ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E2-95-110

A.B.Arbuzov, E.A.Kuraev, N.P.Merenkov[1], L.Trentadue[2]

PAIR PRODUCTION IN SMALL ANGLE BHABHA SCATTERING

Submitted to «ЖЭТФ»

[1]Physical-Technical Institute, Kharkov, 310108, Ukraine
[2]Dipartimento di Fisica, Universita di Parma, 43100 Parma, INFN Sezione di Roma II, via della Ricerca Scientifica, 1, 00133 Roma, Italy

1995

1 Introduction

The electron–positron scattering process (Bhabha process) at small angles was chosen for the mobile precise luminosity measurement at LEP I [1]. The measurement technique provides the accuracy of the order 0.1%, or even better [2]. The adequate theoretical calculation of the Bhabha cross section was absent up to recent time [3,4]. The radiative corrections (RC) due to the emission of virtual, soft and real hard photons and pairs are to be calculated up to the tree–loop level. In our previous paper [3] the program of analytical calculations was performed, the leading ($\sim (\alpha L/\pi)^{1,2,3}$) contributions as well as the next–to–leading ($\sim \alpha/\pi$, $(\alpha/\pi)^2 L$) ones were calculated explicitly for the processes with the emission of photons ($L = \ln Q^2/m_e^2$, $Q^2 \sim 10$ $(\mathrm{GeV}/c)^2$ is the momentum transferred squared, θ is the scattering angle). As for pair production processes the contributions due to the emission of virtual, soft and real hard pairs were considered, but the production of real hard pairs was calculated only in the collinear kinematics (CK). In this paper we give the systematic studying of the hard pair emission in the semi–collinear kinematics (SCK). We present also the total contribution to the observable Bhabha cross–section due to the pair production

$$e^-(p_1) + e^+(p_2) \rightarrow e^-(q_1) + e^+(q_2) + e^-(p_-) + e^+(p_+), \tag{1}$$

which takes into account the cuts on the detection of the scattered electron and positron. We accept the convention [1-3] to consider as an event of the Bhabha process the one when the angles of the simultaneously registered particles hitting the opposite detectors lay in the range:

$$\theta_{\mathrm{min}} < \theta_e < \theta_{\mathrm{max}} = \rho\theta_{\mathrm{min}}, \qquad \pi - \rho\theta_{\mathrm{min}} < \theta_{\bar{e}} < \pi - \theta_{\mathrm{min}}, \tag{2}$$

($\theta_{\mathrm{min}} \sim 3^\circ$, $\rho \gtrsim 1$) in respect to the beam directions. The second condition is imposed on the energy fractions of the scattered electron and positron:

$$x_e x_{\bar{e}} > x_c, \qquad x_{e,\bar{e}} = 2\varepsilon_{e,\bar{e}}/\sqrt{s}, \qquad s = 4\varepsilon^2, \tag{3}$$

where ε is the energy of the initial electron (or positron), $\varepsilon_{e,(\bar{e})}$ is the energy of the scattered electron (positron), the centre–of–mass (CM) reference frame is here and further implied.

Our method of the real hard pair production cross–section calculation within the logarithmic accuracy consists in the separation of the contributions of the collinear and semi–collinear kinematical regions [5,6]. In the first one (CK) we suggest that both electron and positron from the created pair go in the narrow cone along to the direction of one of the charged particles (the projectile (scattered) electron $\vec{p}_1$ ($\vec{q}_1$) or the projectile (scattered) positron $\vec{p}_2$ ($\vec{q}_2$)):

$$\widehat{\vec{p}_+\vec{p}_-} \sim \widehat{\vec{p}_-\vec{p}_i} \sim \widehat{\vec{p}_+\vec{p}_i} < \theta_0 \ll 1, \qquad \varepsilon\theta_0/m \gg 1, \qquad \vec{p}_i = \vec{p}_1, \vec{p}_2, \vec{q}_1, \vec{q}_2 . \tag{4}$$

The contribution of the CK contains the terms of the order $(\alpha L/\pi)^2$ and $(\alpha\pi)^2 L$. In the semi–collinear region only one of conditions (4) on the angles is fulfilled:

$$\widehat{\vec{p}_+\vec{p}_-} < \theta_0, \quad \widehat{\vec{p}_\pm\vec{p}_i} > \theta_0\,; \quad \text{or} \quad \widehat{\vec{p}_-\vec{p}_i} < \theta_0, \quad \widehat{\vec{p}_+\vec{p}_i} > \theta_0\,; \tag{5}$$
$$\text{or} \quad \widehat{\vec{p}_-\vec{p}_i} > \theta_0, \quad \widehat{\vec{p}_+\vec{p}_i} < \theta_0 .$$

The contribution of the SCK contain the terms of the from:

$$\left(\frac{\alpha}{\pi}\right)^2 L \ln\frac{\theta_0}{\theta}, \qquad \left(\frac{\alpha}{\pi}\right)^2 L, \tag{6}$$

$\theta = \widehat{\vec{p}_-\vec{q}_1}$ is the scattering angle. The auxiliary parameter θ_0 disappears in the total sum of the CK and SCK contributions. We systematically omit the terms without large logarithms, they have the order $(\alpha/\pi)^2 \cdot \text{Const} \sim 10^{-5}$.

We restrict ourselves to the case when an electron–positron pair is created. The effects due to the other pair creation ($\mu^+\mu^-$, $\pi^+\pi^-$ etc.) are at least one order less and could be neglected as will be seen from the obtained numbers. All possible mechanisms of the pair creation (the singlet and non–singlet ones) as well as the identity of the particles in the final state are taken into account. In the case of the small angle Bhabha scattering only a part from the total 36 Feynman diagrams at the tree level is relevant — the scattering–type diagrams. Besides that, we convinced in the cancellations of the interference between the amplitudes describing the production of pairs moving along the electron direction and the positron one. This fact is known as the up–down cancellation.

The sum of the contributions due to the virtual pair emission (due to the vacuum polarization insertions in the virtual photon Green function) and of the ones due to the real soft pair emission does not contain cubic ($\sim L^3$) terms but depends on the auxiliary parameter $\Delta = \delta\varepsilon/\varepsilon$ ($m_e \ll \delta\varepsilon \ll \varepsilon$, ε is the energy sum of the soft pair components). The Δ-dependence disappears in the total sum after adding the contributions due to the real hard pair production. Before summing one

has to integrate the hard pair contributions over the energy fractions of the pair components as well as over the ones of the scattered electron and positron:

$$\Delta = \frac{\delta\varepsilon}{\varepsilon} < x_1 + x_2, \qquad x_c < x = 1 - x_1 - x_2 < 1 - \Delta, \tag{7}$$
$$x_1 = \frac{\varepsilon_+}{\varepsilon}, \qquad x_2 = \frac{\varepsilon_-}{\varepsilon}, \qquad x = \frac{q_1^0}{\varepsilon},$$

where $\varepsilon_\pm$ are the energies of the positron and electron from the created pair. We consider for definiteness the case when the created hard pair moves close to the direction of the initial (or scattered) electron.

The paper is organized as follows: in the second part we consider the emission of the hard pair in the collinear kinematics. The results turned out to be very close to the ones obtained by one of us (N.P.M) in paper [6] for the case of the pair production in the electron–nuclei scattering and applied to the case of the small angle Bhabha scattering in [4]. For the completeness we present very briefly the derivation and give the result correcting some misprints in [6]. In the third part we consider the semi–collinear kinematical regions. The differential cross–section is obtained there and integrated over the angles and the energy fractions of the pair components. In the fourth part we give the expression of the RC contribution due to the pair production to the experimental cross–section. The results are illustrated numerically in tables and discussed in the Conclusions.

2 The Collinear Kinematics

There are four different CK regions: when the created pair goes along the direction of the initial (scattered) electron or positron. We will consider in detail only two of them corresponding to the initial and the final electron directions. For the case of the pair emission along the initial electron it is useful to decompose the particle momenta into the longitudinal and transverse components:

$$p_+ = x_1 p_1 + p_+^\perp, \qquad p_- = x_2 p_1 + p_-^\perp, \qquad q_1 = x p_1 + q_1^\perp, \tag{8}$$
$$x = 1 - x_1 - x_2, \qquad q_2 \approx p_2, \qquad p_+^\perp + p_-^\perp + q_1^\perp = 0,$$

where $p_i^\perp$ are the transverse in respect to the initial electron beam direction two dimensional momenta of the final particles. It is convenient to introduce the dimensionless quantities for the relevant kinematical invariants:

$$z_i = \left(\frac{\varepsilon\theta_i}{m}\right)^2, \quad z_1 = \left(\frac{p_-^\perp}{m}\right)^2, \quad z_2 = \left(\frac{p_+^\perp}{m}\right)^2, \quad 0 < z_i < \left(\frac{\varepsilon\theta_0}{m}\right)^2 \gg 1, \tag{9}$$
$$A = \frac{(p_+ + p_-)^2}{m^2} = (x_1x_2)^{-1}\left[(1-x)^2 + x_1^2x_2^2(z_1 + z_2 - 2\sqrt{z_1z_2}\cos\phi)\right],$$
$$A_1 = \frac{2p_1p_-}{m^2} = x_2^{-1}\left[1 + x_2^2 + x_2^2z_2\right], \qquad A_2 = \frac{2p_1p_+}{m^2} = x_1^{-1}\left[1 + x_1^2 + x_1^2z_1\right],$$
$$C = \frac{(p_1 - p_-)^2}{m^2} = 2 - A_1, \qquad D = \frac{(p_1 - q_1)^2}{m^2} - 1 = A - A_1 - A_2,$$

where ϕ is the azimuthal angle between the planes $(\vec{p}_1 p_+^\perp)$ and $(\vec{p}_1 p_-^\perp)$.

Keeping only the terms from the summed over the spin states matrix element module squared which give non-zero contributions to the cross-section in the limit $\theta_0 \to 0$ we find that only 8 from the total 36 tree level Feynman diagrams are essential. They are drawn in fig. 1.

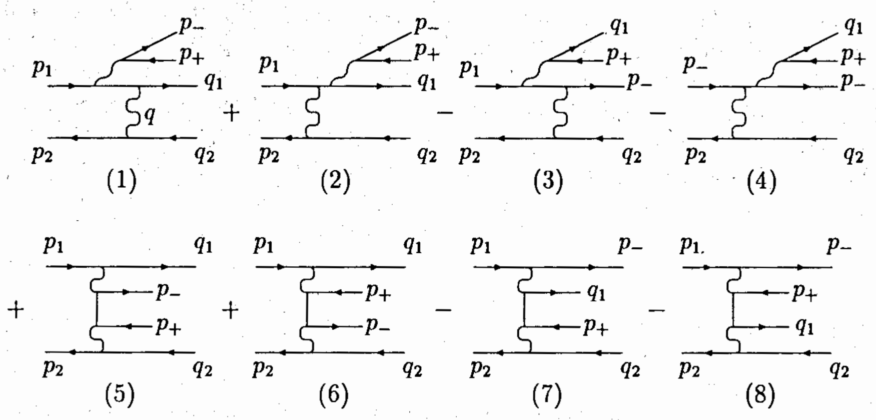

Fig. 1. The Feynman diagram which give logarithmically reinforced contributions in the kinematical region when the created pair goes along the electron direction. The signs represent the Fermi–Dirac statistics of the interchanged fermions.

The result has the factorized form (in agreement with the factorization theorem [8]):

$$\sum_{\text{spins}} |M|^2\Big|_{p_+,p_-\|p_1} = \sum_{\text{spins}} |M_0|^2\, 2^7\pi^2\alpha^2 \frac{I}{m^4}, \tag{10}$$

where one of the multipliers corresponds to the matrix element in the Born approximation (without pair production):

$$\sum_{\text{spins}} |M_0|^2 = 2^7\pi^2\alpha^2\left(\frac{s^4 + t^4 + u^4}{s^2t^2}\right), \tag{11}$$
$$s = 2p_1p_2, \qquad t = -Q^2x, \qquad u = -s - t,$$

and quantity I, which is named as the collinear factor, coincides with the expression for I_a obtained in paper [6]. We put it here in terms of our kinematical variables:

$$I = (1-x_2)^{-2}\left(\frac{A(1-x_2) + Dx_2}{DC}\right)^2 + (1-x)^{-2}\left(\frac{C(1-x) - Dx_2}{AD}\right)^2 \tag{12}$$
$$+ \frac{1}{2xAD}\left[\frac{2(1-x_2)^2 - (1-x)^2}{1-x} + \frac{x_1x - x_2}{1-x_2} + 3(x_2 - x)\right]$$
$$+ \frac{1}{2xCD}\left[\frac{(1-x_2)^2 - 2(1-x)^2}{1-x_2} + \frac{x - x_1x_2}{1-x} + 3(x_2 - x)\right]$$
$$+ \frac{x_2(x^2 + x_2^2)}{2x(1-x_2)(1-x)AC} + \frac{3x}{D^2} + \frac{2C}{AD^2} + \frac{2A}{CD^2} + \frac{2(1-x_2)}{xA^2D}$$
$$- \frac{4C}{xA^2D^2} - \frac{4A}{D^2C^2} + \frac{1}{DC^2}\left[\frac{(x_1 - x)(1 + x_2)}{x(1-x_2)} - 2\frac{1-x}{x}\right].$$

Rearranging the phase volume of the final particles as follows:

$$\mathrm{d}\Gamma = \frac{\mathrm{d}^3q_1\,\mathrm{d}^3q_2}{(2\pi)^6 2q_1^0 2q_2^0}(2\pi)^4\delta^4(p_1x + p_2 - q_1 - q_2) \tag{13}$$
$$\times\, m^4 2^{-8}\pi^{-4} x_1x_2\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}z_1\,\mathrm{d}z_2\frac{\mathrm{d}\phi}{2\pi},$$

and using the table of integrals over the variables of the created pair, which is given in Appendix A, we obtain

$$\bar{I} = \int_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\int_0^{z_0}\mathrm{d}z_1\int_0^{z_0}\mathrm{d}z_2\, I = \frac{L_0}{2xx_1x_2}\Big\{D_1\big(L_0 + 2\ln\frac{x_1x_2}{x}\big) \tag{14}$$
$$+ D_2\ln\frac{(1-x_2)(1-x)}{xx_2} + D_3\Big\}, \qquad L_0 = \ln\left(\frac{\varepsilon\theta_0}{m}\right)^2,$$
$$D_1 = 2xx_1x_2\left(\frac{1}{(1-x)^4} + \frac{1}{(1-x_2)^4}\right) - \frac{(1-x_2)^2}{(1-x)^2} - \frac{(1-x)^2}{(1-x_2)^2} + 1$$
$$+ \frac{(x+x_2)^2}{2(1-x)(1-x_2)} + \frac{3(x_2-x)^2}{2(1-x)(1-x_2)} - \frac{x^2 + x_2^2}{(1-x)(1-x_2)}$$

$$-2xx_2\left(\frac{1}{(1-x)^2}+\frac{1}{(1-x_2)^2}\right), \qquad D_2=\frac{2(x^2+x_2^2)}{(1-x)(1-x_2)},$$

$$D_3=\frac{2xx_1x_2}{(1-x_2)^2}\left(-\frac{8}{(1-x_2)^2}+\frac{(1-x)^2}{xx_1x_2}\right)+\frac{2xx_1x_2}{(1-x)^2}\left[\frac{x_2}{xx_1}\right.$$
$$+\left.\frac{2(x_1-x_2)}{xx_1(1-x)}-\frac{8}{(1-x)^2}+\frac{1}{xx_1x_2}-\frac{4}{x(1-x)}\right]+6+4x\left[\frac{x_2-x_1}{(1-x)^2}\right.$$
$$-\left.\frac{x_1}{x(1-x)}\right]+\frac{4(xx_2-x_1)}{(1-x_2)^2}-\frac{4(1-x_2)x_1x_2}{(1-x)^3}+\frac{8xx_1x_2^2}{(1-x)^4}$$
$$-\frac{xx_2^2}{(1-x_2)^4}+\frac{x_2}{(1-x_2)^2}\left[4(1-x)+\frac{2(x-x_1)(1+x_2)}{1-x_2}\right].$$

Performing the similar manipulations in the case when the pair moves in the direction of the scattered electron, integrating the obtained sum over the energy fractions of the pair components, and finally adding the contribution of the two remaining CK regions (when the pair goes along the positron directions) we obtain:

$$\mathrm{d}\sigma_{\mathrm{coll}}=\frac{\alpha^4\,\mathrm{d}x}{\pi Q_1^2}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}L\Big\{R_0(x)\left(L+2\ln\frac{\lambda^2}{z}\right)(1+\Theta) \tag{15}$$
$$+4R_0(x)\ln x+2\Theta f(x)+2f_1(x)\Big\}, \qquad \lambda=\frac{\theta_0}{\theta_{\mathrm{min}}},$$

$$\Theta\equiv\Theta(x^2\rho^2-z)=\begin{cases}1, & x^2\rho^2>z,\\ 0, & x^2\rho^2\le z,\end{cases}$$

$$R_0(x)=\frac{2}{3}\frac{(1+x)^2}{1-x}+\frac{(1-x)}{3x}(4+7x+4x^2)+2(1+x)\ln x,$$

$$f(x)=-\frac{107}{9}+\frac{136}{9}x-\frac{2}{3}x^2-\frac{4}{3x}-\frac{20}{9(1-x)}+\frac{2}{3}[-4x^2-5x+1$$
$$+\frac{4}{x(1-x)}]\ln(1-x)+\frac{1}{3}[8x^2+5x-7-\frac{13}{1-x}]\ln x-\frac{2}{1-x}\ln^2 x$$
$$+4(1+x)\ln x\ln(1-x)-\frac{2(3x^2-1)}{1-x}\mathrm{Li}_2(1-x),$$

$$f_1(x)=-x\Re e f(\frac{1}{x})=-\frac{116}{9}+\frac{127}{9}x+\frac{4}{3}x^2+\frac{2}{3x}-\frac{20}{9(1-x)}+\frac{2}{3}[-4x^2$$
$$-5x+1+\frac{4}{x(1-x)}]\ln(1-x)+\frac{1}{3}[8x^2-10x-10+\frac{5}{1-x}]\ln x$$
$$-(1+x)\ln^2 x+4(1+x)\ln x\ln(1-x)-\frac{2(x^2-3)}{1-x}\mathrm{Li}_2(1-x),$$

$$\mathrm{Li}_2(x)\equiv-\int\limits_0^x\frac{\mathrm{d}y}{y}\ln(1-y), \qquad Q_1=\varepsilon\theta_{\mathrm{min}}, \qquad L=\ln\frac{zQ^2}{m^2},$$

Some misprints, which take place in the expressions for $f(x)$ and $f_1(x)$ in [4,6], are corrected here.

3 The Semi–Collinear Kinematics

We will restrict ourselves again to the case when the created pair goes close to the electron momentum (the initial or final one). The similar consideration takes place in the CM system in the case when the pair follows the positron momentum. There are three different semi–collinear regions, which contribute to the cross–section in the frame of the required accuracy. The first region includes the events with a very small invariant mass of the created pair:

$$4m^2 \ll (p_+ + p_-)^2 \ll |q^2|,$$

and the pair escapes the narrow cones (defined by θ_0) along both projectile and scattered electron momentum directions. We agree to name this SCK region as $\vec{p}_+ \parallel \vec{p}_-$. Only the diagrams fig. 1(1) and fig. 1(2) contribute in this region and the reason is the smallness of the virtuality of the pair producing photon.

The second SCK region includes the events when the invariant mass of the created positron and the scattered electron is small: $4m^2 \ll (p_+ + q_1)^2 \ll |q^2|$, with the restriction that the positron should escape the narrow cone along the initial electron momentum direction. We name it as $\vec{p}_+ \parallel \vec{q}_1$ and note that only two diagrams fig. 1(3) and fig. 1(4) contribute here.

The third SCK region includes the events when the created electron goes inside the narrow cone along the initial electron momentum direction but the created positron does not. We name it as $\vec{p}_- \parallel \vec{p}_1$. Only the diagrams fig. 1(7) and fig. 1(8) are relevant there.

The differential cross–section has the following form:

$$\mathrm{d}\sigma=\frac{\alpha^4}{8\pi^4 s^2}\frac{|M|^2}{q^4}\frac{\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}x}{x_1x_2x}\,\mathrm{d}^2p_+^\perp\,\mathrm{d}^2p_-^\perp\,\mathrm{d}^2q_1^\perp\,\mathrm{d}^2q_2^\perp\,\delta(1-x_1-x_2-x) \tag{16}$$
$$\times\,\delta^{(2)}(p_+^\perp+p_-^\perp+q_1^\perp+q_2^\perp), \qquad |M|^2=-L_{\lambda\rho}p_{2\lambda}p_{2\rho},$$

where x_1 (x_2), x and $p_+^\perp$ ($p_-^\perp$), $q_1^\perp$ are the energy fractions and the perpendicular momenta of the created positron (electron) and the scattered electron respectively; $s=(p_1+p_2)^2$ and $q^2=-Q^2=(p_2-q_2)^2=-\varepsilon^2\theta^2$ are the centre-of-mass energy squared and the momentum transferred squared; the leptonic tensor $L_{\lambda\rho}$ has different forms for different SCK regions.

3.1 $\vec{p}_+ \parallel \vec{p}_-$ region

For the region of small $(p_++p_-)^2$ we can use the leptonic tensor obtained in paper [6]. Keeping only the relevant terms we present it in the form:

$$
\begin{aligned}
\frac{P^4}{8}L_{\lambda\rho} = {} & \frac{4P^2q^2}{(1)(2)}\left[-(p_1p_1)_{\lambda\rho}-(q_1q_1)_{\lambda\rho}+(p_1q_1)_{\lambda\rho}\right] \\
& -4(p_+p_-)_{\lambda\rho}\left(1-\frac{q^2P^2}{(1)(2)}\right)-\frac{4}{(1)}\left[q^2(p_1q_1)_{\lambda\rho}-2(p_1p_+)(q_1p_-)_{\lambda\rho}\right. \\
& \left.-2(p_1p_-)(q_1p_1)_{\lambda\rho}\right]-\frac{4}{(2)}\left[P^2(p_1q_1)_{\lambda\rho}-2(p_+q_1)(p_1p_-)_{\lambda\rho}\right. \\
& \left.-2(p_-q_1)(p_1p_+)_{\lambda\rho}\right]-\frac{32(p_1p_+)(p_1p_-)}{(1)^2}(q_1q_1)_{\lambda\rho}-\frac{32(q_1p_+)(q_1p_-)}{(2)^2}(p_1p_1)_{\lambda\rho} \\
& +\frac{8(p_1q_1)_{\lambda\rho}}{(1)(2)}\left[P^2(p_1q_1)-2(p_1p_+)(p_-q_1)-2(p_1p_-)(q_1p_+)\right],
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
&P=p_++p_-, \qquad (aa)_{\lambda\rho}=a_\lambda a_\rho, \qquad (ab)_{\lambda\rho}=a_\lambda b_\rho+a_\rho b_\lambda, \\
&q=p_1-q_1-P, \qquad (1)=(p_1-P)^2-m^2, \qquad (2)=(p_1-q)^2-m^2.
\end{aligned}
$$

After some algebraic transformations the expression for $|M|^2$ entering the cross-section could be put in the form:

$$
\begin{aligned}
\frac{1}{q^4}|M|^2 = -\frac{2s^2}{q^4P^4}\Bigg\{ & -\frac{4P^2q^2}{(1)(2)}\left[(1-x_1)^2+(1-x_2)^2\right] \\
& +\frac{128}{(1)^2(2)^2}\left[(q_1p)\,(p_+p_1)-x(p_1p)\,(q_1p_+)\right]^2\Bigg\},
\end{aligned}
\tag{18}
$$

where $p=p_--x_2p_+/x_1$, $(q_2^\perp)^2=-q^2$. In the considered region we can use the relations

$$
(1)=-\frac{1-x}{x_1}2(p_1p_+), \qquad (2)=\frac{1-x}{x_1}2(q_1p_+).
\tag{19}
$$

It is useful to represent all invariants in the terms of the Sudakov variables (energy fractions and perpendicular momenta), namely

$$
\begin{aligned}
&q_1^2=\frac{1}{x_1x_2}((p^\perp)^2+m^2(1-x)^2), \quad 2(q_1p_+)=\frac{1}{xx_1}(xp_+^\perp-x_1q_1^\perp)^2, \\
&2(p_1p_+)=\frac{1}{x_1}(p_+^\perp)^2, \quad 2(p_1p)=\frac{2}{x_1^2}p^\perp p_+^\perp, \quad 2(q_1p)=\frac{2}{x_1^2}(p^\perp[xp_+^\perp-x_1q_1^\perp]), \\
&p^\perp=x_1p_-^\perp-x_2p_+^\perp.
\end{aligned}
\tag{20}
$$

The large logarithm appears in the cross-section after the integration over $p^\perp$. In order to carry out this integration we can use the relation

$$
\delta^{(2)}\,\mathrm{d}^2p_+^\perp\,\mathrm{d}^2p_-^\perp=\frac{1}{(1-x)^2}\,\mathrm{d}^2p^\perp,
\tag{21}
$$

which is valid in the region $\vec{p}_+\parallel\vec{p}_-$. After the integration we derive the contribution to the cross-section of the first SCK region:

$$
\begin{aligned}
\mathrm{d}\sigma_{\vec{p}_+\parallel\vec{p}_-} = {} & \frac{\alpha^4}{\pi}L\,\mathrm{d}x\,\mathrm{d}x_2\frac{\mathrm{d}(q_2^\perp)^2}{(q_2^\perp)^2}\cdot\frac{\mathrm{d}(q_1^\perp)^2}{(q_1^\perp+q_2^\perp)^2} \\
& \times\frac{\mathrm{d}\phi}{2\pi}\cdot\frac{1}{(q_1^\perp+xq_2^\perp)^2}\left[(1-x_1)^2+(1-x_2)^2-\frac{4xx_1x_2}{(1-x)^2}\right],
\end{aligned}
\tag{22}
$$

where ϕ is the angle between the two dimensional vectors $q_1^\perp$ and $q_2^\perp$.

At this stage it is necessary to use the restrictions on the two dimensional momenta $q_1^\perp$ and $q_2^\perp$. They appear when the contribution of CK region (which in this case represents the narrow cones with the opening angle θ_0 along the momentum directions of both initial and the scattered electron) is excluded.

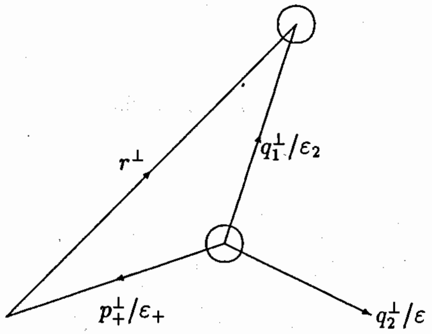

Fig. 2. The kinematics of an event in the angular perpendicular plane corresponding to the SCK region $\vec{p}_+ \parallel \vec{p}_-$.

The kinematics of an event in the perpendicular plane is shown in fig. 2. The circles of the radius θ_0 represent the forbidden collinear regions. The elimination of that regions gives the following restrictions:

$$|\frac{p_+^{\perp}}{\varepsilon_+}| > \theta_0, \qquad |r^{\perp}| = |\frac{p_+^{\perp}}{\varepsilon_+} - \frac{q_1^{\perp}}{\varepsilon_2}| > \theta_0 \tag{23}$$

where ε_+ and ε_2 are the energies of the created positron and the scattered electron respectively. In order to exclude $p_+^{\perp}$ from the above equation we use the conservation of the perpendicular momentum in the region $p_+ \parallel \vec{p}_-$:

$$q_1^{\perp} + q_2^{\perp} + \frac{1-x}{x_1} p_+^{\perp} = 0. \tag{24}$$

It is useful to introduce dimensionless variables $z_{1,2} = (q_{1,2}^{\perp})^2/(\varepsilon\theta_{\min})^2$, where $\theta_{\min}$ is the minimal angle at which the scattered particles (electron and positron) are recorded by the detector. Here we consider only the symmetrical circular detectors. The conditions (23) can be rewritten as follows

$$\begin{cases} 1 > \cos\phi > -1 + \frac{\lambda^2(1-x)^2 - (\sqrt{z_1}-\sqrt{z_2})^2}{2\sqrt{z_1 z_2}}, & |\sqrt{z_1}-\sqrt{z_2}| < \lambda(1-x), \\ 1 > \cos\phi > -1, \quad |\sqrt{z_1}-\sqrt{z_2}| > \lambda(1-x), & \lambda = \theta_0/\theta_{\min}, \end{cases} \tag{25}$$

$$\begin{cases} 1 > \cos\phi > -1 + \frac{\lambda^2 x^2(1-x)^2 - (\sqrt{z_1}-x\sqrt{z_2})^2}{2x\sqrt{z_1 z_2}}, & |\sqrt{z_1}-x\sqrt{z_2}| < \lambda x(1-x), \\ 1 > \cos\phi > -1, \quad |\sqrt{z_1}-x\sqrt{z_2}| > \lambda x(1-x). \end{cases} \tag{26}$$

Restriction (26) excludes the phase space, corresponding to the narrow cone along the direction of the initial electron, while eq. (26) — along the scattered electron.

The conditions of the LEP I experiment are:

$$\theta_0 \gg \frac{m}{\varepsilon} \approx 10^{-5} \quad \text{and} \quad \theta_{\min} \sim 10^{-2}. \tag{27}$$

That is the reason for considering $\lambda \ll 1$. The procedure of the integration of the differential cross–section over regions (26) and (26) is described in detail in Appendix B. Here we give the contribution of the SCK region $\vec{p}_+ \parallel \vec{p}_-$ to the cross–section provided that only the scattered electrons with the energy fraction x exceeded x_c could be recorded.

$$\sigma_{\vec{p}_+\parallel\vec{p}_-} = \frac{\alpha^4}{\pi Q_1^2}\mathcal{L}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}\int\limits_{x_c}^{1-\Delta}\mathrm{d}x\int\limits_0^{1-x}\mathrm{d}x_2\Big[\frac{(1-x_1)^2+(1-x_2)^2}{(1-x)^2} - \frac{4xx_1x_2}{(1-x)^2}\Big]\Big\{(1+\Theta)\ln\frac{z}{\lambda^2} + \Theta\ln\frac{(x^2\rho^2-z)^2}{x^2(x\rho^2-z)^2} + \ln\Big|\frac{(z-x^2)(\rho^2-z)(z-1)}{(z-x)^2(z-x^2\rho^2)}\Big|\Big\}, \qquad \mathcal{L} = \ln\frac{\varepsilon^2\theta_{\min}^2}{m^2}, \tag{28}$$

where $Q_1^2 = \varepsilon^2\theta_{\min}^2$, $\rho = \theta_{\max}/\theta_{\min}$ ($\theta_{\max}$ is the maximal angle of the final particle registration), $\Theta \equiv \Theta(x^2\rho^2 - z)$, $z \equiv z_2$. Auxiliary parameter Δ entering eq. (28) defines the minimal energy of the created hard pair: $2m/\varepsilon \ll \Delta \ll 1$. Note that we replaced L by $\mathcal{L}$ because we do not differ them at the one–logarithmic level.

3.2 $\vec{p}_+ \parallel \vec{q}_1$ *region*

As was already mentioned in the SCK region $\vec{p}_+ \parallel \vec{q}_1$ only diagrams fig. 1(3) and fig. 1(4) contribute. The leptonic tensor in this case could be derived from eq. (17) by the substitution $p_- \leftrightarrow q_1$, and the matrix element squared could be written as:

$$|M|^2_{\vec{p}_+\parallel\vec{q}_1} = -\frac{4s^2}{q_1'^2\vec{q}_2^2}\cdot\frac{1}{(1')(2)}\Big\{(1-x_1)^2 + (1-x_2)^2 \tag{29}$$

$$+\frac{32}{q'^2_1\tilde{q}^2_2}\cdot\frac{1}{(1')(2)}\left[(p_1p_+)(p_-p')-x_2(p_-p_+)(p_1p')\right]^2\Big\},$$

where

$$p'=q_1-p_+x/x_1,\qquad q'^2_1=(q_1+p_+)^2,$$
$$(2)=2(p_+p_-)(1-x_2)/x_1,\qquad (1')=-2(p_1p_+)(1-x_2)/x_1.$$

The integration of the matrix element over $(p_1^\perp)^2$ and $(p_-^\perp)^2$ could be carried out analogously to the previous case, and the contribution of the $\vec{p}_+\parallel\vec{q}_1$ region could be presented in the following form:

$$\mathrm{d}\sigma_{\vec{p}_+\parallel\vec{q}_1}=\frac{\alpha^4}{\pi}L\,\mathrm{d}x\,\mathrm{d}x_2\frac{\mathrm{d}(q_2^\perp)^2}{(q_2^\perp)^2}\cdot\frac{\mathrm{d}(q_1^\perp)^2}{(q_1^\perp)^2}$$
$$\times\frac{\mathrm{d}\phi}{2\pi}\cdot\frac{1}{(q_1^\perp+xq_2^\perp)^2}\cdot\frac{x^2}{(1-x_2)^2}\left[\frac{(1-x)^2+(1-x_1)^2}{(1-x_2)^2}-\frac{4xx_1x_2}{(1-x_2)^2}\right].\qquad(30)$$

The restriction on the phase space, coming from the exclusion of the collinear region when the created pair flies inside the narrow cone along the scattered electron, leads to the relation:

$$\left|\frac{p_-^\perp}{\varepsilon_-}-\frac{q_1^\perp}{\varepsilon_2}\right|>\theta_0.\qquad(31)$$

In eq. (31) we have to exclude $p_-^\perp$ using the conservation of the perpendicular momentum in the case under consideration: $p_-^\perp+q_2^\perp+q_1^\perp(1-x_2)/x=0$. In the terms of dimensionless variables z_1, z_2 and the angle ϕ eq. (31) could be rewritten as:

$$\begin{cases}1>\cos\phi>-1+\frac{\lambda^2x^2x_2^2-(\sqrt{z_1}-x\sqrt{z_2})^2}{2x\sqrt{z_1z_2}}, & |\sqrt{z_1}-x\sqrt{z_2}|<\lambda xx_2,\\ 1>\cos\phi>-1, & |\sqrt{z_1}-x\sqrt{z_2}|>\lambda xx_2.\end{cases}\qquad(32)$$

The integration of the differential cross-section (30) over the region defined in eq. (32) leads to the following result for the contribution of the $\vec{p}_+\parallel\vec{q}_1$ SCK region:

$$\sigma_{\vec{p}_+\parallel\vec{q}_1}=\frac{\alpha^4}{\pi Q_1^2}\mathcal{L}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}\int\limits_{x_c}^{1-\Delta}\mathrm{d}x\int\limits_0^{1-x}\mathrm{d}x_2\left[\frac{(1-x)^2+(1-x_1)^2}{(1-x_2)^2}\right.\qquad(33)$$

$$\left.-\frac{4xx_1x_2}{(1-x_2)^4}\right]\left\{\ln\frac{z}{\lambda^2}+\ln\frac{(\rho^2-z)(z-1)}{x_2^2\rho^2}\right\}.$$

3.3 $\vec{p}_-\parallel\vec{p}_1$ region

In the SCK region $\vec{p}_-\parallel\vec{p}_1$ only diagrams fig. 1(7) and fig. 1(8) contribute to the cross-section within the required accuracy. The leptonic tensor in this case could be derived from eq. (17) by the substitution $p_1\leftrightarrow -p_+$, and the matrix element squared has the form:

$$|M|^2_{\vec{p}_-\parallel\vec{p}_1}=-\frac{4s^2}{q'^2_2\tilde{q}^2_2}\cdot\frac{1}{(1)(2')}\Big\{(1-x)^2+(1-x_1)^2$$
$$+\frac{32}{q'^2_2\tilde{q}^2_2}\cdot\frac{1}{(1)(2')}\left[x_1(p_1\tilde{p})(p_1p_+)+x(p_+\tilde{p})(q_1p_1)\right]^2\Big\},\qquad(34)$$

where

$$\tilde{p}=p_--x_2p_1,\qquad q'^2_2=(p_1-p_-)^2,$$
$$(2')=-2(p_1q_1)(1-x_2),\qquad (1)=-2(p_1p_+)(1-x_2).$$

The integration of the matrix element over $(p_+^\perp)^2$ and $(p_-^\perp)^2$ leads to the differential cross-section:

$$\mathrm{d}\sigma_{\vec{p}_-\parallel\vec{p}_1}=\frac{\alpha}{4\pi}L\,\mathrm{d}x\,\mathrm{d}x_2\frac{\mathrm{d}(q_2^\perp)^2}{(q_2^\perp)^2}\cdot\frac{\mathrm{d}(q_1^\perp)^2}{(q_1^\perp)^2}\qquad(35)$$
$$\times\frac{\mathrm{d}\phi}{2\pi}\cdot\frac{1}{(q_1^\perp+q_2^\perp)^2}\left[\frac{(1-x)^2+(1-x_1)^2}{(1-x_2)^2}-\frac{4xx_1x_2}{(1-x_2)^4}\right].$$

The restriction due to the exclusion of the collinear region when the created pair flies inside a narrow cone along the initial electron has the form:

$$\frac{|p_+^\perp|}{\varepsilon_1}>\theta_0,\qquad p_+^\perp+q_1^\perp+q_2^\perp=0,\qquad(36)$$

or

$$\begin{cases}1>\cos\phi>-1+\frac{\lambda^2x_1^2-(\sqrt{z_1}-\sqrt{z_2})^2}{2\sqrt{z_1z_2}}, & |\sqrt{z_1}-\sqrt{z_2}|<\lambda x_1,\\ 1>\cos\phi>-1, & |\sqrt{z_1}-\sqrt{z_2}|>\lambda x_1.\end{cases}\qquad(37)$$

The integration of the differential cross-section (35) over the region defined in eq. (37) leads to the following result:

$$\sigma_{\vec{p}_-\|\vec{p}_1} = \frac{\alpha^4}{\pi Q_1^2}\mathcal{L}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}\int\limits_{x_c}^{1-\Delta}\mathrm{d}x\int\limits_0^{1-x}\mathrm{d}x_2\left[\frac{(1-x)^2+(1-x_1)^2}{(1-x_2)^2} - \frac{4xx_1x_2}{(1-x_2)^2}\right]\left\{\Theta\ln\frac{z}{\lambda^2}+\Theta\ln\frac{(x^2\rho^2-z)^2}{x_1^4x^4\rho^4}+\ln\left|\frac{\rho^2(z-x^2)}{z-x^2\rho^2}\right|\right\}. \tag{38}$$

The total contribution of the semi–collinear kinematics to the cross-section is the sum of eqs. (28), (33), and (38):

$$\sigma_{\text{s-coll}} = \sigma_{\vec{p}_+\|\vec{p}_-} + \sigma_{\vec{p}_+\|\vec{q}_1} + \sigma_{\vec{p}_-\|\vec{p}_1}. \tag{39}$$

4 The Total Contribution Due to the Real and Virtual Pair Production

In order to obtain the finite expression for the cross-section we have to add to eq. (39) the contribution of the collinear kinematical region (eq. (15)) as well as the ones due to the production of virtual and soft pairs. Taking into account the leading and next–to–leading terms we can write the full hard pair contribution in the following form:

$$\sigma_{\text{hard}} = \frac{\alpha^4}{\pi Q_1^2}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}\int\limits_{x_c}^{1-\Delta}\mathrm{d}x\left\{L^2R(x)+\mathcal{L}[\Theta f(x)+f_1(x)]\right. \tag{40}$$

$$+\mathcal{L}\int\limits_0^{1-x}\mathrm{d}x_2\left[\left(\Theta\ln\frac{(x^2\rho^2-z)^2}{x^2}+\ln\left|\frac{(z-x^2)(\rho^2-z)(z-1)x^2}{z-x^2\rho^2}\right|\right)\varphi\right.$$

$$-\left(\Theta\ln(x\rho^2-z)^2+\ln(z-x)^2\right)\varphi(x,x_2)$$

$$\left.\left.-\left(\Theta\ln(x_1^2x^2\rho^4)+\ln x_2^2\right)\varphi(x_2,x)\right]\right\}, \quad L=\ln\frac{Q_1^2z}{m^2}, \quad \mathcal{L}=\ln\frac{Q_1^2}{m^2},$$

where

$$\varphi=\varphi(x,x_2)+\varphi(x_2,x),$$

$$\varphi(x_2,x)=\frac{(1-x)^2+(x+x_2)^2}{(1-x_2)^2}-\frac{4xx_2(1-x-x_2)}{(1-x_2)^4},$$

$$R(x)=\frac{1}{3}\cdot\frac{1+x^2}{1-x}+\frac{1-x}{6x}(4+7x+4x^2)+(1+x)\ln x.$$

Integrating over x_2 in the right side of eq. (40) we obtain the final expression for the cross-section of hard pair production at the small angle electron–positron scattering:

$$\sigma_{\text{hard}} = \frac{\alpha^4}{\pi Q_1^2}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}\int\limits_{x_c}^{1-\Delta}\mathrm{d}x\left\{L^2(1+\Theta)R(x)+\mathcal{L}[\Theta F_1(x)+F_2(x)]\right\}, \tag{41}$$

$$F_1(x)=d(x)+C_1(x), \qquad F_2(x)=d(x)+C_2(x),$$

$$d(x)=\frac{1}{1-x}\left(\frac{8}{3}\ln(1-x)-\frac{20}{9}\right),$$

$$C_1(x)=-\frac{113}{9}+\frac{142}{9}x-\frac{2}{3}x^2-\frac{4}{3x}-\frac{4}{3}(1+x)\ln(1-x)$$
$$+\frac{2}{3}\cdot\frac{1+x^2}{1-x}\left[\ln\frac{(x^2\rho^2-z)^2}{(x\rho^2-z)^2}-3\mathrm{Li}_2(1-x)\right]+\left(8x^2+3x-9-\frac{8}{x}\right.$$
$$\left.-\frac{7}{1-x}\right)\ln x+\frac{2(5x^2-6)}{1-x}\ln^2 x+\beta(x)\ln\frac{(x^2\rho^2-z)^2}{\rho^4},$$

$$C_2(x)=-\frac{122}{9}+\frac{133}{9}x+\frac{4}{3}x^2+\frac{2}{3x}-\frac{4}{3}(1+x)\ln(1-x)$$
$$+\frac{2}{3}\cdot\frac{1+x^2}{1-x}\left[\ln\left|\frac{(z-x^2)(\rho^2-z)(z-1)}{(x^2\rho^2-z)(z-x)^2}\right|+3\mathrm{Li}_2(1-x)\right]$$
$$+\frac{1}{3}\left(-8x^2-32x-20+\frac{13}{1-x}+\frac{8}{x}\right)\ln x+3(1+x)\ln^2 x$$
$$+\beta(x)\ln\left|\frac{(z-x^2)(\rho^2-z)(z-1)}{x^2\rho^2-z}\right|, \qquad \beta=2R(x)-\frac{2}{3}\cdot\frac{1+x^2}{1-x}.$$

Formula (41) describes the small angle high energy cross-section of process (1) provided that the created hard (with the energy more than Δ) pair flies along the direction of the initial electron 3–momentum, and we have now to double σ_H to take into account the production of a hard pair flying along the direction of the initial positron beam.

I order to pick out the dependence on the parameter Δ in σ_H we will use the following relation:

$$\int\limits_1^{\rho^2}\mathrm{d}z\int\limits_{x_c}^{1-\Delta}\mathrm{d}x\Theta(x^2\rho^2-z)=\int\limits_1^{\rho^2}\mathrm{d}z\Big[\int\limits_{x_c}^{1-\Delta}\mathrm{d}x-\int\limits_{x_c}^{1}\mathrm{d}x\bar{\Theta}\Big], \tag{42}$$

$$\bar{\Theta}=1-\Theta(x^2\rho^2-z).$$

Therefore

$$\int\limits_1^{\rho^2}\mathrm{d}z\int\limits_{x_c}^{1-\Delta}\Theta\frac{\mathrm{d}x}{1-x}=\int\limits_1^{\rho^2}\mathrm{d}z\Big[\ln\frac{1-x_c}{\Delta}-\int\limits_{x_c}^{1}\frac{\mathrm{d}x}{1-x}\bar{\Theta}\Big], \tag{43}$$

$$\int\limits_1^{\rho^2}\mathrm{d}z\int\limits_{x_c}^{1-\Delta}\mathrm{d}x\Theta\frac{\ln(1-x)}{1-x}=\int\limits_1^{\rho^2}\mathrm{d}z\Big[\frac{1}{2}\ln^2(1-x_c)-\frac{1}{2}\ln^2\Delta-\int\limits_{x_c}^{1}\mathrm{d}x\frac{\ln(1-x)}{1-x}\bar{\Theta}\Big]. \tag{44}$$

The contribution to the cross-section of the small angle Bhabha scattering connected with the real soft (with the energy less than $\Delta\cdot\varepsilon$) and the virtual pair production is defined [2] by the formula:

$$\sigma_{\text{soft+virt}}=\frac{4\alpha^4}{\pi Q_1^2}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}\Big\{L^2\Big(\frac{2}{3}\ln\Delta+\frac{1}{2}\Big)+\mathcal{L}\Big(-\frac{17}{6}+\frac{4}{3}\ln^2\Delta \tag{45}$$

$$-\frac{20}{9}\ln\Delta-\frac{4}{3}\zeta_2\Big)\Big\}.$$

Using eqs. (43) and (44) it is easy to check that the auxiliary parameter Δ is canceled in the sum $\sigma_{\text{tot}}=2\sigma_{\text{hard}}+\sigma_{\text{soft+virt}}$, and we can write the total contribution σ_{tot} as follows:

$$\sigma_{\text{tot}}=\frac{2\alpha^4}{\pi Q_1^2}\int\limits_1^{\rho^2}\frac{\mathrm{d}z}{z^2}\Big\{L^2(1+\frac{4}{3}\ln(1-x_c)-\frac{2}{3}\int\limits_{x_c}^{1}\frac{\mathrm{d}x}{1-x}\bar{\Theta})+\mathcal{L}\Big[-\frac{17}{3} \tag{46}$$

$$-\frac{8}{3}\zeta_2-\frac{40}{9}\ln(1-x_c)+\frac{8}{3}\ln^2(1-x_c)+\int\limits_{x_c}^{1}\frac{\mathrm{d}x}{1-x}\bar{\Theta}\cdot(\frac{20}{9}-\frac{8}{3}\ln(1-x))\Big]$$

$$+\int\limits_{x_c}^{1}\mathrm{d}x[L^2(1+\Theta)\bar{R}(x)+\mathcal{L}(\Theta C_1(x)+C_2(x))]\Big\},$$

$$\bar{R}(x)=R(x)-\frac{2}{3(1-x)}.$$

The right side of eq. (46) is the master expression for the small angle Bhabha scattering cross-section connected with the pair production. It is finite and could be used for numerical estimations. Note that the leading term is described by the electron structure function $D_e^{\bar{e}}(x)$ which represents the probability to find a positron inside an electron with virtuality Q^2 provided that the electron loses the energy part $(1-x)$ [9].

In table 1 we present the ratio of the RC contribution due to the pair production σ_{tot} (eq. (46)) to the normalization cross-section σ_0,

$$\sigma_0=\frac{4\pi\alpha^2}{\varepsilon^2\theta^2_{\text{min}}}. \tag{47}$$

Table 1. The ratio $S=\sigma_{\text{tot}}/\sigma_0$ in percents, as a function of x_c, for NN ($\rho=1.74$, $\theta_{\text{min}}=1.61$ rad) and WW ($\rho=2.10$, $\theta_{\text{min}}=1.50$ rad) counters, $\sqrt{s}=2\varepsilon=M_Z=91.187$ GeV.

x_c	0.2	0.3	0.4	0.5	0.6	0.7	0.8
S_{NN}, %	-0.018	-0.022	-0.026	-0.029	-0.033	-0.038	-0.046
S_{WW}, %	-0.013	-0.019	-0.024	-0.029	-0.035	-0.042	-0.052

In table 2 we illustrate the comparison between the non-leading contribution in eq. (46) (containing $\mathcal{L}^1=\ln Q_1^2/m^2$) and the total one (containing $\mathcal{L}^2$ and $\mathcal{L}^1$).

Table 2. Values of R_{NN} and R_{WW} as functions of x_c, where R represents the ratio of the non-leading contribution in eq. (46) in respect to the total one, for NN and WW counters.

x_c	0.2	0.3	0.4	0.5	0.6	0.7	0.8
R_{NN}	0.036	-0.122	-0.194	-0.238	-0.268	-0.335	-0.465
R_{WW}	0.179	-0.021	-0.088	-0.120	-0.179	-0.271	-0.415

5 Conclusions

Thus the result derived in this paper combined together with the results derived earlier in [1,2] give the full and successive analytical description of the small angle electron–positron scattering cross–section at LEP I energies accompanied by the one and two photon radiation as well as by the pair production. The description takes into account the leading and next–to–leading logarithmic approximations and gives the possibility to find the cross–section with the accuracy not worse than 0.1% provided that the scattered electron and positron are recorded by symmetrical circular detectors. Using the above derivation it is possible to carry out the calculations also for non–symmetrical detectors.

Numerical calculations of the virtual and real pair production RC contributions shows their compensation at the level of 10^{-3} percents for the given angular apertures and x_c range. Table 2 shows that the next–to–leading contribution could be comparable with the leading one. Their ratio is sensitive to x_c and angle ranges. The similar situation for the leading and next–to–leading contributions to the small angle Bhabha cross–section takes place in the case of the double bremsstrahlung process $e^- + e^+ \to e^- + e^+ + \gamma + \gamma$ [4].

We have to note that in a realistic case one has to take into account that detectors can not distinguish a single particle event from the event when two or more particles hit the same point of the detector simultaneously. In that case the obtained results could be changed: starting from the presented differential cross–sections one has to integrate imposing the needed experimental restrictions.

We want to emphasize also that the method of calculations and a lot of the derived results could be used for the calculations of the radiative corrections to the small x deep inelastic scattering as well as to the normalization processes cross–sections at HERA. We hope to consider these questions in next publications.

Acknowledgement

We are grateful to V.A.Fadin, L.N.Lipatov for fruitful discussions and criticism. The work was supported in part by INTAS grant 93–1867.

Appendix A

We give here the list of the relevant integrals for the collinear kinematical region, calculated within the logarithmical accuracy. The definitions of eq. (9) are used. We imply in the left side of the relations below the general operation:

$$\langle\langle\ldots\rangle\rangle \equiv \int\limits_0^{z_0}\mathrm{d}z_1\int\limits_0^{z_0}\mathrm{d}z_2\int\limits_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}(\ldots), \tag{A.1}$$

and suggest $z_0 = (\varepsilon\theta_0/m)^2 \gg 1,\ \ L_0 = \ln z_0 \gg 1$. The details of the calculations could be found in Appendix of paper [5]. The results are:

$$\begin{aligned}
&\langle\Big(\frac{x_2D+(1-x_2)A}{DC}\Big)^2\rangle = \frac{L_0}{(1-x_2)^2}\Big\{L_0 + 2\ln\frac{x_1x_2}{x} - 8 \\
&\quad + \frac{(1-x)^2(1-x_2)^2}{xx_1x_2}\Big\}, \qquad \langle\frac{1}{DC}\rangle = \frac{L_0}{x_1x_2(1-x_2)}\big[\frac{1}{2}L_0 + \ln\frac{x_1x_2}{x}\big], \\
&\langle\Big(\frac{x_2A_1 - x_1A_2}{AD}\Big)^2\rangle = \frac{L_0}{(1-x)^2}\Big\{L_0 + 2\ln\frac{x_1x_2}{x} - 8 + \frac{(1-x)^2}{xx_1x_2} - \frac{4(1-x)}{x}\Big\}, \\
&\langle\frac{x_1A_2 - x_2A_1}{AD^2}\rangle = \frac{(x_1-x_2)L_0}{xx_1x_2(1-x)}, \qquad \langle\frac{1}{D^2}\rangle = \frac{L_0}{xx_1x_2}, \\
&\langle\frac{1}{AD}\rangle = \frac{-L_0}{x_1x_2(1-x)}\big[\frac{1}{2}L_0 + \ln\frac{x_1x_2}{x}\big], \qquad \langle\frac{1}{C^2D}\rangle = \frac{-L_0}{x_1(1-x_2)^3}, \\
&\langle\frac{1}{AC}\rangle = \frac{-L_0}{x_1x_2^2}\big[L_0 + 2\ln\frac{x_1x_2}{x} + 2\ln\frac{xx_2}{(1-x)(1-x_2)}\big], \qquad \langle\frac{1}{A^2D}\rangle = \frac{-L_0}{(1-x)^3}, \\
&\langle\frac{A}{C^2D^2}\rangle = \frac{x_2L_0}{x_1(1-x_2)^4}, \qquad \langle\frac{C}{A^2D^2}\rangle = \frac{-x_2L_0}{(1-x)^4}, \\
&\langle\frac{A}{CD^2}\rangle = \frac{-L_0}{x_1(1-x_2)^2}\big[\frac{1}{2}L_0 + \ln\frac{x_1x_2}{x}\big] + L_0\frac{x_2x - x_1}{xx_1x_2(1-x_2)^2}, \\
&\langle\frac{C}{AD^2}\rangle = \frac{-L_0}{x_1(1-x)^2}\big[\frac{1}{2}L_0 + \ln\frac{x_1x_2}{x}\big] - L_0\Big(\frac{x_1 - x_2}{x_1x_2(1-x)^2} + \frac{1}{xx_2(1-x)}\Big).
\end{aligned} \tag{A.2}$$

Appendix B

Here we derive eq. (28) starting from eq. (38) by integration over regions (26) and (26), taking into account the aperture of the detectors. Let us note at first that

$$\begin{aligned}
&\frac{\mathrm{d}(q_2^\perp)^2}{(q_2^\perp)^2}\cdot\frac{\mathrm{d}(q_1^\perp)^2}{(q_1^\perp + q_2^\perp)^2}\cdot\frac{\mathrm{d}\phi}{2\pi(q_1^\perp + xq_2^\perp)^2} = \frac{1}{Q_1^2}\cdot\frac{\mathrm{d}z_2}{z_2}\cdot\frac{\mathrm{d}z_1\,\mathrm{d}\phi}{2\pi(1-x)(z_2x - z_1)} \\
&\quad\times\Big[-\frac{1}{z_1 + z_2 + 2\sqrt{z_1z_2}\cos\phi} + \frac{1}{(z_1/x) + xz_2 + 2\sqrt{z_1z_2}\cos\phi}\Big].
\end{aligned} \tag{B.1}$$

Integrating the right side of eq. (B.1) we have to keep in mind that the first term in the brackets is sensitive to region (26) and the second to region (26). The aperture of the symmetrical circular detector is shown in fig. 3.

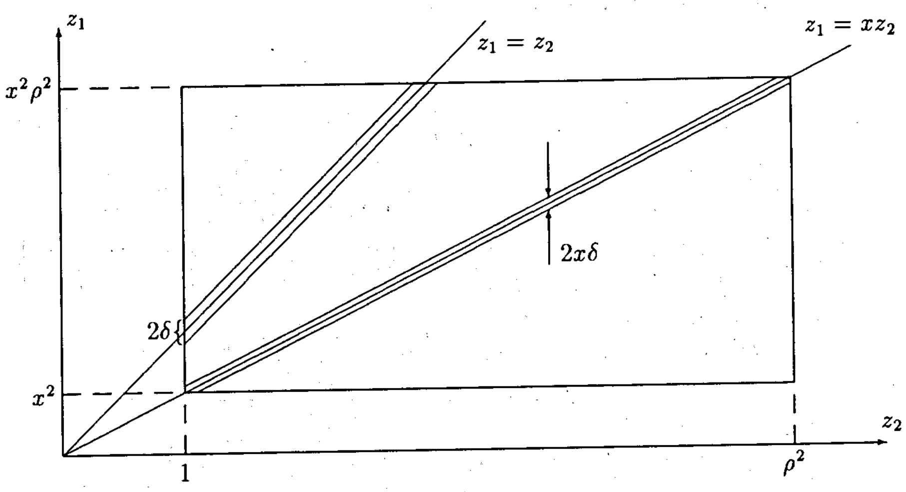

Fig. 3. The aperture of the symmetrical circular detector for the integration over z_1 and z_2 in the case when only the initial electron loses energy for the pair creation; $\delta = 2\sqrt{z_2}\lambda(1-x)$.

We present in detail only the integration of the first term in the brackets in the right side of eq. (B.1). If $|\sqrt{z_1} - \sqrt{z_2}| < \lambda(1-x)$ then the angular integration gives

$$\frac{1}{2\pi}\int\frac{d\phi}{z_1+z_2+2\sqrt{z_1z_2}\cos\phi} = \frac{2}{\pi}\int\limits_0^{\phi\max}\frac{d\phi}{z_1+z_2+2\sqrt{z_1z_2}\cos\phi}$$
$$= \frac{2}{\pi\sqrt{a^2-b^2}}\,\mathrm{atan}\left(\frac{a-b}{\sqrt{a^2-b^2}}\tan\frac{\phi}{2}\right)\Bigg|_0^{\phi\max}, \qquad \text{(B.2)}$$

where

$$\phi\max = \mathrm{acos}\left(-1+\frac{\lambda^2(1-x)^2-(\sqrt{z_1}-\sqrt{z_2})^2}{2\sqrt{z_1z_2}}\right),$$

$$a = z_1 + z_2, \qquad b = 2\sqrt{z_1z_2}.$$

Because of the smallness of the values $\lambda^2(1-x)^2$ and $|\sqrt{z_1}-\sqrt{z_2}|$ in comparison with z_1 and z_2 we can rewrite the last term in eq. (B.2) in the following form:

$$J = \frac{1}{\pi}\cdot\frac{1}{\sqrt{z_2}\,|\sqrt{z_1}-\sqrt{z_2}|}\,\mathrm{atan}\frac{|\sqrt{z_1}-\sqrt{z_2}|}{\sqrt{\lambda^2(1-x)^2-(\sqrt{z_1}-\sqrt{z_2})^2}}. \qquad \text{(B.3)}$$

Let $z_2 > z_1$, than in the region under consideration we have $\sqrt{z_1} > \sqrt{z_2} - \lambda(1-x)$, and we can carry out the subsequent integration over z_1 in eq. (B.1) by taking $z_1 = z_2$ in the factor $(xz_2 - z_1)^{-1}$ and introducing new variable $t = \lambda(1-x)(\sqrt{z_2}-\sqrt{z_1})$ in J. Thus we obtain:

$$J = 2\frac{2}{\pi}\int\limits_0^1\frac{dt}{t}\,\mathrm{atan}\frac{t}{\sqrt{1-t^2}} = 2\ln 2, \qquad \text{(B.4)}$$

where the additional factor 2 is due to the contribution when $z_1 > z_2$. From fig. 3 we see that the region $|\sqrt{z_1}-\sqrt{z_2}| < \lambda(1-x)$ contributes only if $z_2 < x^2\rho^2$. That is why we have to write the contribution corresponding $|\sqrt{z_1}-\sqrt{z_2}| < \lambda(1-x)$ in eq. (B.1) as:

$$\frac{1}{Q_1^2}\int\limits_1^{\rho^2}\frac{dz_2}{z_2^2}\frac{1}{(1-x)^2}2\Theta\ln 2, \qquad \Theta = \Theta(x^2\rho^2 - z). \qquad \text{(B.5)}$$

If now $|\sqrt{z_1}-\sqrt{z_2}| > \lambda(1-x)$ the angular integration is trivial and the subsequent integration over z_1 and z_2 is reduced to the integration of the function $\{(z_1 - xz_2)|z_1 - z_2|\}^{-1}$ over the rectangle $1 < z_2 < \rho^2$, $x^2 < z_1 < x^2\rho^2$ without the narrow strip of the width 2δ $(\delta = 2\sqrt{z_2}\lambda(1-x))$. The result reads:

$$\frac{1}{Q_1^2}\int\limits_1^{\rho^2}\frac{dz_2}{z_2^2}\frac{1}{(1-x)^2}\left\{\ln\left|\frac{(z_1-x^2)(x\rho^2-z_2)}{(x-z_2)(z_2-x^2\rho^2)}\right| + \Theta\left(-2\ln 2\right.\right.$$
$$\left.\left. + \ln\frac{(x^2\rho^2-z_2)^2}{(x\rho^2-z_2)^2x^2}\right)\right\}. \qquad \text{(B.6)}$$

It easy to see that the full contribution of the first term in the brackets in eq. (B.1) is reduced to eq. (B.6) without $-2\Theta\ln 2$ in the latter.

The integration of the second term in the brackets in eq. (B.1) can be done in the full analogy. The result could be written as follows:

$$\frac{1}{Q_1^2}\int\limits_1^{\rho^2}\frac{dz_2}{z_2^2}\frac{1}{(1-x)^2}\left\{\ln\frac{z_2}{\lambda^2}+\ln\left|\frac{(\rho^2-z_2)(z_2-1)}{(x-z_2)(z_2-x\rho^2)}\right|\right\}, \tag{B.7}$$

and formula (28) becomes obvious.

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Received by Publishing Department
on March 3, 1995.