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RHO SELF-ENERGY IN THE ISOSPIN-ASYMMETRIC PION MEDIUM

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I. INTRODUCTION

The fundamental question of how the hadron properties are modified in hot and dense nuclear matter is the currently central problem in relativistic heavy-ion collisions. The rho meson dynamics is crucially important here because it may be related to the observables. Really, one hopes to explore the dilepton production in the $\pi^+\pi^-$ annihilation [1] because the pion electromagnetic form factor is almost completely dominated by the ρ meson below an invariant mass of about 1 GeV [2], which strengthens the well known and widely used vector dominance model [3].

The rho properties under extreme condition are ambiguous, especially in the region below the chiral phase transition. Above the critical temperature, which probably coincides with the deconfinement temperature, the ρ meson should disappear from the hadronic spectrum of excitations as predicted by both the chiral mean field models [1] and lattice calculations [4]. The ρ properties below the chiral phase transition depend on the physical picture of the "matter" constituents and their interactions with the rho meson [5]. The models based on quark degrees of freedom, such as QCD sum rules [6], the effective Lagrangians of the Nambu-Jona-Lasinio type [7], or the models based on the conventional hadronic degrees of freedom [8–10] show different qualitative and quantitative predictions of the in-medium modifications of the rho properties.

For a deeper understanding of the role of the conventional hadronic interactions on the ρ property modification at extreme conditions, which should be considered as background for more exotic interactions, it seems to be important to study the simplest system - a dense and hot pion gas of a strongly interacting matter with a small baryon density, which is often expected to be produced in the central region in relativistic heavy-ion collisions. Gale and Kapusta [11] have analyzed the temperature modification of the ρ self-energy in the one-loop order (order g_ρ^2) at a vanishing pion chemical potential. They found a modest increase in the ρ width and mass with temperature, which means that if a high energy experiment shows a substantial modification of the dilepton spectrum with an invariant mass in the ρ region, it may be some indication of a more exotic interaction.

The model of Gale and Kapusta is extended by Koch [13] who considers the pion system in a chemical non-equilibrium state described by a positive chemical potential μ_π. The chemical potential is associated with the total pion density of the pion gas, and it is supposed that μ_π has the same value for π^+ and π^-. Previously, this idea has been put forward by Kataja and Ruuskanen [14] for explanation of the observed enhancement of pions at low transverse momentum in relativistic heavy ion collisions [15] as a consequence of the Bose-Einstein statistics. In Ref. [13], it is found that the incorporation of the pion chemical potential μ_π gives a strong enhancement of the muon pair yield in the low invariant mass region, provided the lepton pairs are produced predominantly via pion annihilation. This might serve as explanation of the so-called dilepton excess [16] observed in the present CERN-SPS heavy-ion experiments [17].

In principle, one can consider an additional degree of freedom in the conventional ρ-π dynamics, namely a possible non-zero total electric or isospin charge of the pionic system. Generally, there is no restriction on the production of a hadronic fireball with a net electric charge in the first deep-inelastic stage in a relativistic heavy-ion collision. Moreover, some experimental data [18] and theoretical speculations [19] point out this possibility. This may be a consequence of the proton-neutron asymmetry of the colliding heavy ions, and the asymmetry increases with increasing atomic weight of the colliding ions. The electric charge of a pionic system is controlled by the "charge" chemical potential μ_Q which should not be confused with the chemical potential used by Koch $\mu_\pi = \mu_\pi^0$ that is a measure of the chemical equilibrium breaking. The chemical potentials for positive and negative pions are $\mu_{\pi^\pm} = \mu_\pi^0 \pm \mu_Q$.

Incorporation of the finite μ_Q into the theory leads to non-trivial effects as, for example, the dilepton enhancement at $2m_\pi$ [20], sharp modification of the Golstone modes [21], and others.

Here, we explore this additional degree of freedom. Our work may be viewed as an extension of the results of Gale and Kapusta [11] to the ρ meson self energy at finite temperature to finite values of the chemical potential μ_Q. Crucial questions of how does the isospin asymmetric system may be produced in experiment and the role of the baryonic degrees of freedom at high temperature are beyond the scope of our present consideration which may be considered as necessary part of the future theory of the pion – rho medium in a hot and dense baryonic isospin–asymmetric environment.

In our recent paper [22], we restrict ourselves to the simplest case when the vector field is taken in its rest frame with $\mathbf{p} = 0$. We find an increase in both the ρ meson mass and the width with increasing temperature and chemical potential μ_Q. In this paper, we generalize our approach to an arbitrary and finite value of the rho momentum. We evaluate the ρ meson self energy by using, as starting point, the conventional π-ρ effective Lagrangian and the functional integral representation for the partition function, which is familiar to the relativistic quantum field theory at finite temperature and charge chemical potential. We pay attention to the gauge invariance of the rho polarization operator and analyze its dependence on the invariant mass M and the spatial momentum $|\mathbf{p}|$ of the ρ meson. We show that this dependence leads to a sensible modification of the shape of the thermal dilepton production rate. Moreover, at large μ_Q we find a definite difference in the pole positions and in the value of the imaginary parts of the self-energy for different polarization states as the functions of M and $|\mathbf{p}|$. This leads to the perceptible asymmetry of the dielectron production when the momentum $\mathbf{t} = \mathbf{p}_+ - \mathbf{p}_-$ is perpendicular or parallel to $\mathbf{p}$, where $\mathbf{p}_\pm$ are the momenta of $e^\pm$.

II. THE MODEL

Our starting point is the effective Lagrangian $\mathcal{L}$ which describes a system of charged pions and neutral vector ρ mesons

$$\mathcal{L} = \frac{1}{2}(D^\nu \phi)^* D_\nu \phi - \frac{1}{2} m_\pi^2 \phi \phi^* - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^2, \tag{1}$$

where ϕ is the complex charged pion field, ρ stands for the vector field with the strength $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$, and $D_\nu = \partial_\nu - i g_\rho \rho_\nu$ is the covariant derivative; μ and ν are the Lorentz indices. The Hamiltonian of the system is related to the Lagrangian of Eq.(1) in the usual way

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi)} \partial_0 \varphi - \mathcal{L} \tag{2}$$

with $\varphi = (\phi, \phi^*, \rho)$. The reference for what follows, at finite temperature $T \neq 0$ and $\mu_{\pi^\pm} = 0$, is the paper of Gale and Kapusta [11].

Let us consider the case when the system admits some conserved electric or isospin charge. We consider the case $\mu_\pi^0 = 0$ and concentrate on the incorporation of μ_Q. The incorporation of the chemical potential

μ_Q leads to a transformation of the Hamiltonian which we use for the calculation of the partition function

$$\mathcal{H} \to \mathcal{H} - \mu_Q J_0, \tag{3}$$

where J_0 is the time component of Noether's current

$$J_\nu \equiv i\frac{1}{2}(\phi^* D_\nu \phi - \phi(D_\nu \phi)^*). \tag{4}$$

The ρ meson propagator in a medium is related to the self-energy

$$\left(D^{-1}\right)^{\mu\nu} = \left(D_0^{-1}\right)^{\mu\nu} + \Pi^{\mu\nu}, \tag{5}$$

where $D_0^{\mu\nu}$ is the free propagator.

In the Euclidean space, the rho meson self-energy may be obtained with the help of the partition function having a functional integral representation of the form [23]

$$\mathcal{Z} = \int \mathcal{D}\pi_\varphi \int_{periodic} \mathcal{D}\varphi \exp\left\{\int_0^\beta d\tau \int_V d\mathrm{x}\left(i\pi_\varphi \frac{\partial\varphi}{\partial\tau} - \mathcal{H} + \mu_Q J_0\right)\right\},$$

where again $\varphi = (\phi, \phi^*, \rho)$, and $\pi_\varphi = \partial\mathcal{L}/\partial(\partial_0\varphi)$ are the relevant conjugate momenta. The integration over π_φ gives

$$\mathcal{Z} = \int_{periodic} \tilde{\mathcal{D}}\rho \mathcal{D}\phi \mathcal{D}\phi^* e^{S_0 + S_{int}}, \tag{6}$$

where $S_0 = S_{0\pi} + S_{0\rho}$ describes the non-interaction part of the total effective action, and S_{int} corresponds to the interaction part, i.e.,

$$\begin{aligned}
S_{0\pi} &= \int_0^\beta d\tau \int_V d\mathrm{x}\Big(\frac{1}{2}|\partial\phi|^2 - \frac{1}{2}(m_\pi^2 - \mu_Q^2)|\phi|^2 - \mu_Q j_0\Big), \\
S_{0\rho} &= \int_0^\beta d\tau \int_V d\mathrm{x}\Big(-\frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho^2 - \frac{1}{2\alpha}(\partial_\mu\rho^\mu)^2\Big); \\
S_{int} &= \int_0^\beta d\tau \int_V d\mathrm{x}\Big(\frac{1}{2}g_\rho^2\rho^2|\phi|^2 + g_\rho(\rho_\mu j^\mu + \mu_Q\rho_0|\phi|^2)\Big)
\end{aligned} \tag{7}$$

where $\tilde{\mathcal{D}}\rho = \mathcal{D}\rho \cdot \det(\partial_4)$ ($\det\partial_4 \equiv \det\left(\frac{\partial\,\partial_\mu\rho^\mu}{\partial\rho_4}\right)$) and $j_\mu = i/2(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*)$; $i\partial_0 = \partial_\tau$, $\rho_0 = i\rho_4$, etc. $S_{0\rho}$ includes the gauge fixing term. We use the Landau gauge with $\alpha \to 0$.

Expanding Eq.(6) in power series in S_{int} and taking the logarithm of both sides, we get in the second order of g_ρ

$$\begin{aligned}
&\ln\mathcal{Z} = \ln\mathcal{Z}_0 + \ln\mathcal{Z}_{int}, \\
&\ln\mathcal{Z}_{int} \simeq \\
&\frac{1}{2}g_\rho^2\Big(< \int d\tau d\mathrm{x}\rho^2|\phi|^2 >_0 + < (\int d\tau d\mathrm{x}(\rho_\nu j_\nu + \mu_Q\rho_0|\phi|^2))^2 >_0\Big),
\end{aligned} \tag{8}$$

where

$$\mathcal{Z}_0 = \int \mathcal{D}\varphi e^{S_0}; \quad < R >_0 \equiv \mathcal{Z}_0^{-1}\int \mathcal{D}\varphi\, R\, e^{S_0}. \tag{9}$$

The polarization operator $\Pi_{\mu\nu}$ is related to the partition function as follows

$$\Pi_{\mu\nu} = -2\frac{\delta\, ln\mathcal{Z}_{int}}{\delta D_0^{\mu\nu}}, \tag{10}$$

III. RHO PROPAGATOR

The calculation of $\ln\mathcal{Z}_{int}$ may be performed by utilizing the methods of Ref. [23] and textbook recipes [24]. After some tedious algebraic exercises we get the following expression for $\Pi_{\mu\nu}$

$$g_\rho^{-2}\Pi^{\mu\nu}(\mu_Q, p) = \delta^{\mu\nu} I_1 - \frac{1}{4}I_2^{\mu\nu} + 4\mu_Q\left(\delta_4^\mu I_3^\nu + \delta_4^\nu I_3^\mu\right) \tag{11}$$

where

$$\begin{aligned}
I_1 &= T\sum_{k_4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{A(k)}\Big(1 + \frac{A(k)^2 - B(k)^2}{A(k)^2 + B(k)^2}\Big) \\
I_2^{\mu\nu} &= T\sum_{k_4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{4k^\mu k^\nu - 4\delta_4^\mu\delta_4^\nu\mu_Q^2}{A_+A_-}\Big(1 + \mathcal{F}(\mu_Q, p, k)\Big) \\
I_3^\nu &= T\sum_{k_4}\int \frac{d^3\mathbf{k}}{(2\pi)^3}k^\nu\frac{B_+A_- + B_-A_+}{(A_+^2 + B_+^2)(A_-^2 + B_-^2)}
\end{aligned} \tag{12}$$

In the above, the fourth component of the momentum four-vectors is the Matsubara frequency, i.e., k_4 or $p_4 = 2\pi T\times$integer. The functions $A_\pm$ and $B_\pm$ depend on the chemical potential as

$$A_\pm = (k_4 \pm \frac{1}{2}p_4)^2 + \omega_\pm^2 - \mu_Q^2, \quad B_\pm \equiv -2\mu_Q(k_4 \pm \frac{1}{2}p_4), \quad \omega_\pm^2 = (\mathbf{k} \pm \frac{1}{2}\mathbf{p})^2 + m_\pi^2,$$

and $A(k) = A_\pm(p=0)$, $B(k) = B_\pm(p=0)$. The function $\mathcal{F}(\mu_Q, p, k)$ is a combination of $A_\pm$ and $B_\pm$

$$\mathcal{F}(\mu_Q,p,k) \equiv \frac{A_+^2 - B_+^2}{A_+^2 + B_+^2} + \frac{A_-^2 - B_-^2}{A_-^2 + B_-^2} + \frac{\left(A_+^2 - B_+^2\right)\left(A_-^2 - B_-^2\right) - 4A_+A_-B_+B_-}{\left(A_+^2 + B_+^2\right)\left(A_-^2 + B_-^2\right)}$$

In the limit of $\mu_Q = 0$, Eq.(11) reduces to the self-energy of Ref. [11], obtained within the finite-temperature Feynman rules. We calculate the self-energy of Eq.(12) by making use of the standard technique [24,25], i.e., the discrete summation is replaced by the contour integral as

$$T\sum_{n=-\infty}^{\infty} f(k_0 = ik_4) = \frac{1}{2\pi i}\int\limits_{-i\infty}^{i\infty} dk_0 \frac{1}{2}[f(k_0) + f(-k_0)] + \frac{1}{2\pi i}\int\limits_{-i\infty+\epsilon}^{i\infty+\epsilon} dk_0 [f(k_0) + f(-k_0)]\frac{1}{e^{\beta k_0} - 1}, \qquad (13)$$

The first term in Eqs.(12) I_1 does not depend on the external momentum p, and its calculation gives

$$I_1 = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2\omega}\Big(1 + N(\omega)\Big), \qquad (14)$$

where $N(\omega) = n(\omega + \mu_Q) + n(\omega - \mu_Q)$, $n(w) = (e^{\omega/T} - 1)^{-1}$ and $\omega^2 = \mathbf{k}^2 + m_\pi^2$. Calculating $I_2^{\mu\nu}$ and I_3^ν we see that only the poles at the points $k_0 = a_{1,2};\ b_{1,2}$, where

$$a_{1,2} = \omega_+ \pm \mu_Q - \frac{1}{2}ip_4, \quad b_{1,2} = \omega_- + \pm\mu_Q + \frac{1}{2}ip_4,$$

contribute to the contour integral. For example, the contribution coming from a_1 to $I_2^{4,4}$ and I_3^4 reads

$$I_{2,a1}^{4,4} = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{4(k_0^2 + \mu_Q^2)}{\omega_+}\frac{1}{(\omega_+ - p_0)^2 - \omega_-^2}\left\{\frac{1}{2} + n(k_0)\right\}_{k_0=a_1},$$

$$I_{3,a1}^{(4)} = 2\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{k_0}{4\omega_+}\frac{1}{(\omega_+ - p_0)^2 - \omega_-^2}\left\{\frac{1}{2} + n(k_0)\right\}_{k_0=a_1},$$

and their contribution to $g_\rho^{-2}\Pi^{\mu\nu}(\mu_Q, p)$ in (11) results in

$$-\frac{1}{4}I_{2,a1}^{4,4} + 8\mu_Q I_{3,a1}^{(4)} =$$

$$2\int \frac{d^3\mathbf{k}}{(2\pi)^3}\left\{-\frac{k_0^2 + \mu_Q^2}{\omega_+} + \frac{2k_0\mu_Q}{\omega_+}\right\}\frac{1}{(\omega_+ - p_0)^2 - \omega_-^2}\left\{\frac{1}{2} + n(k_0)\right\}_{k_0=a1}$$

$$= -2\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\omega_+}\frac{(\omega_+ - p_0/2)^2}{(\omega_+ - p_0)^2 - \omega_-^2}\left\{\frac{1}{2} + n(\omega_+ + \mu_Q)\right\}; \quad p_0 = ip_4.$$

The final result for the components of $\Pi^{\mu\nu}$ is

$$\Pi^{44} = -\frac{g_\rho^2}{(2\pi)^2}\int\limits_0^\infty \frac{k^2dk}{\omega}\left\{\frac{4\omega^2 - p_4^2}{4kq}ln(a) + \frac{ip_4\omega}{kq}ln(b) - 2\right\}\{1 + N(\omega)\},$$

$$\Pi^{4,j} = -\frac{p_4 p^j}{\mathbf{p}^2}\Pi^{44}$$

$$\Pi^{ij} = \delta^{ij}\mathcal{A} + \frac{p^i p^j}{\mathbf{p}^2}\mathcal{B} \qquad (15)$$

where

$$a = \frac{(p_4^2 + \mathbf{p}^2 - 2k|\mathbf{p}|)^2 + 4p_4^2\omega^2}{(p_4^2 + \mathbf{p}^2 + 2k|\mathbf{p}|)^2 + 4p_4^2\omega^2}, \quad b = \frac{(p_4^2 + \mathbf{p}^2)^2 - 4(ip_4\omega + 2k|\mathbf{p}|)^2}{(p_4^2 + \mathbf{p}^2)^2 - 4(ip_4\omega - 2k|\mathbf{p}|)^2},$$

$$\mathcal{A} = -\frac{1}{2}\frac{g_\rho^2}{4\pi^2}\int\limits_0^\infty \frac{k^2dk}{\omega}\Big(\frac{2(p_4^2 - \mathbf{p}^2)}{\mathbf{p}^2} - \frac{ip_4\omega(p_4^2 + \mathbf{p}^2)}{k|\mathbf{p}|^3}ln(b) + \frac{p_4^2(p_4^2 - 4\omega^2) - \mathbf{p}^2(4k^2 - \mathbf{p}^2 - 2p_4^2)}{4k|\mathbf{p}|^3}ln(a)\Big)\{1 + N(\omega)\},$$

$$\mathcal{B} = -\frac{1}{2}\frac{g_\rho^2}{4\pi^2}\int\limits_0^\infty \frac{k^2dk}{\omega}\Big(-\frac{2(3p_4^2 - \mathbf{p}^2)}{\mathbf{p}^2} + \frac{ip_4\omega(\mathbf{p}^2 + 3p_4^2)}{k|\mathbf{p}|^3}ln(b) - \frac{3p_4^2(p_4^2 - 4\omega^2) - \mathbf{p}^2(4k^2 - 2p_4^2 - \mathbf{p}^2)}{4k|\mathbf{p}|^3}ln(a)\Big)\{1 + N(\omega)\}, \qquad (16)$$

We see that all dependence on the chemical potential μ_Q and the temperature T are absorbed into the Bose-factor $N(\omega)$. The substitution of

$$\frac{1}{2}\{n(\omega + \mu_Q) + n(\omega - \mu_Q)\} \to n(\omega)$$

in the above equations leads to the result of Ref. [11].

In the Minkowski space, the self–energy $\Pi^{\mu\nu}$ may be expressed in the form

$$\Pi^{\mu\nu} = F P_L^{\mu\nu} + G P_T^{\mu\nu}, \qquad (17)$$

where G and F are the so-called "longitudinal" and "transverse" masses, and $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ are the longitudinal and transverse projection tensors:

$$\begin{aligned} &P_T^{00} = P_T^{0i} = P_T^{i0} = 0, \; P_T^{ij} = \delta^{ij} - p^i p^j/\mathbf{p}^2, \\ &P_L^{\mu\nu} = p^\mu p^\nu/p^2 - g^{\mu\nu} - P_T^{\mu\nu}. \end{aligned} \qquad (18)$$

The tensor structure of $P_L^{\mu\nu}$ and $P_T^{\mu\nu}$ confirms the current conservation or, the transversality of $\Pi^{\mu\nu}$ with respect to the external momentum. The final expression for the ρ propagator in the Landau gauge in medium reads

$$D^{\mu\nu} = -\frac{P_L^{\mu\nu}}{p^2 - m_\rho^2 - F} - \frac{P_T^{\mu\nu}}{p^2 - m_\rho^2 - G}. \qquad (19)$$

For the concrete application we must perform the analytical continuation from the discrete Matsubara frequencies to the Minkowski space: $p_0 = ip_4 \to p_0 = E + i\delta$, and calculate the divergent part of self-energy regularizing it with counterterms. We use the dimensional regularization as in Refs [11,22] and find

$$\begin{aligned} &Re\, F_{vac}(M^2) = Re\, G_{vac}(M^2) = \\ &\frac{g_\rho^2}{48\pi^2}\left(M^2(1 - 4m_\pi^2/M^2)^{3/2}\left\{ln\frac{1 + (1 - 4m_\pi^2/M^2)^{1/2}}{1 - (1 - 4m_\pi^2/M^2)^{1/2}}\right\} + 8m_\pi^2 + C\right) \\ &C = -(1 - \frac{4m_\pi^2}{m_\rho^2})^{3/2} ln(\frac{(1 - 4m_\pi^2/m_\rho^2)^{1/2} + 1}{(1 - 4m_\pi^2/m_\rho^2)^{1/2} - 1} - \frac{8m_\pi^2}{m_\rho^2} \end{aligned} \qquad (20)$$

$$\begin{aligned} &Im\, F_{vac}(M^2) = Im\, G_{vac}(M^2) = \\ &\qquad -\frac{g_\rho^2}{48\pi^2} M^2 (1 - 4m_\pi^2/M^2)^{3/2} \Theta(M^2 - 4m_\pi^2), \end{aligned} \qquad (21)$$

$$\begin{aligned} G_{R,mat} = \frac{1}{2}\frac{g_\rho^2}{4\pi^2}\int_0^\infty \frac{k^2 dk}{\omega} N(\omega)\Big[\frac{2(E^2 + \mathbf{p}^2)}{\mathbf{p}^2} - \frac{M^2 E\omega}{k\mathbf{p}^3} ln|b| - \\ \frac{E^2(4\omega^2 + E^2) - \mathbf{p}^2(4k^2 - \mathbf{p}^2 + 2E^2)}{4k\mathbf{p}^3} ln|a|\Big] \end{aligned} \qquad (22)$$

$$F_{R,mat} = \frac{M^2}{\mathbf{p}^2}\frac{1}{2}\frac{g_\rho^2}{4\pi^2}\int_0^\infty \frac{k^2 dk}{\omega} N(\omega)\Big[\frac{4\omega^2 + E^2}{2k|\mathbf{p}|} ln|a| + \frac{2E\omega}{k|\mathbf{p}|} ln|b| - 4\Big]$$

$$\begin{aligned} G_{I,mat} = \frac{1}{2}\frac{g_\rho^2}{4\pi}\int_0^\infty \frac{k^2 dk}{\omega} N(\omega) \\ \frac{E^2(4\omega^2 + E^2) - \mathbf{p}^2(4k^2 - \mathbf{p}^2 + 2E^2) - 4M^2 E\omega}{4k|\mathbf{p}|^3}\zeta \end{aligned} \qquad (23)$$

$$F_{I,mat} = -\frac{1}{2}\frac{M^2}{\mathbf{p}^2}\frac{g_\rho^2}{2\pi}\int_0^\infty \frac{k^2 dk}{\omega} N(\omega)\frac{(2\omega - E)^2}{4k|\mathbf{p}|}\zeta, \qquad (24)$$

$$\zeta = \Theta(k - k_-) \cdot \Theta(k_+ - k),$$

where $k_\pm = \left|E(1 - 4m_\pi^2/M^2)^{1/2} \pm |\mathbf{p}|\right|$ and "mat" and "vac" denote the matter dependent contribution at finite T, μ_Q, and vacuum contribution at $T = 0, \mu_Q = 0$, respectively.

IV. SELF-ENERGY

From the above we see that the medium effect can be manifested (i) in the shift of the rho meson propagator pole position because of the real part of the self-energy modification; (ii) in modification of the imaginary part of the self-energy which is responsible for the decay width; and (iii) in the difference in (i) and (ii) for different polarization. All these phenomena depend on T, μ_Q and the rho momentum p. At large value of the spatial momentum $|\mathbf{p}|$ all medium corrections vanish because this case corresponds to the short range correlation where the many-body effects become rather small. At $|\mathbf{p}|$=0 we have no preferential direction and difference in (i) and (ii) is absent.

The matter corrections as functions of T and $|\mathbf{p}|$ at $\mu_Q = 0$ have been studied in Ref. [11]. It is shown that the corrections decrease rapidly with decreasing T. So, we limit ourselves to presenting the results at large T. In our calculations we use: $m_\pi = 139.6$ MeV, $m_\rho = 770$ MeV, $g_\rho^2/4\pi = 2.93$.

We start with the discussion of the dispersion relation or dependence of the ρ energy on the spatial momentum. In the mediumm, this dependence is different from that in the case of the free meson: $\omega_{\rho 0}^2 = m_\rho^2 + \mathbf{p}^2$. Moreover, the dispersion relations are different for different polarization states: the longitudinal and transverse dispersions are defined by the functions $ReG(p_0, \mathbf{p})$ and $ReF(p_0, \mathbf{p})$, respectively, and should be found as solution of the equations

$$\begin{aligned} \omega_L^2 &= \mathbf{p}^2 + m_\rho^2 + F_R(\omega_L, |\mathbf{p}|, T, \mu_Q) \\ \omega_T^2 &= \mathbf{p}^2 + m_\rho^2 + G_R(\omega_T, |\mathbf{p}|, T, \mu_Q) \end{aligned} \tag{25}$$

At the point $|\mathbf{p}|$=0, $\omega_{L,T}$ are just the "in-medium" ρ mass.

In describing the matter modification for longitudinal and transverse dispersions it is more illuminating to specify the difference $\Delta\omega_{L,T} = \omega_{L,T} - \omega_{\rho 0}$ which is shown in Fig.1 at different values of the chemical potential μ_Q =0, 60 and T = 150 MeV. We find that (a) $\Delta\omega_{L,T}$ increase with the chemical potential, (b) decrease with increasing momentum $|\mathbf{p}|$, and (c) the matter modifications for different polarization are similar in shape but they do not coincide exactly.

We find that the conventional π–ρ dynamics predicts increasing "in-medium" mass. This contradicts the QCD-sum rules conclusion on decrease in all in-medium masses [5,6]. We see that finite and large μ_Q=120 MeV leads to the corrections which are about two times as large as those at μ_Q =0.

In parallel with dispersions it is interesting to look at the pole positions $M_{L,T}(\mathbf{p})$ defined as $M_{L,T}^2(\mathbf{p}) = \omega_{L,T}^2(\mathbf{p}) - \mathbf{p}^2$ which coincide with $\omega_{L,T}$ at $|\mathbf{p}|$ =0. Fig.2 shows the difference in the shifts of the pole position $\Delta M_{L,T} = M_{L,T} - m_\rho$ for different polarization states: ΔM_L decreases faster with increasing $|\mathbf{p}|$. This is because of the inequality $ReF < ReG$ at $|\mathbf{p}|$ >0.

Fig.3 shows the M-dependence of the imaginary parts of the self-energy on the ρ invariant mass M at fixed temperature and different μ_Q and p. We find increasing in $Im\,G$, F with M, which leads to inrease in the damping constants. We find some difference between $Im\,F$ and $Im\,G$ predected by Eqs.(23) and (24) where $|ImF| > |ImG|$ at $|\mathbf{p}|$. The largest difference is in the region of $M \sim 0.3 - 0.5$ GeV for finite values of $|\mathbf{p}|$. This is illustrated in Fig.4 where we show the ratio $Im\,G/Im\,F$ as a function of M at different $|\mathbf{p}|$ and μ_Q. Here and in the following discussion we have restricted ourselves to the case μ_Q =0, 120 MeV. The results with μ_Q=120 MeV may be considered as upper limit of possible effects.

V. THE THERMAL DILEPTON PRODUCTION RATE

Now we try to understand: may the predicted matter correction be seen in the dielectron production rate? Recall that the thermal dilepton production rate in the vector dominance model is related to the imaginary part of the ρ propagator as follows [11,27]

$$E_+E_-\frac{dR}{d^3\mathbf{p}_+d^3\mathbf{p}_-} = \frac{1}{(2\pi)^6}\frac{e^4}{g_\rho^2}\frac{m_\rho^4}{M^2}((1-\xi)W_L + (1+\xi)W_T)\,n(E), \tag{26}$$

with

$$\begin{aligned} &\xi = 1 - (\mathbf{t}^2 - (\mathbf{t}\cdot\mathbf{p})^2/\mathbf{p}^2)/M^2; \\ &W_L = \frac{-F_I}{(M^2 - m_\rho^2 - F_R)^2 + F_I^2}, \quad W_T = \frac{-G_I}{(M^2 - m_\rho^2 - G_R)^2 + G_I^2}; \\ &n(E) = (e^{E/T} - 1)^{-1}, \end{aligned} \tag{27}$$

where $p = p_+ + p_-$ is the total pair momentum $p = (E, \mathbf{p})$, $p^2 = M^2$, and $t = p_+ - p_-$. The function ξ depends on the angle between vectors $\mathbf{t}$ and

$\mathbf{q}$ and varies through 0 at $\theta_{\hat{\mathbf{q}}\hat{\mathbf{p}}} = \pi/2$ to 1 at $\theta_{\hat{\mathbf{q}}\hat{\mathbf{p}}} = 0$. Integrating Eq.(26) over the lepton directions we find the integrated rate as a function of the invariant mass

$$\frac{dR}{dM} = \frac{1}{12\pi^4}\frac{e^4 m_\rho^4}{g_\rho^2}\int dE\sqrt{\frac{E^2}{M^2}-1}\,(W_L + 2W_T)\,n(E) \qquad (28)$$

Fig.5 shows the differential rate given by Eq.(26) as a function of M at different $|\mathbf{p}|$ at the points $\xi = 0$, where both the polarizations contribute with the same weight. These points correspond to events where the leptons have equal energies $E_+ = E_- = E/2$ and the angle between their momenta is related to M and E as $\sin(\theta_{+-}/2) = M/E$. One can see that at small $|\mathbf{p}|$ we have standard "two–bumps" structure of the rate where the first bump is the combination of the threshold increase and suppression factors in Eq.(26), and the second bump is the consequence of the resonance–like behavior of the rate. At large $|\mathbf{p}|$, the first bump is suppressed by the Bose factor $n(\sqrt{M^2+\mathbf{p}^2/T})$ which leads to the total rate suppression as well. We confirm the conclusion of Ref. [11] that the shape of the distribution at fixed $|\mathbf{p}|$ is not sensitive to the temperature at $\mu_Q = 0$.

At small $|\mathbf{p}|$ we have no difference between two polarizations and the matter correction to the pole shift at finite μ_Q is largest. But, comparing the calculated distributions at different μ_Q we find a sensible deformation of the rate shape. At small M it is explained by the Bose factors proportional to $(\exp((M/2-\mu_Q)/T-1)^{-1}$ in Eqs.(23), (24) which increases the rate at small M where it is proportional to ImF, G. At $M \sim m_\rho$ the pole position shifts to the largest value with μ_Q increasing. But because of the strong suppression factor $\exp(-E/T)/M^2$, the position of the second bump increases moderately and as result we find some effective suppression of the bump which remains in the same position, approximately.

At large $|\mathbf{p}|$ the difference in F_{mat} and G_{mat} leads to a different contribution of the longitudinal and transverse polarizations and, as a consequence, the net rate has an additional "deformation".

Fig.6 shows the integrated rate as a function of invariant mass. We see that its shape is akin to the shape of a differential rate in the region $|\mathbf{p}|$=0.4–0.5 GeV which gives the lagest contribution to the integral of Eq.(28).

The effect of the difference in the longitudinal and transverse polarization contributions can be best manifested from the asymmetry of the differential distributions

$$A_{\perp\parallel} = \frac{dR(\mathbf{t}\perp\mathbf{p}) - dR(\mathbf{t}\parallel\mathbf{p})}{dR(\mathbf{t}\perp\mathbf{p}) + dR(\mathbf{t}\parallel\mathbf{p})} = \frac{W_L - W_T}{W_L + 3W_T}, \qquad (29)$$

In Fig.7, we plot the asymmetry. At small $|\mathbf{p}|$ we have $W_L \approx W_T$ and the asymmetry vanishes. In the case of finite $\mathbf{p}$, at $M \to 2m_\pi$ we find $A_{\perp\parallel} \to 0.5(\Delta M_T^2 - \Delta M_L^2)/m_\rho^2 \simeq 0$. On the other hand, one can find that $A_{\perp\parallel}$ has the second zero at the point $M \simeq m_\rho + \Delta M(0) + \delta(\mathbf{p})$ where $\delta(\mathbf{p})$ is a smooth decreasing function of $\mathbf{p}^2$ with $\delta(\mathbf{p})/m_\rho < 10^{-2}$. So, the asymmetry reaches a maximum between the two zeroes: $2m_\pi < M < m_\rho$ because of $ImG < ImF$ at $M < m_\rho$. We find that the asymmetry increases with $|\mathbf{p}|$ and it may be as much as 0.25 for $\mu_Q = 120$ MeV, whereas at $\mu_Q = 0$ it is about three times smaller.

Probably, this is the most interesting medium effect of our consideration which can provide a fresh insight into the dilepton production as a probe for the hadron properties at extreme conditions.

VI. SUMMARY

In summary, we have calculated the ρ meson self–energy in the pion medium at finite temperatures and charge chemical potential which is responsible for the difference between π^+ and π^- densities in matter. The calculation is performed within the functional integral representation for the partition function in second order of g_ρ^2. We find that the pole positions and the imaginary parts of the self–energy are modified in the medium and this modification is different for different polarization states. We show that the shift of the pole positions is too modest to be seen in dilepton production rate. However, incorporation of the large chemical potential changes the shape of the rates: both differential and integral. Another non trivial effect is the predicted asymmetry in the differential dielectron distribution which increases with μ_Q. But we would not like to overestimate our results because at the present stage, the predicted effects have rather a methodical importance than a direct relation to experimental data. For the latter, on the one hand we have to complete our consideration by the space-time evolution of the hadronic system with taking into account the kinetic theory. On the other hand, more or less realistic calculation should take into account the baryonic degrees of freedom and their contribution to the complete picture. Really, the most likely source of the large pion charge chemical potential is

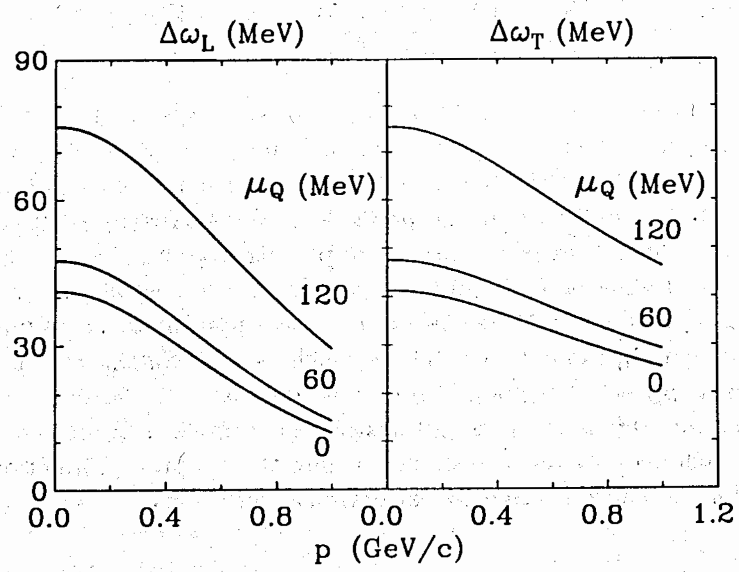

FIG. 1. The matter modifications for longitudinal and transverse dispersions $\Delta\omega_{L,T} = \omega_{L,T} - \omega_0$ at different values of the chemical potential $\mu_Q = 0, 60, 120$ MeV; $T = 150$ MeV.

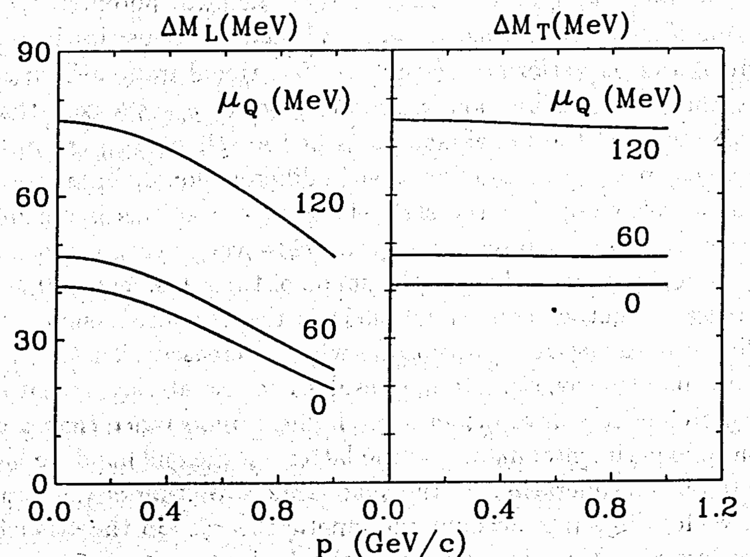

FIG. 2. The shift of the pole positions for different polarization states $\Delta M_{L,T} = M_{L,T} - m_\rho$ at different values of the chemical potential $\mu_Q = 0, 60, 120$ MeV; $T = 150$ MeV.

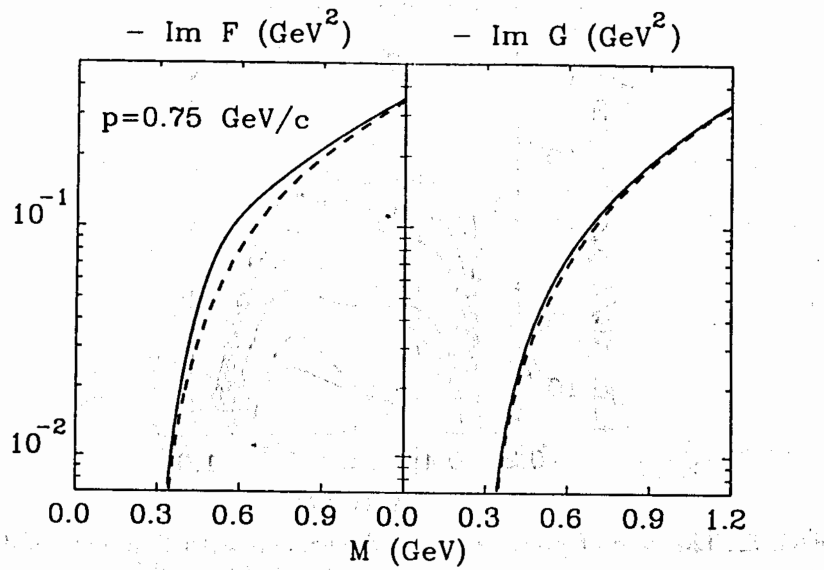

FIG. 3. The imaginary parts of the polarization operator $-ImF$ and $-ImG$ as functions of the invariant mass M at the fixed temperature $T = 150$ MeV and ρ momentum $|\mathbf{p}| = 0.75$ GeV.

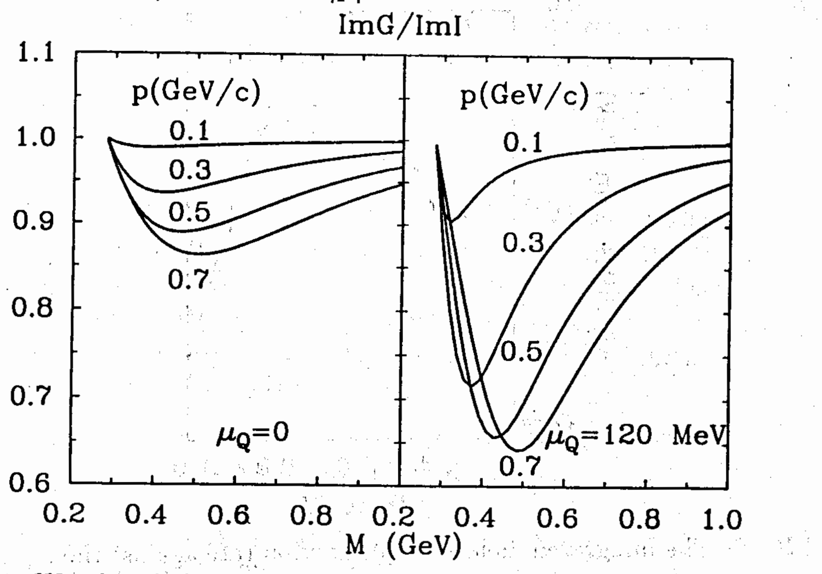

FIG. 4. We plot the ratio $Im\,G/Im\,F$ against the invariant mass M at different $|\mathbf{p}|$ and $\mu_Q = 0$, 120 MeV. $T = 150$ MeV.

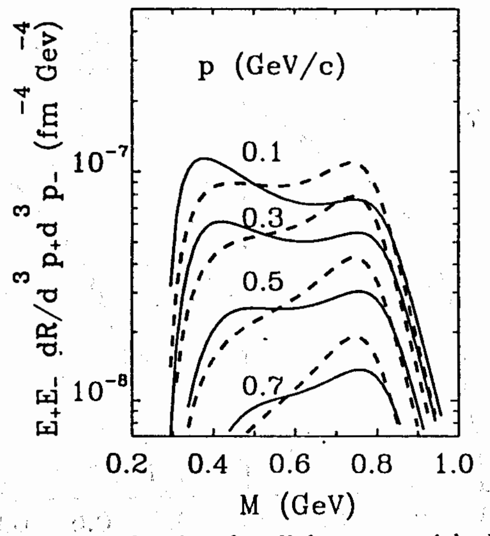

FIG. 5. The rate of production for dielectrons with different total momenta, against their invariant mass M. For $T = 150$ MeV, the prediction for $\mu_Q = 0$, 120 MeV appears as dashed and solid curves, respectively.

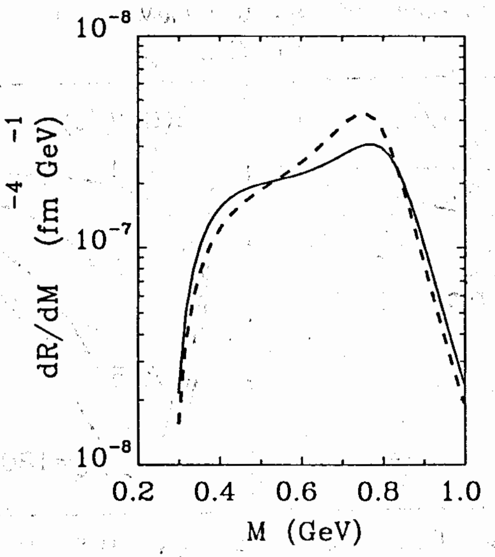

FIG. 6. The integrated dielectron production rate against their invariant mass M. Notations are the same as in Fig. 5.

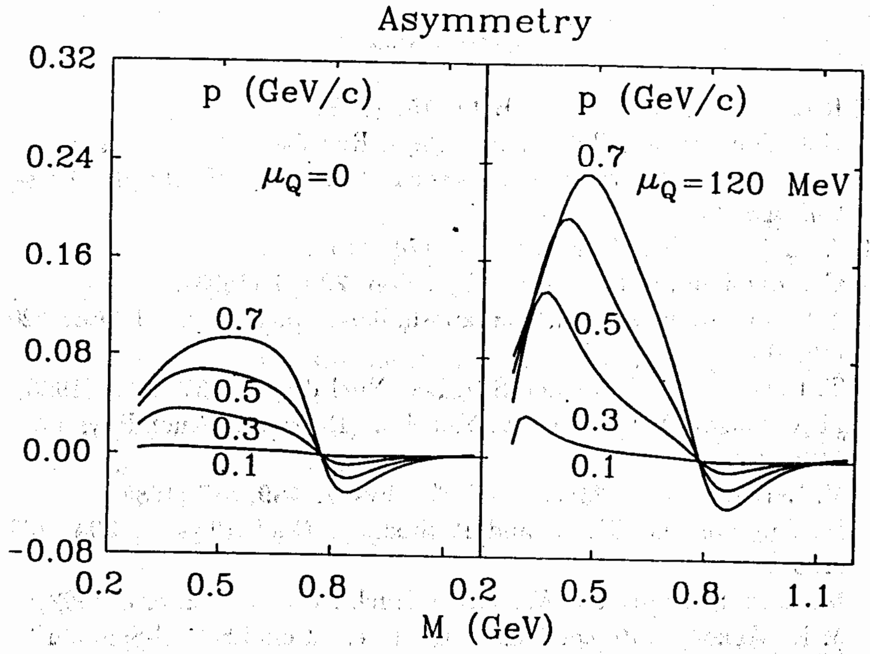

FIG. 7. We plot the asymmetry $A_{\perp\|}$ against the invariant mass M at different $|\mathbf{p}|$ and $\mu_Q = 0$, 120 MeV. $T = 150$ MeV. The asymmetry $A_{\perp\|}$ is defined in the text.

the neutron–proton asymmetry of the colliding heavy ions. In this case, the pions are produced in a rich baryon environment where the interaction of the rho meson and baryons cannot be neglected. From this point of view, our present work may be considered as a necessary step towards generalization of the vector dominance model to the case of the hot and dense nuclear isospin–asymmetric systems.

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REFERENCES

[1] R.D. Pisarski, Phys. Lett. B **11**, 157 (1982).
[2] G.J. Gounaris and J.J Sakurai, Phys. Rev. Lett. **21**, 244 (1968).
[3] J.J. Sakurai, Currents and mesons (University of Chicago Press, Chicago 1969).
[4] A. Goksch, Phys. Rev. Lett. **67**, 1701 (1991).
[5] C. Adami and G.E. Brown, Phys. Rep. **234**, 1 (1893).
[6] A.I. Bochkarev and M.E. Shaposhnikov, Nucl. Phys. B **268**, 220 (1986);
T. Hatsuda, Y. Koike, and S.H. Lee, Nucl. Phys. B **374**, 221 (1993);
M.K. Volkov, V.I. Zakharov, Yad. Fiz. (Russian J. Nucl. Phys.) **57**, 1106 (1994).
[7] V. Bernard and U. Meisner, Nucl. Phys. A **489**, 647 (1988);
M. Jaminon, G. Ripka, and P. Stassart, Nucl. Phys. A **504**, 733 (1989);
M. Lutz, S. Klimt, and W. Weise, Nucl. Phys. A **542**, 521 (1992);
M.K. Volkov, in Proceedings of the International School-Seminar'93 "Hadrons and Nuclei from QCD" Tsuruga, Vladivostok and Sapporo (ed. by K.Fujii et al), 1993, p. 238; (ed.)
T.I. Gulamov and A.I. Titov, Yad. Fiz. (Russian J. Nucl. Phys.) **58**, 337 (1995).
[8] C. Gale and J.I. Kapusta, Phys. Rev. C **35**, 2107 (1987).
[9] C.L. Korpa and S. Pratt, Phys. Rev. Lett. **64**, 1502 (1990).
C.L. Korpa, L. Xiong, C.M. Ko, and P.J. Siemens, Phys. Lett. B **246**, 333 (1990);
C.M Ko, L.H. Xia, and P.J. Siemens, Phys. Lett. B **231**, 16 (1989).
[10] H, Herrmann, B.L. Friman, and W. Nörenberg, Z. Phys. A **343**, 119 (1992).
[11] J.L. Kapusta and C. Gale, Nucl. Phys. B **357**, 65 (1991).
[12] J.L. Kapusta, Phys. Rev. D **24**, 426 (1981) 426.
[13] P. Koch, Phys. Lett. B **288**, 187 (1992); Z. Phys. C **57**, 283 (1993);
B. Kämpfer, P. Koch, and O.P. Pavlenko, Phys. Rev. C **49**, 1132 (1994).
[14] M. Kataja and P.V. Ruuskanen, Phys. Lett. B **243**, 181 (1990).

[15] J.W. Harris et. al., in Proceedings of the International Workshop on the Gross Properties of Nuclei and Nuclear Exitations XV, Hirshegg, Austria, 1987, 67.
[16] E.V. Shuryak and L. Xiong, preprint SUNY-NGT-94-14.
[17] A. Mazzoni, Nucl. Phys. A **566**, 95c (1994).
[18] M. Gonin, Nucl. Phys. A **553**, 799c (1993)c.
[19] M.I. Gorenstein and S.N. Yang, Phys. Rev. C **44**, 2875 (1991);
D. Bandyopadhyay, M.I. Gorenstein, H. Stöcker, W. Greiner, H. Sorge, Z. Phys. C **58**, 461 (1993).
[20] H.A. Weldon C.W. Phys. Lett. B **274**, 133 (1992).
[21] T.D. Cohen and W. Broniowski, Preprint INT, Univ. of Washington - DOE/ER/40561-173-INT-94-00-77.
[22] T.I. Gulamov, A.I. Titov and B. Kämpfer, Preprint FZR Rossendorf, FZR-66, December 1994.
[23] C.W. Bernard, Phys. Rev. D **9**, 3312 (1974).
[24] J.I. Kapusta, Finite temperature field theory (Cambridge Univ. Press, 1989).
[25] M.B. Kislinger and P.D. Morley, Phys. Rep. **51**, 65 (1979).
[26] C. Quigg, Gauge theories of the strong, weak and electromagnetic interactions (Benjamin/Cummings, Menlo Park, CA, 1983).
[27] H.A. Weldon, Phys. Rev. D **42**, 2384 (1990);
[28] B. Kämpfer, O.P. Pavlenko, M.I. Gorenstein, A. Peshier, and G. Soff, preprint FZR-94-50

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