\partial Q^2}\langle \bar{D}^h_\Delta(Q^2)\rangle_n = -\alpha_s(Q^2)\gamma^n_{qq}\langle \bar{D}^h_\Delta(Q^2)\rangle_n \qquad (6)$$

$$Q^2\frac{\partial}{\partial Q^2}\begin{pmatrix} \langle \bar{D}^h_\Sigma(Q^2)\rangle_n \\ \langle \bar{D}^h_G(Q^2)\rangle_n \end{pmatrix} =$$
$$= -\alpha_s(Q^2)\begin{pmatrix} \gamma^n_{qq}(\alpha) & 2f\gamma^n_{Gq} \\ \gamma^n_{qG} & \gamma^n_{GG} \end{pmatrix} \times \begin{pmatrix} \langle \bar{D}^h_\Sigma(Q^2)\rangle_n \\ \langle \bar{D}^h_G(Q^2)\rangle_n \end{pmatrix} \quad (7)$$

that under inverse Mellin transformation take a form of integro- differential Altarelli-Parisi equations. The second order anomalous dimensions $\gamma^{n(1)}_{ij}$ for the case of inclusive annihilation (IA) were calculated in [13]- [15]. It is easy to check with allowance for of the results of [17],[18] (see also [11]) that the structure of the analytic solutions of evolution equations (6) and (7) is analogous to that one of the solutions, found for the moments of the deep inelastic scattering (DIS) structure functions, i.e. (we follow the notation of [16] and show the results for the leading order only):

$$\langle \bar{D}^h_\Delta(Q^2)\rangle_n = [\frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)}]^{d_{NS}}\langle \bar{D}^h_\Delta(Q^2_0)\rangle_n \qquad (8)$$

$$\langle \bar{D}^h_\Sigma(Q^2)\rangle_n =$$
$$= [(1-\alpha_n)\langle \bar{D}^h_\Sigma(Q^2_0)\rangle_n - \varepsilon_n\langle \bar{D}^h_G(Q^2_0)\rangle_n]\cdot[\frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)}]^{d^n_+}$$
$$+[\alpha_n\langle \bar{D}^h_\Sigma(Q^2_0)\rangle_n + \varepsilon_n\langle \bar{D}^h_G(Q^2_0)\rangle_n]\cdot[\frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)}]^{d^n_-}, \qquad (9)$$

and an analogous to (9) equation for gluon moment (see [16]). Here $d^n_\pm = \frac{\lambda^n_\pm}{2\beta_0}$, and the expressions for $\lambda^n_\pm$ through the anomalous dimentions and for the coefficients $\alpha_n, \tilde{\alpha}_n$ and ε_n are given in [16].

An important feature of the analytical solutions (8) and (9) is that they contain the explicit dependence on the boundary conditions at a reference point Q^2_0 from which the Q^2 evolution starts. These boundary conditions are concentrated in the quantities $\langle \bar{D}^h_G(Q^2_0)\rangle_n$ and $\langle \bar{D}^h_q(Q^2_0)\rangle_n$ ($q = \Sigma, NS$) that are moments of the $\bar{D}^h_i(z, Q^2_0)$ functions ($i = NS, \Sigma, G$) describing the distributions of hadrons h inside quarks and gluons at the reference point Q^2_0.

Here we shall use the method of QCD analysis based on the method of expanding the fragmentation functions, as functions of the z-variable, in a series of the polynominals [19] $P^\alpha_k(z) = \sum^k_{n=0} A^\alpha_{kn}z^n$, orthogonal with some weight function $\omega^\alpha(z)$ in the interval $0 \le z \le 1$:

$$\int_0^1 w^\alpha(z)P^\alpha_k(z)P^\alpha_l(z)dz = \delta_{kl}$$

Our approach is close to the Parisi-Sourlas [20] method that consists in application of the Jacobi polynomials. They are defined by the weight function $\omega^\alpha(x) = x^{\alpha_1}(1-x)^{\alpha_2}$; ($\alpha = \{\alpha_1, \alpha_2\}$) that reproduce well the main part of the x-dependence of deep inelastic lepton-hadron scattering structure functions $F_i(x, Q^2)$.

The expansion of the fragmentation function (or structure function) in orthogonal polynomials has the form

$$\bar{D}(z, Q^2) = w^\alpha(z)\lim_{N_{MAX}\to\infty}\sum^{N_{MAX}}_{k=0} a_k(Q^2)P^\alpha_k(z). \qquad (10)$$

So if the form of the weight function $\omega^{\alpha}(z)$ reproduces well the main part of the z-dependence of the $D(z,Q^2)$, then only a few number N_{MAX} of terms in the sum in the right-hand side of (10) is really needed for reproducing of this function by a series of polynomials.

The coefficients $a_k(Q^2)$ in the series of (10) can be represented as a linear combination of the moments [19],[20]

$$a_k(Q^2) = \int_0^1 dz P_k^{\alpha}(z) \cdot D(z,Q^2) = \sum_{n=0}^{k} A_{kn}^{\alpha} M(n+2,Q^2), \tag{11}$$

which can be verified by applying the orthogonality relation to (10) with the use of $P_k^{\alpha}(z) = \sum_{n=0}^{k} A_{kn}^{\alpha} z^n$.

According to [19], [20], the transition to the QCD analysis is achieved by a substition of the moments in (11) by the analytical expresions for $M^{QCD}(n,Q^2)$ found in the framework of the QCD perturbation theory, i.e. by formulae (8)-(9).

In what follows we shall use the generalization of this method, proposed in ref.[21], and applied in ref.[3] for the leading order analysis of the TASSO data on production of inclusive hadrons in e^+e^- interactions.

This generalization, according to [3], consists in application of the polynomials defined by the weight function of an arbitrary, more general than in a case of Jacobi polynomials form . For this reason, we shall take the weight function, as in [3], in a form of a sum of exponents.

3 Results and discussion

The final expession to be used in what follows will be

$$\bar{D}(z,Q^2) = \omega^{\alpha}(z) \cdot \sum_{k=0}^{N_{MAX}} \sum_{n=0}^{k} A_{kn}^{\alpha} \cdot M^{QCD}(n+2,Q^2) \cdot P_k^{\alpha}(z), \tag{12}$$

with the substitution of the explicit expressons for $M^{QCD}(n,Q^2)$, calculated in ref.[13]-[15] up to the second order in α_s.

We have taken for our analysis $N_{MAX} = 8$, which provides a reasonable accuracy of the method. The fit to the combined data of the same as in ref.[6],[7] set of TASSO (here we have included TASSO data for all 4 energies, i.e. for $\sqrt{s} = E_{beam} = 14, 22, 35, 44\ GeV$, so the range Q^2 used in our present analysis would be: $196\ GeV^2 \leq Q^2 \leq M_z^2$ ($\simeq 8312\ GeV^2$)), MARK-II, CELLO, AMY, ALEPH and DELPHI collaborations was done for the region of $0.18 < z < 0.80$ with $N_f = 5$ and with the inclusion of contributions of singlet (3) and nonsinglet (4) combinations of quark contributions.

The data of each experiment k were multiplied as in [6], [7] by the normalization factor N_k. The systematic and normalization uncertainties were taken into account in the same way as in [6],[7] by introducing a point-to-point error for each data point.

The fit to these data has given the results for α_s in agreement within the errors with those of Ref. [6],[7].

$$\Lambda_{\overline{MS}}^{(5)} = 151^{+101}_{-73}\ MeV \tag{13}$$

or

$$\alpha_s(M_z^2) = 0.111^{+0.009}_{-0.009}. \tag{14}$$

The parameters of quark distributions defined at the reference point $Q_0^2 = 110\ GeV^2$ were found to be:

1. Singlet distribution: $S(z) = A(5) \cdot exp(-A(6) \cdot z)$; $A(5) = 30.17 \pm 0.68$; $A(6) = 6.182 \pm 0.047$;

2. Nonsinglet distribution: $NS(z) = A(9) \cdot exp(-A(10) \cdot z)$; $A(9) = 30.03 \pm 2.9$; $A(10) = 5.68 \pm 0.19$.

3. The fit has shown that the parameters of gluon contribution $G(z) = A(1) \cdot exp(-A(2) \cdot z)$ could not be defined from the fit of the data with $z > 0.18$. So MINUIT has shown no need in a gluon contribution at the reference point $Q_0^2 = 110\ GeV^2$. (But it should be emphasized that due to the evolution equations (7), the gluon contribution is generated by these equations at any other point Q^2 not equal to Q_0^2). So in our analysis, it was set to zero at a chosen reference point $Q_0^2 = 110\ GeV^2$: $G(z, Q_0^2) = 0$.

The values of the normalisation factors N_k for each experiment k were found to be equal to that found in Ref. [6],[7] with the errors less than 2%. The obtained value of χ^2 is: $\chi^2 = 73$. The number of experimental points included in fit is 97 (one point from TASSO ($z = 0.55, \sqrt{s} = 35\ GeV^2$ with the contribution to χ^2 around 14 was omitted in fitting procedure). So we can conclude that the quality of the description is quite good.

The found value of α_s can be considered as a preliminary one , because the data with lower values of x were not included into analysis. In the region $x < 0.18$ the longitudinal component of the fragmentation function gives an important contribution. Still it should be noted that it agrees well with the value of α_s found from the analysis of the scaling violation in deep inelastic structure functions [8], [9].

A possibility of the presence of power corrections (m^2/Q^2 or m/Q) due to heavy quark effects (see [22]) or other nonperturbative effects was exemined also. It was found that the inclusion of these power corrections into a fit procedure (in a way as it was discussed in [22]) results in a small (5÷7%) shift of the central value of $\Lambda^{(5)}_{\overline{MS}}$, what gives a negligibly, small correction to a value of α_s (as compared with the value of its error).

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References

[1] U.Amaldi, W. de Boer and H.Fürstenau, Phys. Lett. B260 (1991) 131; U.Amaldi, W. de Boer, P.H.Frampton, H.Fürstenau and J.T. Liu, Phys. Lett.B260 (1992) 374.

[2] N.B.Skachkov, JINR Communication E2-90-301, 1990.

[3] E.P.Velicheva, S.P.Kurlovich, N.B.Skachkov, JINR Rapid Communications N1[47]-91, p.5-9; In "Standard Model and Beyond: from LEP to UNC and LHC" p.247, Wold Scientific, Singapure, 1991.

[4] A.Korytov et al., DELPHI NOTE: DELPHI 91-16 Phys 87 (April 1991).

[5] K.Kato, K.Kitani, T.Munehisa, H.Okada and Y.Shimizu, Phys. Rev. Lett. 50 (1983) 389;
K.Kato, T.Munehisa, H.Okada and Y.Shimizu, Prog. Theor.Phys. 70(1983)840.

[6] W. de Boer and T.Kussmaul, Univ. of Karlsruhe preprint, KA-IEKP/92-11. (1992); KA-IEKP/93-08, (1993);

[7] DELPHI Collab., P.Abreu et al., Phys. Lett. B311 (1993) 408.

[8] BCDMS Collab., A.C.Benvenuti et al., Phys.Lett. B195 (1987) 97;
ibid. B233(1989)490.

[9] V.G.Krivokhizhin et al., Zeit. Phys. C36 (1987) 51; ibid. C48 (1990) 347.

[10] R.Brandelik et al., Phys.Lett. B114 (1982) 65.

[11] G.Altarelli et al., Nucl.Phys. B160 (1979) 301; G.Altarelli, Phys.Rep. 81 (1982) 1.

[12] A.H.Mueller, Phys.Rev. D18 (1978) 3705.

[13] G.Curci, W.Furmanski, R.Petronzio, Nucl.Phys. B175 (1980) 27;
W.Furmanski, R.Petronzio, Phys.Lett. B97 (1982) 437.

[14] E.G.Floratos, C.Kounnas, R.Lacaze, Nucl.Phys. B192 (1981) 417.

[15] T.Munehisa, H.Okada, K.Kudoh, K.Kitani, Prog. Theor. Phys. 67 (1982) 609.

[16] A.J.Buras, Rev.Mod.Phys. 52 (1980) 199.

[17] I.F.Owens, Phys.Lett. B76 (1978) 85.

[18] T.Uematsu, Phys.Lett. B79 (1978) 97.

[19] F.J.Yndurain, Phys.Lett. B74 (1978) 421.

[20] G.Parisi, N.Sourlas, Nucl.Phys. B151 (1979) 421.

[21] S.P.Kurlovich, A.V.Sidorov, N.B.Skachkov, JINR Communication E2-89-655, Dubna JINR (1989).

[22] P.Nason, B.R. Webber, Nucl.Phys. B421 (1994) 473.

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