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ISOTOPIC STRUCTURE OF THE ELECTROMAGNETIC CURRENT OF $e^+e^- \to M\overline{M}$ AND $\tau^- \to \nu_\tau M^0 M^-$ PROCESSES

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1 Introduction

An information on the electromagnetic (EM) structure of $\pi^\pm$ and K- mesons in the time-like momentum transfer region can be obtained, as it is well known, from measured cross-sections on $e^+e^- \to MM$ ($M = \pi$ or K) processes at the colliding e^+e^-- beam experiments [1,2]. However, by means of a such procedure only the absolute value of corresponding complex form factors (FF) at the time-like region, dependent on the momentum transfer squared t, is determined. Phases of the EM FF's of $\pi^\pm$ and K, though they appear to be physical quantities, can not be measured in a straightforward way.

On the other hand, t behaviour of the pseudoscalar meson FF phases for $t \gtrsim 4m_M^2$, where m_M is the pseudoscalar meson mass, is very important to be known from the point of view of a verification of dispersion relation methods [3] and various models of the pseudoscalar meson EM structure, like e.g. the unitary and analytic VMD model [4] and others [5,6].

We note, that a situation with the pion EM FF is slightly different from the kaon EM FF's. As a consequence of isospin properties of the hadron EM current, the pion EM FF appears to be essentially isovector, whereas the kaon EM structure is characterized by means of the isovector and isoscalar FF's simultaneously as follows

$$F_{K^+}(t) = \frac{1}{2}[F^{(s)}(t) + F^{(v)}(t)]; \quad F_{K^0}(t) = \frac{1}{2}[F^{(s)}(t) - F^{(v)}(t)]. \tag{1}$$

As a result, in the case of pions a knowledge of $|F_\pi(t)|^2$ is equivalent just to a knowledge of the isovector FF $F_\pi(s)$ and by using the CVC hypothesis [7] one can predict [8,17] the effective mass spectrum behaviour of pions in $\tau^- \to \nu_\tau\pi^-\pi^0$ decay, in fact by a model independent way.

A completely different situation is in the case of kaons. The decay amplitude of the $\tau^- \to \nu_\tau K^-K^0$ process is specified by the isovector part of the kaon EM FF's in the time-like region, more precisely, by the absolute value of the latter. However, one can not draw out a model-independent information on the isovector part of the kaon EM FF's only from data on the $e^+e^- \to K^+K^-$ and $e^+e^- \to K^0\bar{K}^0$ reactions. Really,

taking into account (1), one can write down relations as follows

$$\begin{aligned} 4|F_{K^+}(t)|^2 &= |F^{(s)}(t)|^2 + |F^{(v)}(t)|^2 + 2\cos\delta(t)|F^{(s)}(t)||F^{(v)}(s)|, \\ 4|F_{K^0}(t)|^2 &= |F^{(s)}(t)|^2 + |F^{(v)}(t)|^2 - 2\cos\delta(t)|F^{(s)}(t)||F^{(v)}(t)|, \end{aligned} \tag{2}$$

where $\delta(t)$ is a relative phase of the $F^{(s)}(t)$ and $F^{(v)}(t)$ as a function of the momentum transfer squared t.

Therefore only in a simultaneous experimental measurement of the following three processes

$$\tau \to \nu_\tau K^-K^0; \quad e^+e^- \to K^+K^- \text{ and } e^+e^- \to K^0\bar{K}^0$$

a complete restoration of isotopic structure of the kaon EM FF's, i.e. a complete determination of $|F^{(s)}(t)|$, $|F^{(v)}(t)|$ and $\cos\delta(t)$, is possible.

Of course, a realization of the latter program is attainable just in the interval

$$1GeV^2 \approx 4m_K^2 \leq t \leq m_\tau^2 \approx 3.2GeV^2, \tag{3}$$

given by a kinematics of the decay $\tau^- \to \nu_\tau K^-K^0$ process.

We note, however, that the upper bound in (3) can be in principle overcome by an experimental investigation of the $\bar{\nu}_e e^- \to K^-K^0$ process of collisions of high-energetic electron antineutrinos with atomic electrons. The threshold of the latter reaction is remarkable high, $E_\nu = 2m_K^2/m_e \approx 1TeV$. But, such antineutrinos in principle can be produced at the corresponding proton accelerators (FERMILAB- Batavia. UNK-Protvino) or in the cosmic rays. At the latter case one is in need of large target-detectors (SUPER-KAMIOKANDE [9] or DUMAND [10]).

In this paper various characteristics of the $\tau^- \to \nu_\tau K^-K^0$ and $\tau^- \to \nu_\tau\pi^-\pi^0$ decay processes are investigated theoretically. Particularly, by using the CVC hypothesis a decay of polarized and unpolarized τ- leptons is analyzed. Taking into account the value BR$(\tau^- \to \nu_\tau K^-K^0) = 0.29 \pm 0.12 \pm 0.03\%$ [11] we can see that one year statistics of a $\tau^+\tau^-$- pair creation on c-τ factories of about 10^6 will allow to study quantitatively the t- dependence of the kaon isovector FF within a reasonable precision, which seems to be very important in testing the unitary and analytic VMD model [4] and the QCD-like description of EM structure of hadrons as well.

The paper is organized as follows. In the next section a general structure of the Dalitz- distribution of $\tau \to \nu_\tau M^- M^0$ decay is derived, particularly with regards to the analysis of CVC- hypothesis consequences. In section 3 the same is investigated in a language of $\cos\vartheta$- distribution in the c.m. system of hadrons produced in final state of that decay process. Polarization effects in $\tau^- \to \nu_\tau M^- M^0$ are studied in section 4. The unitary and analytic VMD model for kaon and pion EM FF's is presented in section 5. The section 6 contains results of our numerical predictions. Conclusions and summary are given in the last section.

2 Dalitz-distribution for $\tau^- \to \nu_\tau M^- M^0$

We start with the following matrix element of the decay $\tau^-(k_1) \to \nu_\tau(k_2) + M^-(p_1) + M^0(p_2)$ process

$$\mathcal{M} = \frac{G_E}{\sqrt{2}} \bar{u}(k_2)\gamma_\mu(1 + a\gamma_5)u(k_1)J_\mu, \tag{4}$$

where W- boson exchange mechanism is assumed to be realized. However, since $m_\tau^2/m_W^2 << 1$, all nonlocal effects in (4) due to the W- boson exchange can be neglected. G_F means the Fermi constant of weak interactions, J_μ is a weak charged current responsible for $W^- \to M^- + M^0$ transition and "a" is a constant of axial-vector interaction in the weak charged current of $\nu_\tau \to \tau$ transition. We note, that the difference $a = 1$ characterizes the admixture of right currents.

For general reasons we assume, that the constant "a" is complex one. Then T-violation in the weak lepton current of $\nu_\tau \to \tau$ transition is allowed to be realized. Of course, the latter very peculiar violation, introduced phenomenologically, is different from the Kobayashi- Maskava mechanism and it does not necessitate an existence of three generations of leptons. Analyzing existing data [12] on the lepton decays $\tau^- \to \nu_\tau l^- \nu_\ell$ (assuming the constant "a" to be real) one finds

$$\frac{a-1}{a+1} < 0.37,$$

which indicates that the large admixture of right currents can be present in the $\nu_\tau \to \tau$ transition.

Now, by using (4) one can obtain for the absolute value of the corresponding matrix element squared (the summation over polarization states of neutrino is carried out) the expression

$$|\overline{\mathcal{M}}|^2 = \frac{G_F^2}{2}[\ell_{\mu\nu} + r_{\mu\nu}]J_\mu J_\nu^* \tag{5}$$

$$\begin{aligned} \ell_{\mu\nu} &= 2(1 + |a|^2)(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1 \cdot k_2) + \\ &+ 2(1 - |a|^2)m_\nu m_\tau g_{\mu\nu} - 4iRe\, a\epsilon_{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma} \end{aligned} \tag{6}$$

$$\begin{aligned} r_{\mu\nu} &= 2i(1 + |a|^2)\epsilon_{\mu\nu\alpha\beta}s_\alpha k_{2\beta} - 4Re\, am_\tau(s_\mu k_{2\nu} + s_\nu k_{2\mu} - g_{\mu\nu}s \cdot k_2) - \\ &- 2im_\nu(1 - |a|^2)\epsilon_{\mu\nu\alpha\beta}s_\alpha k_{1\beta} + 4im_\nu Im\, a(k_{1\mu}s_\nu - s_\mu k_{1\nu}) \end{aligned} \tag{7}$$

where s is a four vector of the τ- lepton spin, $s \cdot k_1=0$ and m_ν, m_τ are masses of the neutrino and τ- lepton, respectively.

We note, that linear in the neutrino mass terms appear in tensors $\ell_{\mu\nu}$ and $r_{\mu\nu}$, as one could expect, under two following conditions:

- the right currents are present in the weak current of $\nu_\tau \to \tau$ transition (i.e. $1 - |a|^2 \neq 0$);
- there is T- violation in the weak current of $\nu_\tau \to \tau$ transition (i.e. $Im\, a \neq 0$).

One can see from the relations (5)-(7), that there are T- odd effects (in the framework of the model presented here) in the $\tau^- \to \nu_\tau M^- M^0$ decays, if the following conditions are fulfilled simultaneously:

- the neutrino mass m_ν is different from zero;
- the decaying τ- leptons are polarized;
- the hadronic tensor $J_\mu J_\nu^*$ contains an antisymmetric part.

However, as a consequence of the CVC hypothesis the current J_α describing the $\tau \to \nu_\tau M^- M^0$ decay is completely determined by only the isovector part of the pseudoscalar meson FF. As a consequence of the latter, the tensor $J_\mu J_\nu^*$ for such decays is symmetric one and T- odd effects should not arise there. Nevertheless, one can observe them also in this case by verifying the relation $|Re\ a| \equiv |a|$, which requires the constant "a" to be pure real.

Further we start with a general expression for differential probability of the $\tau^- \to \nu_\tau M^- M^0$ decay of unpolarized τ- leptons as follows

$$\begin{aligned} d\Gamma &= (2\pi)^4 \int \frac{|\overline{\mathcal{M}}|^2}{2m_\tau} \delta(k_1 - k_2 - p_1 - p_2) \frac{d^3k_2}{(2\pi)^3 2E_\nu} \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} = \qquad (8) \\ &= \frac{|\overline{\mathcal{M}}|^2}{64\pi^3} \frac{dEdE_\nu}{m_\tau} = \frac{|\overline{\mathcal{M}}|^2}{128\pi^3} \frac{dEdk^2}{m_\tau^2} = \\ &= \frac{|\overline{\mathcal{M}}|^2}{256\pi^3} m_\tau dxdy \end{aligned}$$

where $k^2 = (k_1 - k_2)^2$ is the effective mass squared of the $M^- M^0$ system, $x = k^2/m_\tau^2$; $y = 2E/m_\tau$ and E is the energy of M^- meson at the τ- lepton rest reference frame.

Then

$$R \le x \le 1, \quad \sqrt{R} \le y \le 1, \quad R = 4m_M^2/m_\tau^2$$

and m_M is the mass of M- meson.

In accordance with the CVC hypothesis, as we have already mentioned, the current describing $W^- \to M^- M^0$ transition is determined by means of only the isovector (I=1) FF in the following way

$$\begin{aligned} J_\alpha &= \sqrt{C_M} F_M^{I=1}(k^2)(p_1 - p_2)_\alpha \qquad (9) \\ k \cdot J &= \sqrt{C_M} F_M^{I=1}(k^2)(m_{M^-}^2 - m_{M^0}^2) = 0; \text{ if } m_{M^-} = m_{M^0} \end{aligned}$$

where C_π=1 for pions and $C_K = 1/\sqrt{2}$ for kaons. As it is well known, the CVC hypothesis is valid with the precision of isotopic invariance of strong interactions of hadrons. Therefore on this level one can neglect a difference in masses of the charged and neutral mesons inside of the corresponding isotopic multiplets (π and K).

By using (9) and (5)-(7), one can derive the following expression for the energy (E and E_ν) distribution of the $\tau^- \to \nu_\tau K^- K^0$ decay probability

$$d\Gamma = G_F^2 \frac{1 + |a|^2}{8\pi^2} C_M |F_M^{I=1}(k^2)|^2 [(E - E_0)^2 + A(k^2)] dk^2 dE, \qquad (10)$$

where

$$E_0 = \frac{m_\tau^2 + k^2 - m_\nu^2}{4m_\tau}$$

$$A(k^2) = \left(\frac{k^2}{4} - m_M^2\right)\left(\frac{m_\tau^2 - k^2 + m_\nu^2}{4m_\tau^2} - \frac{m_\nu}{2m_\tau}\frac{1 - |a|^2}{1 + |a|^2}\right).$$

Just from the relation (10) it follows that energies E and E_ν play a completely different role in $\tau^- \to \nu_\tau M^- M^0$ decay. The E_ν- dependence (or k^2- dependence, what is the same) of the decay probability is essentially determined by the k^2- dependence of $|F_M^{I=1}(k^2)|^2$, i.e. it has a dynamical origin. On the other hand a dependence of $d^2\Gamma/dk^2dE$ on the energy E of M^-- meson is demonstrated in an explicit form as follows

$$(E - E_0)^2 + A(k^2). \qquad (11)$$

The latter E- dependence is a consequence of the two following fundamental circumstances:

- the conservation of the weak current J_μ, $k \cdot J$=0
- the vector nature of interactions of leptonic and hadronic weak currents.

Besides, an absence of a charged Higgs boson mechanism is very important too.

Since quantities E_0 and $A(k^2)$ are known functions of k^2, then one can predict also E- dependence of the probability $d^2\Gamma/dk^2dE$ of $\tau^- \to \nu_\tau M^- M^0$ decay. Therefore one can say that the latter dependence has a kinematical origin unlike the dynamical E_ν- dependence.

In the m_ν=0 limit the expressions for E_0 and $A(k^2)$ are simplified:

$$\begin{aligned} E_0 &= \frac{k^2 + m_\tau^2}{4m_\tau} = \frac{1}{2}(m_\tau - E_\nu) > 0, \qquad (12) \\ A(k^2) &= \frac{(k^2 - 4m_M^2)(m_\tau^2 - k^2)}{16m_\tau^2} = (k^2 - 4m_M^2)\frac{E_\nu}{8m_\tau} \ge 0, \end{aligned}$$

i.e. at the borders of the physical region of k^2 (for $k^2 = 4m_M^2$ and $k^2 = m_\tau^2$) the function $A(k^2)$ is equal to zero. A maximal value of $A(k^2)$ is achieved inside of the physical region of k^2 just for $k^2 = m_\tau^2 + 4m_M^2/2$ and it takes the value as follows

$$A_{max} = (\frac{m_\tau}{8})^2(1 - \frac{4m_M^2}{m_\tau^2})^2 = \begin{cases} 0.0469 GeV^2 & for\ \tau^- \to \nu_\tau \pi^- \pi^0 \\ 0.0235 GeV^2 & for\ \tau^- \to \nu_\tau K^- K^0 \end{cases} \qquad (13)$$

3 $\cos\vartheta$- dependence of differential probability

An analysis of consequences of the CVC hypothesis is especially simplified, if instead of E- dependence of the differential probability of the $\tau^- \to \nu_\tau M^- M^0$ decay (at the τ-lepton rest frame) $\cos\vartheta$- dependence of that probability at the c.m. system of $M^- M^0$ system (ϑ is the angle between the M^- meson three momentum at the c.m. system of $M^- M^0$ and a direction of the total three momentum of $M^- M^0$ at the τ- lepton rest reference frame) is investigated.

Energies of particles under consideration at the c.m. system of $M^- M^0$ are expressed by the following relations

$$\widetilde{E_\nu} = \frac{m_\tau^2 - k^2}{2\sqrt{k^2}}, \quad \widetilde{E_\tau} = \frac{m_\tau^2 + k^2}{2\sqrt{k^2}}, \quad \widetilde{E} = \sqrt{\frac{k^2}{4} - m_M^2}. \tag{14}$$

Starting from (10) one can write down the $\cos\vartheta$- dependence of the differential probability of $\tau^- \to \nu_\tau M^- M^0$ decay in the following form (m_ν=0)

$$d^2\Gamma = G_F^2 \frac{1+|a|^2}{512\pi^3 m_\tau^3} k^2 (m_\tau^2 - k^2)^2 (1 - \frac{4m_M^2}{k^2})^{3/2} C_M |F_M^{I=1}(k^2)|^2 (1 + \cos^2\vartheta \frac{m_\tau^2 - k^2}{k^2}) dk^2 d\cos\vartheta. \tag{15}$$

It follows from the latter expression, that:

- the differential probability $d^2\Gamma/dk^2 d\cos\vartheta$ is by a specific (quadratic) $\cos\vartheta$- dependence characterized
$$\cos^2\vartheta + \frac{k^2}{m_\tau^2 - k^2} \tag{16}$$
and so, it is symmetric according to the change $\cos\vartheta \to -\cos\vartheta$;
- the differential probability $d^2\Gamma/dk^2 d\cos\vartheta$ turns out to be zero at the boundary values of k^2 ($k^2 = 4m_M^2$ and $k^2 = m_\tau^2$);
- a relative magnitude of $\cos^2\vartheta$- contribution and the constant term in (16) is determined only by a magnitude of k^2, i.e. it does not need to be known the exact form of EM FF $F_M^{I=1}(k^2)$.

The angle dependence coefficient $\alpha = (m_\tau^2 - k^2)/k^2$ of the differential probability $d^2\Gamma/dk^2 d\cos\vartheta$, defined by the general expression (15), takes values from the intervals

as follows

$$(k^2 = m_\tau^2)\ 0 \le \alpha \le 39 \quad (k^2 = 4m_\pi^2), \ \text{for } \tau^- \to \nu_\tau \pi^- \pi^0; \tag{17}$$
$$(k^2 = m_\tau^2)\ 0 \le \alpha \le 2.2 \quad (k^2 = 4m_K^2), \ \text{for } \tau^- \to \nu_\tau K^- K^0.$$

As a result with increased values of k^2 a smoothness of the angle dependence of the differential probability is observed, i.e. a relative role of the angle dependence coefficient is enlarged.

Now by using the x- variable, defined in the relation (8), one can rewrite the decay probability in the following form

$$\frac{1}{\Gamma_e}\frac{d^2\Gamma}{dx\, d\cos\vartheta} = \frac{3}{4} x (1-x)^2 (1 - \frac{R}{x})^{3/2} (1 + \frac{1-x}{x}\cos^2\vartheta) C_M |F_M^{I=1}(k^2)|^2, \tag{18}$$

where $\Gamma_e = G_F^2 (1+|a|^2) m_\tau^5 / 384\pi^3$ is the total decay probability of the $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ process (with unpolarized particles).

By a $\cos\vartheta$ integration in (18) one gets the expression for the effective mass spectrum of $M^- M^0$ system created at the $\tau^- \to \nu_\tau M^- M^0$ decay

$$\frac{1}{\Gamma_e}\frac{d\Gamma}{dx} = \frac{1}{2}(1+2x)(1-x)^2 (1 - \frac{R}{x})^{3/2} C_M |F_M^{I=1}(k^2)|^2. \tag{19}$$

In accordance with (18) the angle distribution properties of M^-- mesons in $\tau^- \to \nu_\tau M^- M^0$ decays, which do not depend on a concrete parametrization of $F_M^{I=1}(k^2)$, are characterized by the following two functions

$$g_1(x) = \frac{3}{4} x (1-x)^2 (1 - \frac{R}{x})^{3/2} \ge 0;$$
and
$$g_2(x) = \frac{3}{4}(1-x)^3 (1 - \frac{R}{x})^{3/2} \ge 0. \tag{20}$$

Both of them are turning to be zero at the borders of the physical region of k^2, however, their maxima, which depend on m_M^2, are in different places (Fig.1 a,b).

By means of the functions (20) one can rewrite the angular distribution of mesons (18) in the following form

$$\frac{1}{\Gamma_e}\frac{d^2\Gamma}{dx\, d\cos\vartheta} = [g_1(x) + g_2(x)\cos^2\vartheta] C_M |F_M^{I=1}(k^2)|^2 =$$
$$= g_1(x)(1 + \frac{1-x}{x}\cos^2\vartheta) C_M |F_M^{I=1}(k^2)|^2. \tag{21}$$

From here, by an integration through x in the whole physical region of k^2, one gets the $\cos\vartheta$- dependence of the differential probability $d\Gamma/d\cos\vartheta$ of the $\tau^- \to \nu_\tau M^- M^0$ decay

$$\frac{1}{\Gamma_e}\frac{d\Gamma}{d\cos\vartheta} = I_1 + cos^2\vartheta \cdot I_2, \tag{22}$$

where

$$I_{1,2} = \int_R^1 g_{1,2}(x) C_M |F_M^{I=1}(k^2)|^2 dx$$

are definite numerical dimensionless numbers depending on $|F_M^{I=1}(k^2)|^2$.

Resultant values of the integrals I_1 and I_2 have to be sensitive to $F(k^2)$ at different regions of k^2 as the maxima of the functions $g_1(x)$ and $g_2(x)$ do not coincide.

It is straightforward to see that the integrals I_1 and I_2 determine a ratio of total probabilities

$$\Gamma(\tau^- \to \nu_\tau M^- M^0)/\Gamma_e = 2(I_1 + \frac{1}{3}I_2). \tag{23}$$

In order to find the E- dependence of the decay probability, first the distribution $d^2\Gamma/dk^2dE$ is transformed to the dimensionless variables x and y defined in (8)

$$\frac{1}{\Gamma_e}\frac{d^2\Gamma}{dxdy} = 6[(y-y_0)^2 + b(x)] C_M |F_M^{I=1}(k^2)|^2, \tag{24}$$

where

$$y_0 = \frac{1+x}{2}, \qquad b(x) = \frac{1-x}{4}(1 - \frac{R}{x}).$$

Then the y- distribution (i.e. the E- distribution) is defined by the following integral over the variable x

$$\frac{1}{\Gamma_e}\frac{d\Gamma}{dy} = 6\int_{x_-(y)}^{x_+(y)} dx[(y-y_0)^2 + b(x)] C_M |F_M^{I=1}(k^2)|^2, \tag{25}$$

where

$$x_\pm(y) = \frac{1}{2}[y \pm \sqrt{y^2 - R}]/(1 - y + \frac{R}{4}).$$

In order to calculate the latter integral one has to know a k^2- dependence of the isovector part of the M^-- meson FF.

One can show immediately, that for arbitrary values of E (besides the threshold region) an integration in (25) for the $\tau^- \to \nu_\tau \pi^- \pi^0$ decay contains a large $\rho(770)$- meson contribution and therefore contributions of heavier ρ- mesons will be in the y- distribution disguised.

On the other hand, for the $\tau^- \to \nu_\tau K^- K^0$ decay the $\rho(770)$- meson appears to be under the threshold and the main contribution will be from the radially excited states of the $\rho(770)$- meson.

4 A decay of polarized τ- leptons

The Dalitz distribution for $\vec{\tau}^- \to \nu_\tau M^- M^0$ decays of polarized τ- leptons is given by the expression

$$d\Gamma = G_F^2 \frac{1+|a|^2}{32\pi^4} dk^2 dE d\Omega_M C_M |F_M^{I=1}(k^2)|^2 \cdot \tag{26}$$
$$\left\{(E-E_0)^2 + A(k^2) - \frac{2Re\, a}{1+|a|^2}[(s\cdot p_1)(E-E_0) + \right.$$
$$\left. + \frac{1}{2}(s\cdot k_2)(E - E_0 + (k^2 - \frac{m_M^2}{4})/m_\tau)]\right\},$$

where $d\Omega_M$ is an element of the space angle of detected meson related to a reference frame given by a three-momentum $\vec{p_\nu}$ and a three-momentum $\vec{s}$ of the τ- lepton polarization at its rest reference frame.

From the latter for the corresponding decay asymmetries one gets

$$A_M = \frac{2Re\, a}{1+|a|^2}\frac{\sqrt{E^2 - m_M^2}(E-E_0)}{(E-E_0)^2 + A(k^2)}, \quad \text{at the correlation } \vec{s}\cdot\vec{n_M} \tag{27}$$

$$A_{\nu_\tau} = \frac{Re\, a}{1+|a|^2}\frac{\sqrt{E_\nu^2 - m_\nu^2}[(E-E_0) + (k^2 - \frac{m_M^2}{4})/m_\tau]}{(E-E_0)^2 + A(k^2)}, \quad \text{at the correlation } \vec{s}\cdot\vec{n_\nu}$$

where $\vec{n_M}$ and $\vec{n_\nu}$ are unit vectors along the three-momenta of M^- and ν_τ.

Since the angle ϑ_ν between three-momenta of ν_τ and M^- varies with a variation E and k^2, then instead of A_ν and A_M it is useful to introduce the following combinations of the latter

$$A_t = A_\nu \sin\vartheta_\nu, \quad A_\ell = A_M + \cos\vartheta_\nu A_\nu, \tag{28}$$

which are made dependent upon components of $\vec{s}$ to be parallel (A_ℓ) and perpendicular (A_t) to the three-momentum of the negative meson M^-. For m_ν=0 $\cos\vartheta_\nu$ takes the form as follows

$$\cos\vartheta_\nu = \frac{m_\tau^2 - 2m_\tau(E+E_\nu) + 2EE_\nu}{2E_\nu\sqrt{E^2 - m_M^2}}. \tag{29}$$

At the limit $m_\nu=0$ asymmetries A_ν and A_M in terms of dimensionless variables x and y take the following form

$$A_M = \frac{2Re\ a}{1+|a|^2}\frac{(y-y_0)\sqrt{y^2-R}}{(y-y_0)^2+b(x)}, \qquad (30)$$
$$A_\nu = \frac{Re\ a}{1+|a|^2}\frac{(y-z_0)(1-x)}{(y-y_0)^2+b(x)},$$

where

$$z_0=(1-3x+R/4)/2 \quad \text{and} \quad \cos\vartheta_\nu=\frac{2x-y-xy}{(1-x)\sqrt{y^2-R}}.$$

The asymmetry A_M at the kinematical interval of values y is changing the sign at the point $y=y_0$.

Both asymmetries A_ν and A_M, which are always different from zero for $\vec{\tau}^- \to \nu_\tau M^- M^0$ decays, are P- odd (though T- even). The latter is caused by a P- violation in lepton current of the $\nu_\tau \to \tau^-$ transition. Therefore both asymmetries are proportional to $Re\ a$.

The asymmetries A_ν and A_M are determined only by a kinematics of $\vec{\tau}^- \to \nu_\tau M^- M^0$ decays, i.e. by the values of x, y and R. They do not depend on FF $F_M^{I=1}(k^2)$. This is a result of a conservation of the weak-vector-current of hadrons. As a result both of these asymmetries in $\vec{\tau}^- \to \nu_\tau M^- M^0$ decays are suitable for a CVC- hypothesis conservation verification.

Let us investigate now ϑ- dependence of those asymmetries at the c.m. system of M^-M^0 to be created in $\vec{\tau}^- \to \nu_\tau M^- M^0$ decay. Components of the four-vector polarization s_α of τ- lepton in the c.m. system of M^-M^0 are connected with three-vector $\vec{s}$ of τ- lepton polarization at the laboratory system in the following way

$$\tilde{s}_0=\vec{s}\cdot\vec{p_\tau}/m_\tau; \quad \tilde{\vec{s}}=\vec{s}+\tilde{\vec{p_\tau}}\frac{\vec{s}\cdot\tilde{\vec{p_\tau}}}{m_\tau(\widetilde{E_\tau}+m_\tau)}, \qquad (31)$$

where $\tilde{\vec{p_\tau}}$ and $\widetilde{E_\tau}$ are three- momentum and energy of τ- lepton at the c.m. system of M^-M^0 (see Fig.2).

Then for asymmetries A_t and A_ℓ, which are induced by the components s_t and s_ℓ of τ- lepton vector polarization, one can obtain the following $\cos\vartheta$- and k^2- dependence (the limit $m_\nu=0$ is considered)

$$A_t = -\frac{2Re\ a}{1+|a|^2}\frac{\sqrt{x}\sin(2\vartheta)}{1-x}/(\cos^2\vartheta+\frac{x}{1-x}) \qquad (32)$$
$$A_\ell = -\frac{2Re\ a}{1+|a|^2}\frac{1+x}{1-x}(\cos^2\vartheta-\frac{x}{1+x})/(\cos^2\vartheta+\frac{x}{1-x}).$$

Thus, particularly, for $k^2=m_\tau^2$ we have

$$A_t = -\sin(2\vartheta)\frac{2Re\ a}{1+|a|^2}, \qquad (33)$$
$$A_\ell = (1-2\cos^2\vartheta)\frac{2Re\ a}{1+|a|^2}$$

and also the relations

$$A_\ell(\cos\vartheta=\pm1) = -\frac{2Re\ a}{1+|a|^2}, \qquad (34)$$
$$A_\ell(\cos\vartheta=0) = \frac{2Re\ a}{1+|a|^2}$$

to be valid for arbitrary values of k^2.

It is demonstrated in Fig.3 a,b, where ϑ- dependence of asymmetries A_t, A_ℓ at different values of k^2 is presented, that all possible values of A_ℓ are from the interval bounded by the values $A_\ell(4m_M^2)$ and $A_\ell(m_\tau^2)$. It follows from the latter that the A_ℓ variation for $\tau^- \to \nu_\tau K^- K^0$ decay is essentially narrower than for $\tau^- \to \nu_\tau \pi^- \pi^0$ one.

The asymmetry A_ℓ, investigated as a function of $\cos\vartheta$, is changing the sign two-times at

$$\cos\vartheta_0=\pm\sqrt{\frac{x}{1+x}} \qquad (35)$$

which depends only on k^2, but it does not depend on the meson mass.

In order to find a ϑ- dependence of asymmetries integrated over k^2, we start with the probability expression for polarized τ- lepton decays in the following form

$$\frac{1}{\Gamma_e}\frac{d^2\Gamma}{dx\,d\cos\vartheta} = \frac{3}{4}x(1-x)^2(1-\frac{R}{x})^{3/2}C_M|F_M^{I=1}(k^2)|^2\left\{1+\cos^2\vartheta\frac{1-x}{x}\right. \qquad (36)$$
$$\left. - s_t\frac{2Re\ a}{1+|a|^2}\frac{\sin(2\vartheta)}{\sqrt{x}} - s_\ell\frac{2Re\ a}{1+|a|^2}\frac{1+x}{x}(\cos^2\vartheta-\frac{x}{1+x})\right\}.$$

From here, particularly, we see immediately that by an investigation of ϑ- dependence of $\vec{\tau}^- \to \nu_\tau M^- M^0$ decay of polarized τ- leptons one can determine not only longitudinal, but also transversal polarization of τ- lepton.

By an integration of (36) over x- variable (i.e. over k^2) one gets

$$\frac{1}{\Gamma_e}\frac{d\Gamma}{d\cos\vartheta} = I_1 + I_2\cos^2\vartheta - \frac{2Re\ a}{1+|a|^2}[s_t\sin(2\vartheta)\cdot I_3 + s_\ell(I_4\cos^2\vartheta + I_5)] \qquad (37)$$

where

$$I_3 = \frac{3}{4}\int_R^1 \sqrt{x}(1-x)^2(1-\frac{R}{x})^{3/2}C_M|F_M^{I=1}(k^2)|^2 dx \qquad (38)$$
$$I_4 = \frac{3}{4}\int_R^1 (1+x)(1-x)^2(1-\frac{R}{x})^{3/2}C_M|F_M^{I=1}(k^2)|^2 dx,$$

$I_5 = -I_1$ and $I_{1,2}$ are defined by the relations (22).

Two independent asymmetries A_t and A_ℓ are related with integrals $I_1,..,I_4$ only by the following way

$$\overline{A_t} = -\frac{2Re\ a}{1+|a|^2}\frac{i_3\sin(2\vartheta)}{1+i_2\cos^2\vartheta};\quad \overline{A_\ell} = \frac{2Re\ a}{1+|a|^2}\frac{1-i_4\cos^2\vartheta}{1+i_2\cos^2\vartheta}, \qquad (39)$$

where $i_2 = I_2/I_1$, $i_3 = I_3/I_1$ and $i_4 = I_4/I_1$.

It is natural that in a calculation of both asymmetries $\overline{A_t}$ and $\overline{A_\ell}$ one is in need of the corresponding M- meson FF's.

An integrated over $\cos\vartheta$ probability of the $\vec{\tau}^- \to \nu_\tau + (M^-M^0)$ decay is determined only by s_ℓ component of the τ- lepton vector polarization

$$\frac{1}{\Gamma_e}\frac{d\Gamma}{dx} = \frac{1}{2}(1-x)(1-\frac{R}{x})^{3/2}C_M|F_M^{I=1}(k^2)|^2[1+2x+s_\ell(-1+2x)\frac{2Re\ a}{1+|a|^2}]. \qquad (40)$$

and the corresponding asymmetry takes the form as follows

$$\widetilde{A_\ell} = \frac{2Re\ a}{1+|a|^2}\frac{2x-1}{2x+1}. \qquad (41)$$

It is straightforward to see that the asymmetry $\widetilde{A_\ell}$ does not depend neither on the FF $F_M^{I=1}(k^2)$, nor on the mass of M- meson.

The k^2- dependence of the asymmetry $\widetilde{A_\ell}$ is presented in Fig.4. For x=1/2, i.e. for k^2=1.58 GeV^2, $\widetilde{A_\ell}$ is changing the sign.

At the $\rho(770)$ $\widetilde{A_\ell}(m_\rho^2)$ = -0.46 and this value is two times smaller than the maximum possible value $\widetilde{A_\ell}$= -0.90, which is achieved for $k^2 = 4m_\pi^2$. However, to make use of the latter results seems to be difficult as there is suppressing factor $(1-\frac{R}{x})^{3/2} = \beta^3$ in (40).

At the threshold of K^-K^0 creation (i.e. for x=0.316) the asymmetry $\widetilde{A_\ell}$= -0.225.

Asymmetries of $\vec{\tau} \to \nu_\tau + \rho$ decays are already effectively used in measurement of a polarization of τ- leptons, created in the $e^+e^- \to Z^0 \to \tau^+\tau^-$ process [13].

It is well known that analyzing potency of the $\vec{\tau}^- \to \nu_\tau\rho^-$ decay is increased, if simultaneously a polarization of ρ^-- meson is registered [14].

5 Formfactors of pseudoscalar mesons in unitary and analytic vector-meson-dominance model

In order to predict the effective mass spectrum behaviour (19) of the M^-M^0 system created at the $\tau^- \to \nu_\tau M^-M^0$ decay and the E - dependence of the decay probability (25) of the same process as well, we employ the unitary and analytic VMD model of the pion and kaon electromagnetic structure [4,15] the parameters of which (with clear physical meaning) are fixed in fitting procedure of existing data, essentially on the $e^+e^- \to M\bar{M}$ processes.

The starting point in a construction of the unitary and analytic model is the canonical VMD model expression for the EM FF's

$$F_M(t) = \sum \frac{f_{vM\bar{M}}}{f_v}\frac{m_v^2}{m_v^2 - t} \qquad (42)$$

where $f_{vM\bar{M}}$ and f_v are vector-meson-meson-antimeson and universal vector-meson coupling constants, respectively, and m_v is the vector-meson mass.

Let t_0 be the lowest threshold for hadron-antihadron production in electron-positron annihilation and t_1 the effective threshold representing contributions of other, specified by the unitarity condition, thresholds of hadronic productions to $F_M(t)$. As it is clearly seen that equation (42) does not contain any information related to these thresholds, which characterize, though not completely, the analyticity of FF's $F_M(t)$. On the other hand, according to (42) the only singularities of $F_M(t)$ are simple poles on the real axis, corresponding to vector mesons, considered to be stable particles. The instability of the latter can be taken into account by including the vector-meson widths Γ_v transforming simply the denominator of (42) into the Breit-Wigner form. However, this "ad hoc" shift of the poles into the complex plane t has no relationship with the threshold cuts in the structure of EM FF. Moreover, such complex poles destroy the important property of real analyticity of FF exhibited by the expression for $F_M(t)$ in eq. (42).

The canonical VMD expression (42) can nevertheless be modified firstly to incorporate implicitly the analytic cut structure represented by the thresholds t_0 and t_1. Secondly the modification can be so carried out that also by an incorporation of the nonzero values of the vector-meson widths $\Gamma_v \neq 0$ the real analytic property and the asymptotic behaviour as predicted (up to logarithmic corrections) by QCD are preserved.

With the aim of that modification one operates the change of variables

$$t = t_0 - 4\frac{t_1 - t_o}{(U - 1/U)^2} \tag{43}$$

in eq. (42), which leads to the form as follows

$$F_M[U(t)] = \left(\frac{1-U^2}{1-U_N^2}\right)^2 \cdot \sum_v \frac{(U_N - U_{v_0})(U_N + U_{v_0})(U_N - 1/U_{v_0})(U_N + 1/U_{v_0})}{(U - U_{v_0})(U + U_{v_0})(U - 1/U_{v_0})(U + 1/U_{v_0})} \frac{f_{vM\bar{M}}}{f_v} \tag{44}$$

where U_N is the value of U for t=0 and U_{v_0} the value of U for $t = m_v^2$, i.e. Γ_v=0 and therefore the subindex 0 in U_{v_0}.

The expansion (44) is in a factorized form, where the overall factor $(1-U^2)^2$ outside of the sum over the vector mesons completely determines the asymptotic behaviour of $F_M[U(t)]$ for $t \to \pm\infty$. The latter is so because $t \to \pm\infty$ on the first sheet of the Riemann surface corresponds to $U \to -1$, according to eq. (43). The terms under the sum determine the analytic behaviour of $F_M[U(t)]$ and they do not contribute to the asymptotic behaviour as for $t \to \infty$ they turn to be real constants.

Further, provided that there are r and s vector mesons ($r + s = v$ in the sum of (44)) with masses remaining the inequalities $t < m_r^2 < t_1$ and $m_s^2 > t_1$ to be true, respectively, then one can derive the following relations

$$U_{r_0} = -U_{r_0}^* \quad U_{s_0} = 1/U_{s_0}^*, \tag{45}$$

where stars mean a complex conjugation. These relations can be used to rewrite (44) into the form which exhibits the reality condition $F_M^*(t) = F_M(t^*)$ explicitly.

Taking all this into account and introducing nonzero vector-meson widths $\Gamma_v \neq 0$ by means of the replacements

$$m_v^2 \to (m_v - i\Gamma_v/2)^2 \quad \text{i.e.} \quad U_{v_0} \to U_v, \tag{46}$$

one gets finally the unitary and analytic model

$$F_M[U(t)] = \left(\frac{1-U^2}{1-U_N^2}\right)^2 \cdot \left\{\sum_r \frac{(U_N - U_v)(U_N - U_v^*)(U_N - 1/U_v)(U_N - 1/U_v^*)}{(U - U_v)(U - U_v^*)(U - 1/U_v)(U - 1/U_v^*)} \frac{f_{rM\bar{M}}}{f_r} + \sum_s \frac{(U_N - U_v)(U_N - U_v^*)(U_N + U_v)(U_N + U_v^*)}{(U - U_v)(U - U_v^*)(U + U_v)(U + U_v^*)} \frac{f_{sM\bar{M}}}{f_s}\right\} \tag{47}$$

of the pseudoscalar meson EM FF's, which depend on parameters like m_v, Γ_v, $f_{vM\bar{M}}/f_v$ to be determined in a comparison of (47) with existing experimental information.

The application of (47) to the FF of π^+ ($F_{\pi^-}[U(t)] = -F_{\pi^+}[U(t)]$) leads to the following form

$$F_{\pi^+}^{I=1}[U(t)] = \left(\frac{1-U^2}{1-U_N^2}\right)^2 \frac{(U - U_z)(U_N - U_p)}{(U_N - U_z)(U - U_p)} \cdot \left\{\sum_r \frac{(U_N - U_\rho)(U_N - U_\rho^*)(U_N - 1/U_\rho)(U_N - 1/U_\rho^*)}{(U - U_\rho)(U - U_\rho^*)(U - 1/U_\rho)(U - 1/U_\rho^*)} \frac{f_{\rho\pi\pi}}{f_\rho} + \sum_{v=\rho',\rho''\rho'''} \frac{(U_N - U_v)(U_N - U_v^*)(U_N + U_v)(U_N + U_v^*)}{(U - U_v)(U - U_v^*)(U + U_v)(U + U_v^*)} \frac{f_{vM\bar{M}}}{f_v}\right\}, \tag{48}$$

where the normalized term $\frac{(U-U_z)(U_N-U_p)}{(U_N-U_z)(U-U_p)}$ (U_z is a zero and U_p is a pole on the positive real axis in the U-plane) represents effectively [16] a very important contribution from the left-hand cut of the second Riemann sheet, which appears as a consequence of the presence of the $\pi\pi$ scattering amplitude in the pion FF unitarity condition.

For kaon EM FF's, as a consequence of particular transformation properties of the kaon EM current with respect to rotations of the isotopic spin space, one can use expressions (1) and (47) is applied [4,15] directly to the isoscalar $F_K^{(s)}(t)$ and the isovector $F_K^{(v)}(t)$ parts as follows

$$F_K^{(s)}[V(t)] = \left(\frac{1-V^2}{1-V_N^2}\right)^2 \cdot$$
$$\cdot\left[\sum_{s=\omega,\phi}\frac{(V_N-V_s)(V_N-V_s^*)(V_N-1/V_s)(V_N-1/V_s^*)}{(V-V_s)(V-V_s^*)(V-1/V_s)(V-1/V_s^*)}\frac{f_{sM\bar{M}}}{f_r}+\right.$$
$$\left.+\frac{(V_N-V_{\phi'})(V_N-V_{\phi'}^*)(V_N+V_{\phi'})(V_N+V_{\phi'}^*)}{(V-V_{\phi'})(V-V_{\phi'}^*)(V+V_{\phi'})(V+V_{\phi'}^*)}\frac{f_{\phi'K\bar{K}}}{f_{\phi'}}\right] \tag{49}$$

$$F_K^{(v)}[U(t)] = \left(\frac{1-U^2}{1-U_N^2}\right)^2 \cdot$$
$$\cdot\left[\frac{(U_N-U_\rho)(U_N-U_\rho^*)(U_N+U_\rho)(U_N+U_\rho^*)}{(U-U_\rho)(U-U_\rho^*)(U+U_\rho)(U+U_\rho^*)}\frac{f_{\rho K\bar{K}}}{f_\rho}+\right.$$
$$\left.+\sum_{v=\rho',\rho'',\rho'''}\frac{(U_N-U_v)(U_N-U_v^*)(U_N+U_v)(U_N+U_v^*)}{(U-U_v)(U-U_v^*)(U+U_v)(U+U_v^*)}\frac{f_{vK\bar{K}}}{f_v}\right]. \tag{50}$$

Though the pure isovector π^+ EM FF and the isovector part of the kaon EM FF's have very similar structure, their behaviour is different, as it is presented in Fig.5 a,b.

6 Discussion of numerical results

Assuming the k^2- dependence of the $F_M^{I=1}(k^2)$ for pions and kaons to be known, one can now predict a behaviour of the effective mass spectrum of pions

$$\frac{1}{\Gamma}\frac{d\Gamma(\tau\to\nu_\tau\pi^-\pi^0)}{dx} = 1/2(1+2x)(1-x)^2(1-\frac{R}{x})^{3/2}2|F_\pi^{I=1}(k^2)|^2 \tag{51}$$

and a behaviour of the effective mass spectrum of kaons

$$\frac{1}{\Gamma}\frac{d\Gamma(\tau^-\to\nu_\tau K^-K^0)}{dx} = 1/4(1+2x)(1-x)^2(1-\frac{R}{x})^{3/2}2|F_K^v(k^2)|^2. \tag{52}$$

Their graphical presentation is in Fig.6 a,b and Fig.7, respectively. One can see immediately from there that the effective mass spectrum of pions takes considerably larger values than the effective mass spectrum of kaons, but its peak is narrower. Both properties are predestined by the interval of values of $F_M^{I=1}(k^2)$, which contributes to the behaviour of (51) and (52).

Further, taking into account the explicit form for the $g_{1,2}(x)$ functions, given by (20), and the isovector parts of the pion and kaon EM FF's given by (48) and (50),

one can calculate integrals $I_{1,2}$ numerically, to be for pions

$$I_1^\pi = 0.30217, \quad I_2^\pi = 1.233267, \tag{53}$$

and for kaons

$$I_1^K = 2\cdot 0.00352, \quad I_2^K = 2\cdot 0.00354. \tag{54}$$

As a result the $\cos\vartheta$- dependence (22) of the differential probability of the $\tau^-\to\nu_\tau M^-M^0$ process is specified. Furthermore, by using (23) the ratio of total probabilities

$$\Gamma(\tau^-\to\nu_\tau\pi^-\pi^0)/\Gamma_e = 1.42612 \tag{55}$$

and

$$\Gamma(\tau^-\to\nu_\tau K^-K^0)/\Gamma_e = 0.00940 \tag{56}$$

are determined automatically to be compared with values of other estimates $\Gamma_\pi/\Gamma_e = 1.32\pm0.05$ [17], $\Gamma_\pi/\Gamma_e = 1.31\pm0.04$ [18], $\Gamma_\pi/\Gamma_e = 1.21$ [19] or experimental measurements $\Gamma_\pi/\Gamma_e = 1.40\pm0.03$ [20] and $\Gamma_K/\Gamma_e = 0.0161\pm0.0068$ [11]. One can see from (55) and (56) that our estimates of $\Gamma\to\nu_\tau M^-M^0/\Gamma_e$ ratios are in an excellent agreement with existing experimental values. Moreover, our pion EM ff model confirms the experimental tendency to increase the branching ratio of the $\tau^-\to\nu_\tau\pi^-\pi^0$ decay. Other theoretical estimates of $\Gamma(\tau^-\to\nu_\tau\pi^-\pi^0)/\Gamma_e$ give smaller values.

By means of the $F_M^{I=1}(k^2)$ presented in Fig.5a,b and the integral expression (25), the E- dependence of the decay probability of the $\tau^-\to\nu_\tau\pi^-\pi^0$ and $\tau^-\to\nu_\tau K^-K^0$ processes is predicted (see Fig.8).

Finally integrals I_3 and I_4 for pions and kaons, given by (38) and determining $\cos\vartheta$- dependence of the differential probability (37) of the decays $\tau^-\to\nu_\tau M^-M^0$ of polarized τ- leptons, are determined to be

$$I_3^\pi = 0.67060 \quad I_4^\pi = 1.83700 \tag{57}$$

and

$$I_3^K = 0.00496 \quad I_4^K = 0.01060. \tag{58}$$

7 Conclusions

The analysis carried out in this paper has shown, that the CVC hypothesis substantially determines the main properties of the $\tau^- \to \nu_\tau M^- M^0$ decays, simplifies the decay amplitudes and gives a series of predictions, which do not depend on dynamics of the decay, i.e. on the behaviour of $F_M^{I=1}(k^2)$.

We have to stress, that the decays $\tau^- \to \nu_\tau M^- M^0$ can appear to be very useful in a solution of the whole set of global problems of the physics of τ- leptons and the physics of weak interactions of leptons with hadrons.

One of a such problem is test of symmetry properties of weak interactions of τ-leptons with hadrons, which has to include an investigation of the following questions:

- the CVC hypothesis;
- the hypothesis of an absence of the second class currents.

It is clear, that there are no contributions of the second class currents into $\tau^- \to \nu_\tau \pi^- \pi^0$ and $\tau^- \to \nu_\tau K^- K^0$ decays as they can not be constructed at all in this case. However, the decay $\tau^- \to \nu_\tau \pi^- \eta$ is completely determined by the second class currents.

- T- invariance of the lepton current of $\nu_\tau \to \tau^-$ transition (by means of eventual deviations from the relation $|a| = |Re\ a|$, which is true only in the case of the pure real constant a);
- a role of possible charged Higgs bosons;
- a locality of weak interactions of the lepton current to be responsible for $\nu_\tau \to \tau^-$ transition with hadrons;
- a size of radiative corrections to the processes under consideration.

In the first stage of an experimental investigation of the $\tau \to \nu_\tau M^- M^0$ decays the main task will be a reliable determination of the isovector parts of the pion and kaon EM FF's at the region of time-like values of the momentum transfer squared. These results can substantially improve proposed models for a description of the EM structure of pions and kaons.

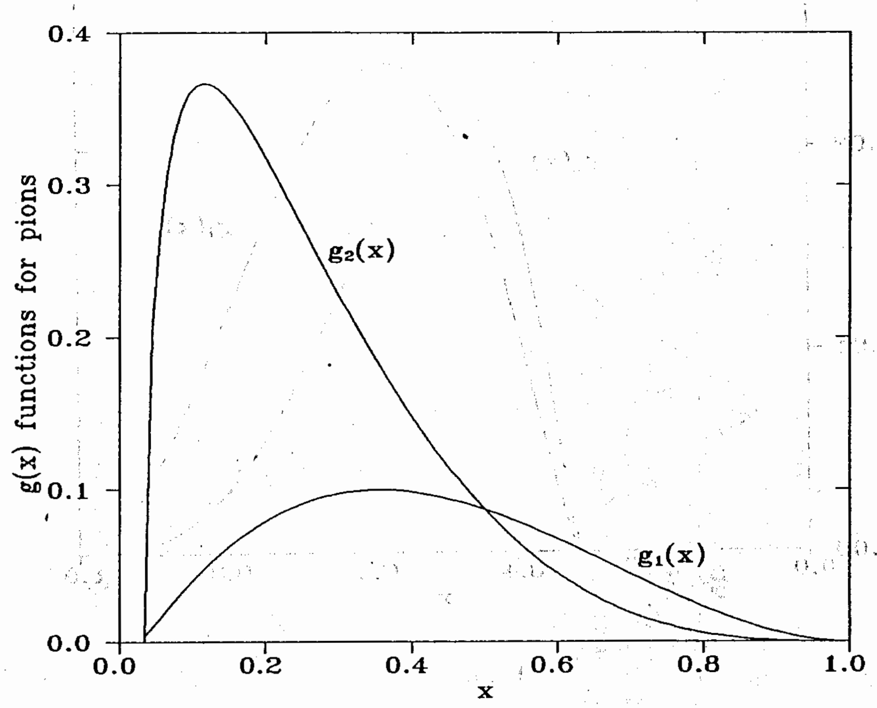

Fig. 1a

Fig.1. The angle distribution properties of M^-- mesons in $\tau^- \to \nu_\tau M^- M^0$ decays, which do not depend on a concrete parametrization of $F_M^{I=1}(k^2)$, are characterized by two functions $g_1(x)$ and $g_2(x)$. Both of them are turning to be zero at the borders of the physical region of k^2, however, their maxima, which depend on m_M^2, are in different places (for π-meson see (a) and for K-meson (b)).

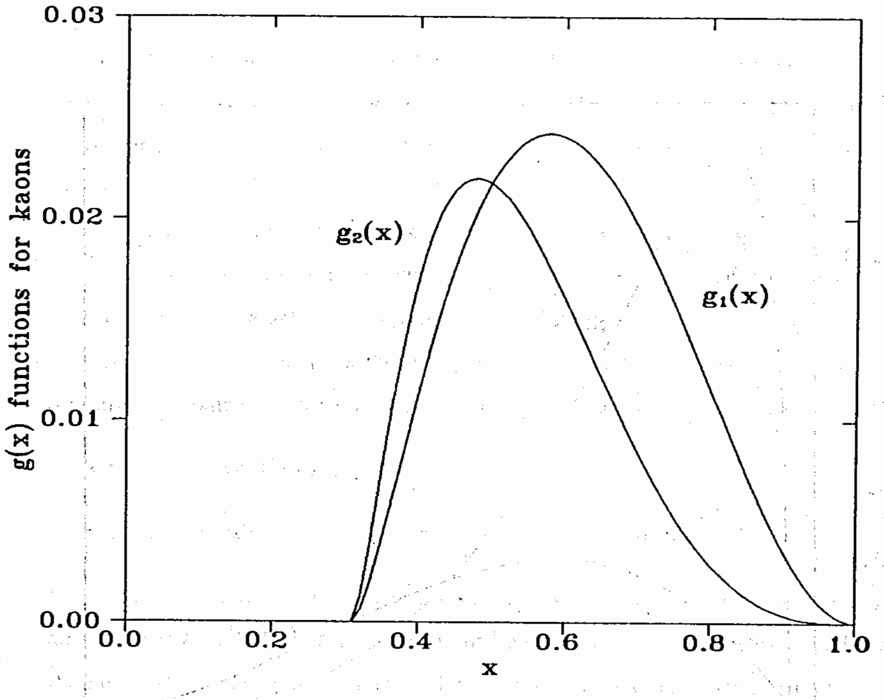

Fig. 1b

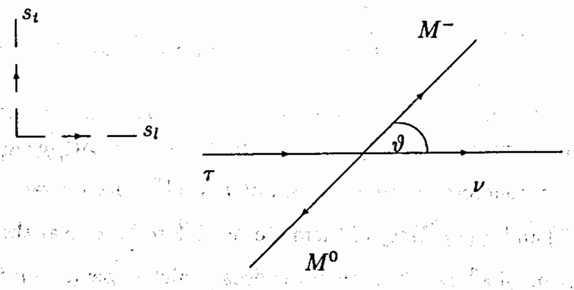

Fig.2. Three-momenta of particles under consideration at the c.m. system of M^-M^0.

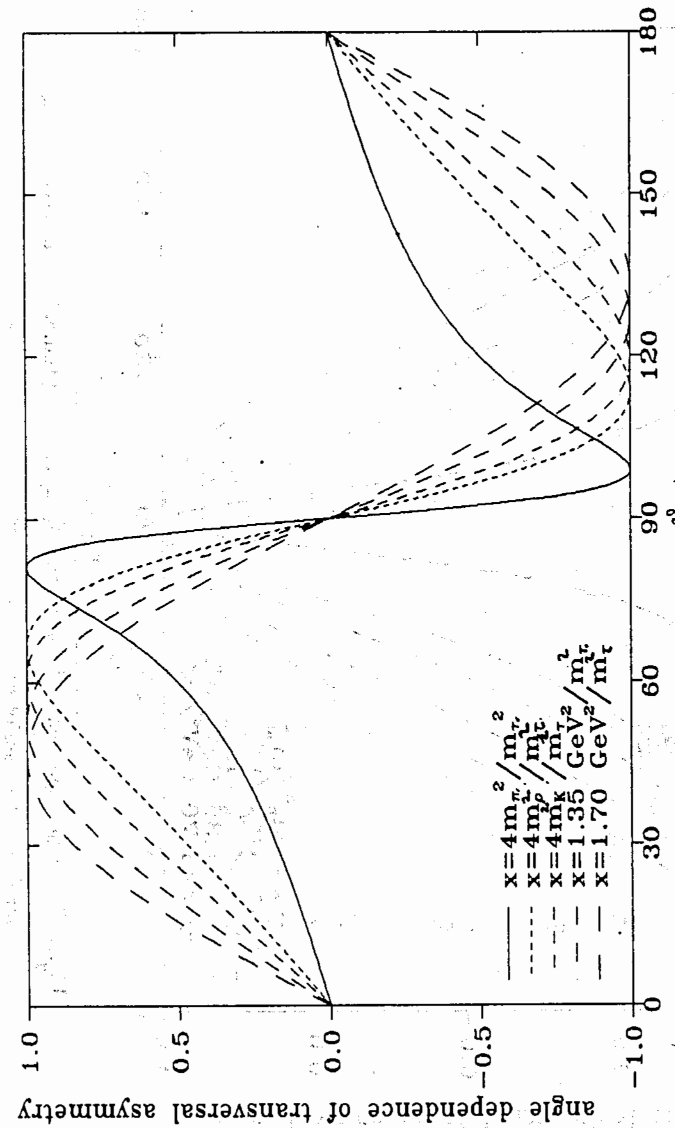

Fig. 3a

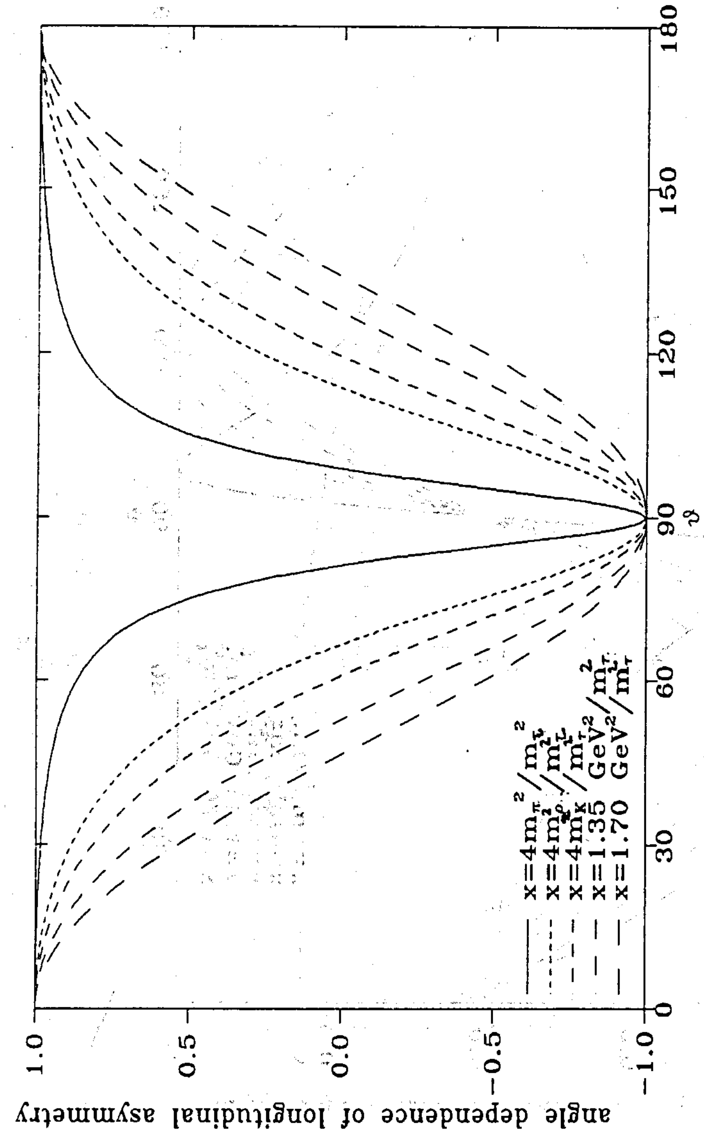

Fig. 3b

Fig.3. The angle dependence of transversal A_t (a) and longitudinal A_ℓ (b) asymmetrics for different values of k^2.

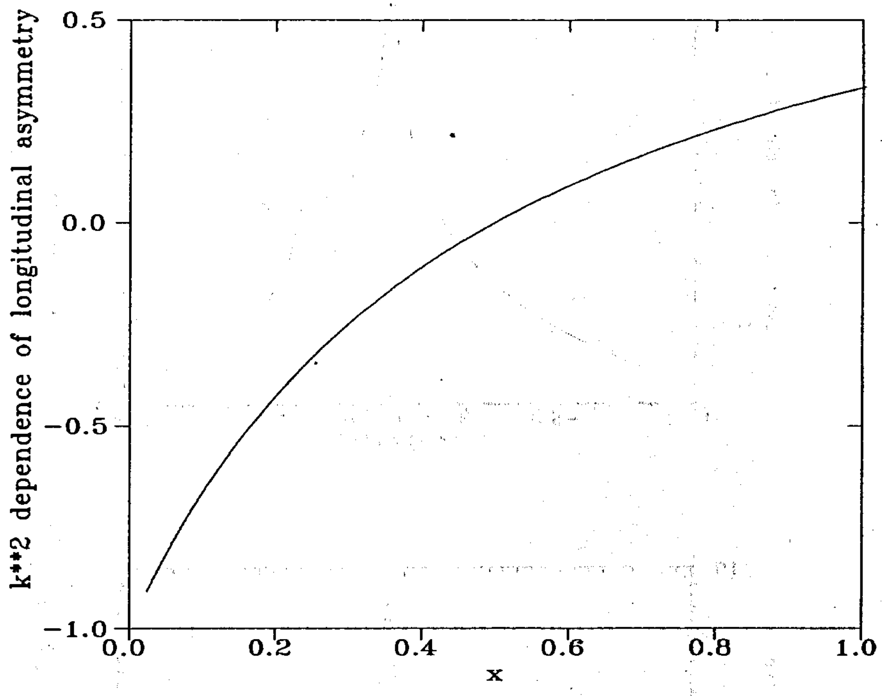

Fig.4. The k^2- dependence of the longitudinal asymmetry $\widetilde{A_\ell}$.

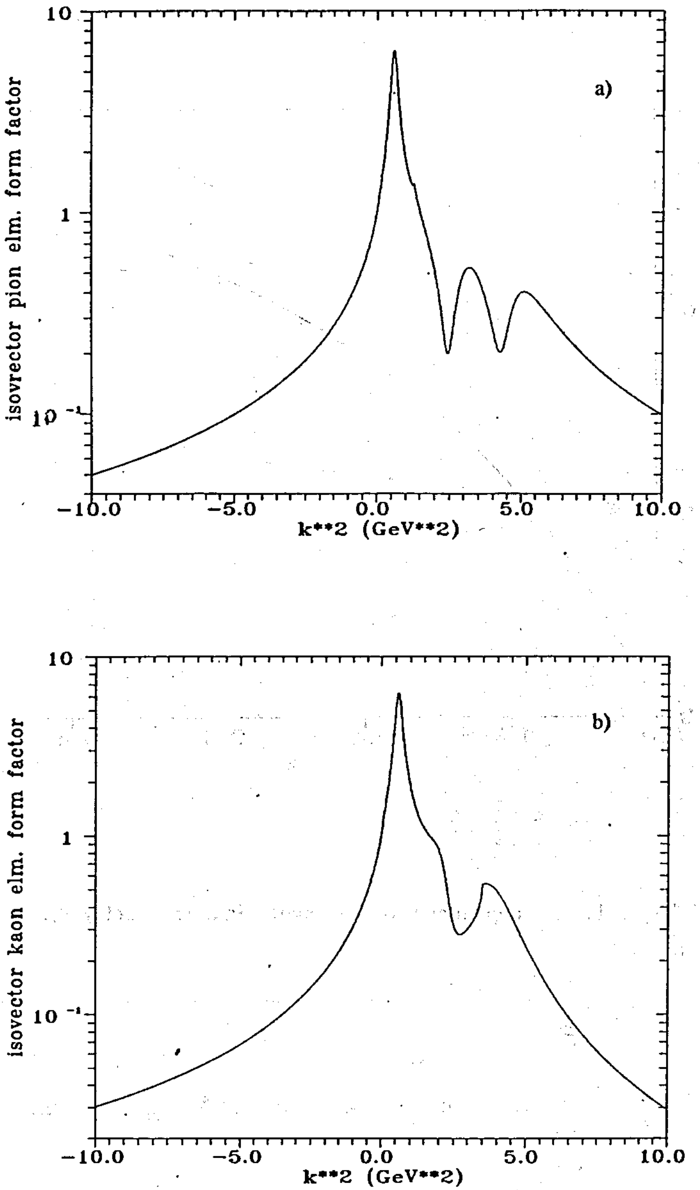

Fig.5. The pure isovector π^+ EM FF (a) and the isovector part of the kaon EM FF (b).

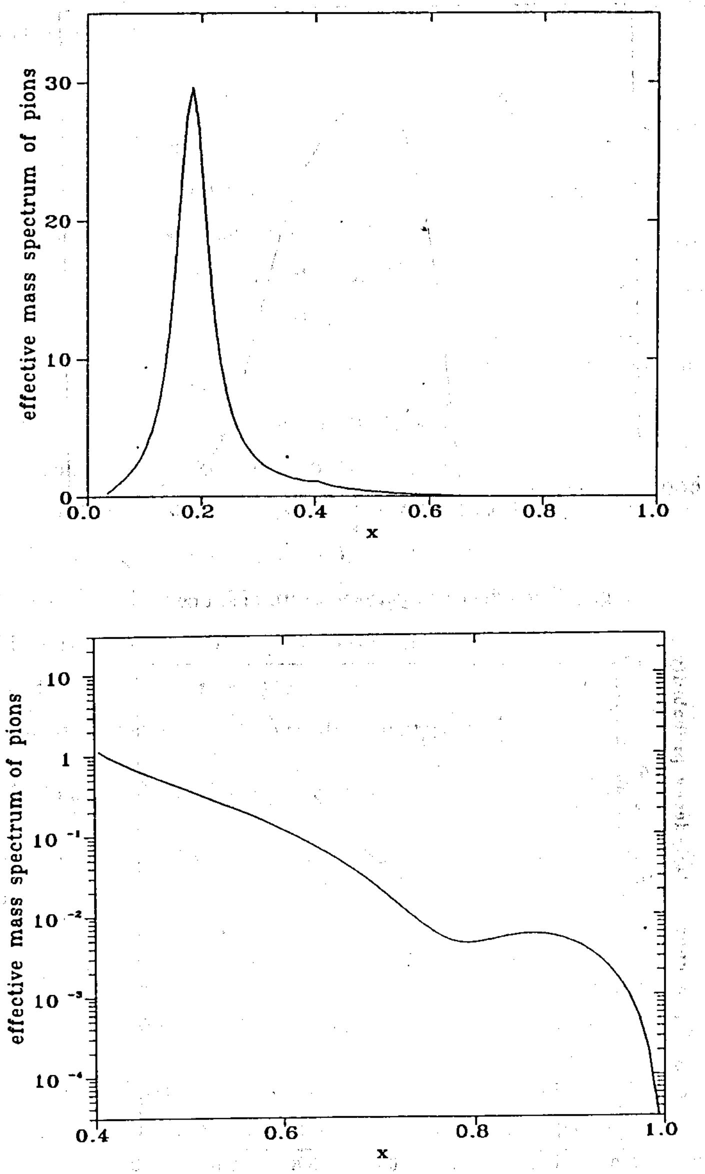

Fig.6. The effective mass spectrum of pions (a). In (b) the interval $0.4 < x < 1.0$ in logaritmic scale is presented, from which the higher ρ- contribution is clearly seen.

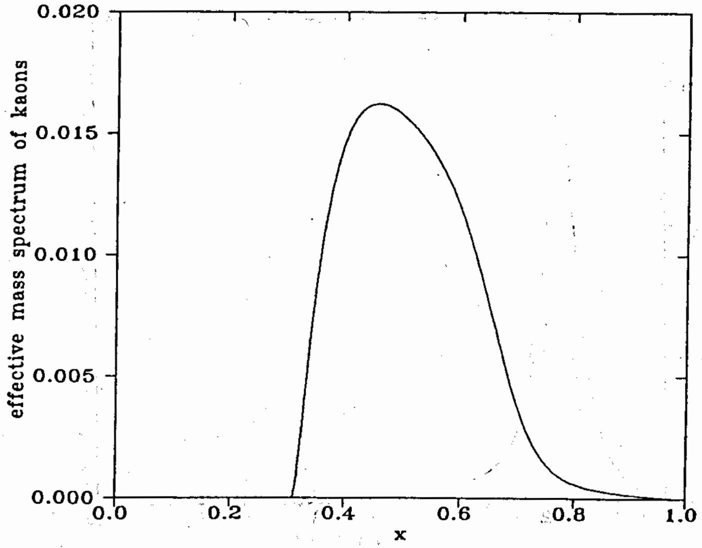

Fig.7. The effective mass spectrum of kaons.

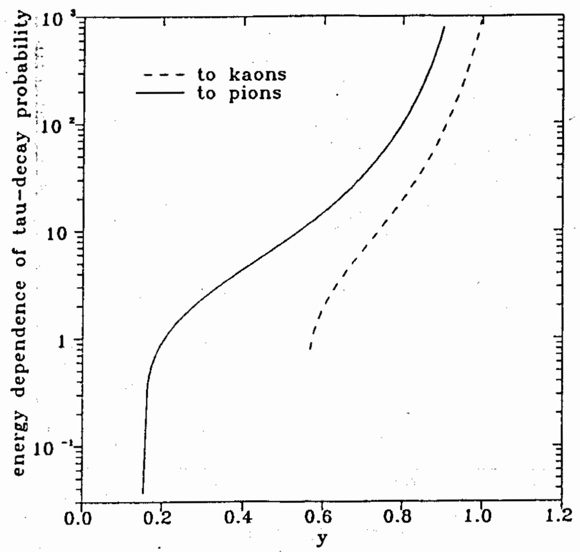

Fig.8. The energy dependence of the decay probability of the $\tau^- \to \nu_\tau \pi^- \pi^0$ and $\tau^- \to \nu_\tau K^- K^0$ processes.

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Изотопическая структура электромагнитного $e^+e^- \to M\overline{M}$ и $\tau^- \to \nu_\tau M^0 M^-$ процесса

Рассматривается распад τ-лептона на два псевдоскалярных мезона, так как этот распад дает полную информацию об изовекторной части соответствующего формфактора. Получена общая структура распределения Далитца, в том числе и углового распределения, с учетом гипотезы частичного сохранения векторного тока. Специальное внимание уделяется поляризационным эффектам. Вычислены различные независимые асимметрии распадов поляризованных τ-лептонов. Используя унитарную и аналитическую ВМД модель для описания электромагнитной структуры псевдоскалярных мезонов, предсказывается спектр масс и энергетическая зависимость вероятностей распада τ-лептона.

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Isotopic Structure of the Electromagnetic Current of $e^+e^- \to M\overline{M}$ and $\tau^- \to \nu_\tau M^0 M^-$ Processes

The τ-decay into two pseudoscalar mesons, as a source of a complete information on the isovector part of the corresponding electromagnetic form factors, is investigated in detail. A general structure of the Dalitz-distribution and the angular distribution, particularly with regard to the consequences of the conserved-vector-current (CVC) hypothesis, is derived. A special attention to polarization effects is devoted and various independent asymmetries for decays of polarized τ-leptons are calculated. By using the unitary and analytic vector-meson-dominant (VMD) model of the pseudoscalar meson electromagnetic structure, the effective mass spectra and the energy dependence of the τ-decay probabilities are predicted.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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