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EXCITED STATES OF MESONS WITHIN THE NJL MODEL WITH THE HOMOGENEOUS VACUUM GLUON FIELD[1]

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Возбужденные состояния мезонов в модели Намбу — Иона-Лазинио с однородным вакуумным глюонным полем

Возбужденные состояния мезонов рассматриваются в рамках обобщения модели Намбу — Иона-Лазинио с учетом (анти)самодуального однородного фонового глюонного поля. Данная вакуумная конфигурация обеспечивает аналитический конфайнмент кварков. Бесцветные моды определяются конфайнмированными глюонами и описываются нелокальными кварковыми токами с полным набором квантовых чисел, включая радиальные и орбитальные числа. Эффективный мезонный лагранжиан соответствует ультрафиолетово-конечной теории. Спектры радиальных и орбитальных возбуждений мезонов являются асимптотически эквидистантными, т.е. имеют реджевское поведение. Для реальных значений орбитального момента они дают количественное описание экспериментальных данных.

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Excited States of Mesons Within the NJL Model with the Homogeneous Vacuum Gluon Field

Excited states of mesons are considered within generalization of the Nambu — Jona-Lasinio model taking into account the (anti)self-dual homogeneous background gluon field. This vacuum configuration ensures an analytical quark confinement. Colorless model are determined by the confined gluons and are described by the nonlocal quark currents with appropriate radial and orbital quantum numbers. The effective meson Lagrangian corresponds to ultraviolet finite theory. Spectrum of radial and orbital meson excitations predicted by the model is asymptotically equidistant, i.e., it has the Regge behavior. For real values of the orbital momentum it describes experimental data quantitatively.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

Vacuum self-dual gluon fields like instanton solutions [1]-[4], stochastic fields [5] or the fields with a constant strength [6, 7, 8] are widely used to explain various features of the low-energy hadron physics. In particular, H. Leutwyler has shown that (anti)self-dual homogeneous gluon field provides an analytical quark confinement [6]. In contrast to pure chromomagnetic or chromoelectric configurations, self- or antiself-dual fields are stable in a sense that the effective potential for these fields is a real function. Unfortunately, numerous attempts to estimate the field strength, minimizing the effective potential, have not given definite result. A general reason for this is quite clear: phase transitions in quantum field systems accompanied by appearance of nonzero vacuum fields occur out of perturbation region and their successful investigation is damped by a lack of nonperturbative methods. At present time, a task to prove an existance of the vacuum field and to estimate its strength, starting with the first principles, seems to be quite complicated.

We will follow another, in some sense phenomenological, point of view. Namely, we suppose, that self- or antiself-dual homogeneous gluon field realizes the QCD vacuum at low energies, and find out the points in hadron physics, where this vacuum field can play an important role. Our consideration is based on the bosonization procedure of the standard NJL-model [9, 10]. At the same time, taking into account the background field both in the quark and gluon propagators requires essential modification of this procedure.

Starting with the Euclidean generating functional of QCD with the background gluon field [2, 11] we construct the color singlet bilocal quark currents. Confined gluon fields ensure a natural expansion of the bilocal quark currents over the nonlocal ones with appropriate radial and orbital quantum numbers. An idea of such expansion was discussed in general form in [12]. Realization of this idea implies an existance of a set of orthonormalized functions. Particular form of these functions reflects specific physical peculiarities of the system. We show that the homogeneous (anti)self-dual vacuum field determines quite definite set – generalized Laguerre polynomials. As a result of the expansion, an interaction of quarks is realized by the current-current terms in the effective Lagrangian, the currents are nonlocal and carry radial and orbital quantum numbers. In contrast to nonrenormalizable local NJL model, our generalization leads to the effective four-fermion theory, that is superrenormalizable due to a nonlocality of the currents.

By means of the standard NJL bosonization we get a representation of generating functional in terms of the local meson fields, interacting with the nonlocal quark currents. These meson fields have a complete set of quantum numbers including radial n and orbital ℓ ones. Effective meson theory is ultraviolet finite due to nonlocal meson interactions. It should be noted that we are based at the representation for generating functional of QCD, which implies an averaging over some parameters of background field (for detailes see [2, 11]). In the case of the homogeneous field we have to average over self- and antiself-dual configurations and over directions of the field. Due to this averaging all amplitudes at hadron level are invariant under space rotations and parity transformation.

Parametrization of the generalized model is quite natural. The quark masses, the four-fermion coupling costant and the background field strength are the free parameters.

Usually, description of the Regge behavior of meson spectra within the Schrödinger equation requires to introduce into the Hamiltonian the term $r^{2/3}$ or to consider relativized versions of the Schrödinger equation. Nonlocal extension of the NJL model gives additional possibilities to investigate the excited meson states within the relativistic QFT approach [12]. In presence of the background field under consideration the quark propagator and the meson-quark vertices are nonlocal. This nonlocality has an important consequence – the spectra of radial and orbital meson excitations are equidistant. First of all, we calculate the masses of excited meson states for asymptotically large values of n and ℓ and demonstrate that the specific form of nonlocality induced by the homogeneous self-dual vacuum field provides the Regge behavior of the spectra. Calculation of the masses for real values of $\ell = 1,2$ gives quantitative (with an accuracy about ten percent) description of experimental data.

Besides that, there is another point, where the vacuum gluon field seems to be quite important. From the quantum mechanical point of view, difference between the masses of the pseudoscalar and vector mesons with identical quark content relates to the contact spin-spin interaction of quarks. In relativistic field theory an interaction of this kind should be realized *via* vector gauge field. An interaction of the quark spin with the vacuum gluon field is taken into account in the quark propagator through the term $\gamma_\mu B_{\mu\nu}\gamma_\nu$, where γ_μ are the Dirac matrices and $B_{\mu\nu}$ is the background field strength tensor. This spin-field interaction leads to mass splitting between the vector and pseudoscalar mesons with identical quark structure (ρ-π, K-K^* and so on). Accounting of this dynamical reason for the mass splitting reduces number of free parameters of the model in comparison with the local NJL-model, where successful description of the meson masses requires four-fermion coupling constants to be different for the pseudoscalar and vector nonets [14].

The paper is organized as follows. In Sect. 2,3 we introduce all definitions and discuss the quark and gluon dynamics in the background field. The collective modes are considered in Sect. 4. The masses of the pseudoscalar and vector meson excitations are calculated in Sect. 5.

2 Euclidean generating functional

After integration over the quantum gluon fields A^a_μ the generating functional for QCD with the background gluon field B^a_μ in the Euclidean metrics can be represented in the form [2, 11]:

$$Z = N\int d\sigma_{\rm vac} Dq D\bar q\ \exp\left\{\int d^4x \sum_f^{N_f} \bar q_f(x)(i\gamma_\mu\hat\nabla_\mu - m_f)q_f(x) + \sum_{n=2}^{\infty} L_n\right\}, \qquad (2.1)$$

where

$$L_n = \frac{g^n}{n!}\int d^4y_1 \dots \int d^4y_n j^{a_1}_{\mu_1}(y_1)\cdot\ldots\cdot j^{a_n}_{\mu_n}(y_n) G^{a_1\dots a_n}_{\mu_1\dots\mu_n}(y_1,\dots,y_n\mid B),$$
$$j^a_\mu(y) = \bar q_f(x)\gamma_\mu t^a q_f(x).$$

The representation (2.1) implies that the vacuum field B^a_μ is characterized by a set of parameters $\{\sigma_{\rm vac}\}$ and the vacuum state is degenerated respecting to change of these parameters. Integration over $d\sigma_{\rm vac}$ means that all amplitudes have to be averaged over $\{\sigma_{\rm vac}\}$. The function $G^{a_1\dots a_n}_{\mu_1\dots\mu_n}$ is the exact (up to the quark loops) n-point gluon Green function in the external field B^a_μ. Here and below we use the notation:

$$\hat\nabla_\mu = \partial_\mu - ig\hat B_\mu\ ,\ \ \hat B_\mu = B^a_\mu t^a,\ \ \check\nabla_\mu = \partial_\mu - ig\check B_\mu\ ,\ \ \check B_\mu = B^a_\mu C^a,$$

where the matrices t^a and C^a are the generators of the color group $SU_c(3)$ in the fundamental and adjoint representations.

We will be interested in $(q\bar q)$-collective modes and consider the form of Eq.(2.1) truncated up to the term L_2

$$Z = N\int d\sigma_{\rm vac} Dq D\bar q \exp\left\{\int d^4x \sum_f^{N_f} \bar q_f(x)(i\gamma_\mu\hat\nabla_\mu - m_f)q_f(x) + L_2\right\}, \qquad (2.2)$$
$$L_2 = \frac{g^2}{2}\int d^4x \int d^4y j^a_\mu(x) G^{ab}_{\mu\nu}(x,y\mid B) j^b_\nu(y)\ .$$

Representations (2.1) and (2.2) are manifestly invariant under the background gauge transformations [15]:

$$q_f(x)\to e^{-i\hat\omega(x)}q_f(x)\ ,\quad \hat A_\mu \to e^{-i\hat\omega(x)}\hat A_\mu e^{i\hat\omega(x)}\ ,$$
$$\hat B_\mu \to e^{-i\hat\omega(x)}\hat B_\mu e^{i\hat\omega(x)} + \frac{i}{g}e^{-i\hat\omega(x)}\partial_\mu e^{i\hat\omega(x)}\ . \qquad (2.3)$$

The standard NJL-model corresponds to the generating functional (2.2) with

$$B^a_\mu = 0 \text{ and } G^{ab}_{\mu\nu}(x,y) = \delta^{ab}\delta_{\mu\nu}\delta(x-y)\ .$$

We will approximate the two-point Green function in Eq.(2.2) by the gluon propagator in the external field. In other words, generalization of the NJL-model consists in taking into account an influence of the background field both on the gluon and quark propagators.

3 Quarks and gluons in the homogeneous background field

The homogeneous (anti)self-dual gluon field has the following form (e.g., see [6, 7, 16, 17])

$$B^a_\mu(x) = B^a_{\mu\nu}x_\nu = n^a B_{\mu\nu}x_\nu\ ,\quad n^2 = 1\ ,$$

where the vector n defines a direction in the color space. The constant tensor $B_{\mu\nu}$ satisfies the following conditions:

$$B_{\mu\nu} = -B_{\nu\mu}\ ,\ \ B_{\mu\rho}B_{\rho\nu} = -B^2\delta_{\mu\nu}\ ,\ \ \tilde B_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}B_{\alpha\beta} = \pm B_{\mu\nu}\ . \qquad (3.1)$$

The gauge invariant quantity B is a tension of the background field.

Any matrix $\hat n = n^a t^a \in SU_c(3)$ can be reduced to the general form

$$\hat n = t^3\cos\xi + t^8\sin\xi\ ,\quad 0\le\xi<2\pi\ , \qquad (3.2)$$

by appropriate global gauge transformation. As soon as the chromomagnetic $\vec H$ and chromoelectric $\vec E$ fields relates to each other like $\vec H = \pm\vec E$ for the (anti)self-dual configuration, one has only two spherical angles (θ,φ) defining a direction of the fields in the x-space. Now we can write down the explicit form of the measure $\mathrm{d}\sigma_{\mathrm{vac}}$:

$$\int \mathrm{d}\sigma_{\mathrm{vac}} = \frac{1}{(4\pi)^2}\int_0^{\pi} \mathrm{d}\theta \sin\theta \int_0^{2\pi} \mathrm{d}\varphi \int_0^{2\pi} \mathrm{d}\xi \sum_{\pm}, \qquad (3.3)$$

where the sign $\sum_{\pm}$ denotes averaging over the self- and antiself-dual configurations, which are assumed to be equiprobable.

The averaging over ξ can be included into the formalizm described below. However, in order to simplify further calculations and clarify a contribution of the background field under consideration into forming the bound quark systems we will omit the integral over ξ in Eq.(3.3) and fix particular vector $n^a = \delta^{a8}$, so that:

$$\hat{n} = t^8 = \mathrm{diag}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right), \quad \hat{B}_{\mu\rho}\hat{B}_{\rho\nu} = -v^2\Lambda^4\delta_{\mu\nu},$$

where we use the notation

$$\Lambda^2 = \sqrt{3}B, \quad v = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right)$$

which are convenient for further calculations.

In the adjoint representation one has

$$\breve{n} = C^8 = \frac{\sqrt{3}}{2}K, \quad \breve{B}_{\mu\rho}\breve{B}_{\rho\nu} = -\frac{3}{4}K^2B^2\delta_{\mu\nu},$$
$$K_{54} = -K_{45} = K_{76} = -K_{67} = i, \quad K^2 = \mathrm{diag}(0,0,0,1,1,1,1,0).$$

The rest of elements of the matrix K are equal to zero. Below we denote $gB^a_\mu \equiv B^a_\mu$.

The quark and gluon propagators satisfy the equations:

$$(i\gamma_\mu\hat{\nabla}_\mu - m_f)S_f(x,y \mid B) = -\delta(x-y), \qquad (3.4)$$
$$(\breve{\nabla}^2\delta_{\mu\nu} + 4i\breve{B}_{\mu\nu})G_{\nu\rho}(x,y \mid B) = -\delta_{\mu\rho}\delta(x-y). \qquad (3.5)$$

The equations (3.4) and (3.5) can be solved using the Schwinger proper time technique (e.g., see [11]). The quark propagator takes the following form

$$S_f(x,y \mid B) = \frac{i\hat{\nabla}_\mu\gamma_\mu + m_f}{\hat{\nabla}^2 - m_f^2 + \sigma_{\mu\nu}\hat{B}_{\mu\nu}}\delta(x-y) = \mathrm{e}^{\frac{i}{2}(x\hat{B}y)}H_f(x-y \mid B)\mathrm{e}^{\frac{i}{2}(x\hat{B}y)}, \qquad (3.6)$$

$$\tilde{H}_f(p \mid B) = \frac{1}{v\Lambda}\int_0^1 \mathrm{d}s\, \mathrm{e}^{-\frac{p^2}{2v\Lambda^2}s}\left(\frac{1-s}{1+s}\right)^{\frac{\alpha_f^2}{4v}}\left[\alpha_f + \frac{1}{\Lambda}p_\mu\gamma_\mu - is\frac{1}{\Lambda}(\gamma f p)\right]$$
$$\times\left[P_\pm + P_\mp\frac{1+s^2}{1-s^2} - \frac{i}{2}(\gamma f\gamma)\frac{s}{1-s^2}\right], \qquad (3.7)$$

where the following notation are introduced (see also Eq.(3.1))

$$P_\pm = \frac{1}{2}(1 \pm \gamma_5), \quad \alpha_f = \frac{m_f}{\Lambda}, \quad (xBy) = x_\mu B_{\mu\nu}y_\nu,$$
$$(pf\gamma) = p_\mu f_{\mu\nu}\gamma_\nu, \quad f_{\mu\nu} = \frac{t^8}{v\Lambda^2}B_{\mu\nu}, \quad f_{\mu\rho}f_{\rho\nu} = -\delta_{\mu\nu}.$$

The upper (lower) sign in the matrix P corresponds to the self-dual (antiself-dual) field.

Let us introduce the variable $\mu = p_\nu\gamma_\nu$. The function $\tilde{H}_f(\mu \mid B)$ is an entire analytical function in the complex μ-plane. It means that there is no pole corresponding to quark mass. In other words, the so-called analytical quark confinement is manifested. The parameter Λ (tension B) determines a characteristic region of the variation of the function $\tilde{H}_f(p|B)$, i.e., it defines the scale of confinement.

The term $\sigma_{\mu\nu}\hat{B}_{\mu\nu}$ in the quark propagator (3.6) can be interpreted as interaction of quark spin with the vacuum field. The second line in Eq.(3.7) relates just to the term $\sigma_{\mu\nu}\hat{B}_{\mu\nu}$ in Eq. (3.6). We shall discuss an influence of this spin-field interaction on meson masses in Sect. 5.

Gluon propagator can be represented in the form (see Eq.(3.5)):

$$G_{\mu\nu}(x,y \mid B) = \mathrm{e}^{\frac{i}{2}(x\breve{B}y)}D_{\mu\nu}(x-y \mid B)\mathrm{e}^{\frac{i}{2}(x\breve{B}y)}, \qquad (3.8)$$
$$D^{ab}_{\mu\nu}(z \mid B) = \delta_{\mu\nu}[K^2]^{ab}D(z \mid \Lambda^2) + R^{ab}_{\mu\nu}(z),$$
$$R^{ab}_{\mu\nu}(z \mid B) = \delta_{\mu\nu}[(1-K^2)D(z \mid 0) + K^2D_0(z \mid \Lambda^2)]]^{ab} + 2if_{\mu\nu}K^{ab}D_1(z \mid \Lambda^2),$$
$$D(z|\Lambda^2) = \frac{1}{(2\pi)^2z^2}\exp\left\{-\frac{\Lambda^2z^2}{4}\right\}. \qquad (3.9)$$

The Fourier transform of the function $D(z \mid \Lambda^2)$ is an entire analytical function in the momentum space. It describes a propagation of confined modes of the gluon fields. We suppose that these confined modes play crucial role in generation of hadron bound states. Other terms $R^{ab}_{\mu\nu}(z)$ do not take any significant part in forming the bound states and will be omitted below.

4 Colorless collective modes.

According to Eq.(3.8) the term L_2 in Eq.(2.2) looks

$$L_2 = \frac{g^2}{2}\iint \mathrm{d}^4x\mathrm{d}^4y\{\bar{q}_f(x)\gamma_\mu t^a\left[\mathrm{e}^{\frac{i}{2}(x\breve{B}y)}\right]^{aa'}q_f(x)\}D^{a'b'}_{\mu\nu}(x-y \mid B)$$
$$\times\{\bar{q}_{f'}(y)\gamma_\nu\left[\mathrm{e}^{\frac{i}{2}(x\breve{B}y)}\right]^{b'b}t^bq_{f'}(y)\}. \qquad (4.1)$$

Using the identity

$$t^a_{kj}\left[\mathrm{e}^{-i\breve{\omega}}\right]^{aa'} = \left[\mathrm{e}^{-i\hat{\omega}}t^{a'}\mathrm{e}^{i\hat{\omega}}\right]_{kj}, \quad (\breve{\omega} = \omega^aC^a\ \hat{\omega} = \omega^at^a),$$

one can get

$$L_2 = \frac{g^2}{2}\iint \mathrm{d}^4x\mathrm{d}^4yJ^a_\mu(x,y)D^{ab}_{\mu\nu}(x-y \mid B)J^a_\mu(y,x),$$
$$J^a_\mu(x,y) = \{\bar{Q}_f(x,y)\gamma_\mu t^aQ_f(x,y)\},$$

where $Q_f(x,y) = \mathrm{e}^{-\frac{i}{2}(x\hat{B}y)}q_f(x)$, $\bar{Q}_f(x,y) = \bar{q}_f(x)\mathrm{e}^{\frac{i}{2}(x\hat{B}y)}$.

Then the Fierz transformation of the color, flavor (we consider $N_f = 3$) and Dirac matrices is performed. Keeping only the scalar J^{aS}, pseudoscalar J^{aP}, vector J^{aV} and axial vector J^{aA} colorless currents we obtain:

$$L_2 = \frac{g^2}{2} \sum_{aJ} C_J \iint \mathrm{d}^4x \mathrm{d}^4y D(x-y \mid \Lambda^2) J^{+}_{aJ}(x,y) J_{aJ}(x,y) \,, \tag{4.2}$$

$$J_{aJ}(x,y) = \bar{q}_f(y) M^a_{ff'} \Gamma^J e^{i(y\hat{B}x)} q_{f'}(x) \,, \tag{4.3}$$

$$\Gamma^S = \mathbf{1} \,, \quad \Gamma^P = i\gamma_5 \,, \quad \Gamma^V = \gamma_\mu \,, \quad \Gamma^A = \gamma_5\gamma_\mu \,,$$

$$C_S = C_P = \frac{1}{9} \,, \quad C_V = C_A = \frac{1}{18} \,,$$

M^a–flavor mixing matrices, being equal to the matrices λ^a or their linear combinations ($a = 0, ..., 8$). The currents (4.3) are the scalars under color group transformations (2.3).

Under transformation ($x \to x + \frac{1}{2}y$, $\;y \to x - \frac{1}{2}y$) representation (4.2) takes the form:

$$L_2 = g^2 \sum_{aJ} C_J \iint \mathrm{d}^4x \mathrm{d}^4y D(y \mid \Lambda^2) J^{+}_{aJ}(x,y) J_{aJ}(x,y) \,,$$

$$J_{aJ}(x,y) = \bar{q}(x) M^a \Gamma^J \mathrm{e}^{-\frac{1}{2}y\overleftrightarrow{\nabla}(x)} q(x) \,,$$

$$\overleftrightarrow{\nabla}_\mu(x) = \overleftarrow{\nabla}_\mu(x) - \overrightarrow{\nabla}_\mu(x) \,, \quad \overrightarrow{\nabla}_\mu(x) = \overrightarrow{\partial}_\mu(x) - i\hat{B}_\mu(x) \,,$$

$$\overleftarrow{\nabla}_\mu(x) = \overleftarrow{\partial}_\mu(x) + i\hat{B}_\mu(x) \,, \quad \partial_\mu(x) = \frac{\partial}{\partial x_\mu} \,.$$

From a quantum mechanical viewpoint, description of the radial and orbital excitations is provided by an appropriate orthogonal set of polynomials. It means that a decomposition of the bilocal currents has the form

$$J_{aJ}(x,y) = \sum_{n\ell} f^{n\ell}_{\mu_1...\mu_\ell}(y) J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) \,, \tag{4.4}$$

$$f^{\ell n}_{\mu_1...\mu_\ell}(y) = L_{n\ell}(y^2) T^{(\ell)}_{\mu_1...\mu_\ell}(n_y) \quad (n_y = y/\sqrt{y^2}) \,.$$

The irreducible tensors of the four-dimensional rotational group $T^{(\ell)}_{\mu_1...\mu_\ell}$ are orthogonal

$$\int_\Omega \frac{\mathrm{d}\omega}{2\pi^2} T^{(\ell)}_{\mu_1...\mu_\ell}(n_y) T^{(k)}_{\nu_1...\nu_k}(n_y) = \frac{1}{2^\ell(\ell+1)} \delta^{\ell k} \delta_{\mu_1\nu_1} ... \delta_{\mu_\ell\nu_\ell} \,, \tag{4.5}$$

and are subjected to the conditions:

$$T^{(\ell)}_{\mu_1...\mu...\nu...\mu_\ell}(n_y) = T^{(\ell)}_{\mu_1...\nu...\mu...\mu_\ell}(n_y) \,, \quad T^{(\ell)}_{\mu...\mu...\mu_\ell}(n_y) = 0,$$

$$T^{(\ell)}_{\mu_1...\mu_\ell}(n_y) T^{(\ell)}_{\mu_1...\mu_\ell}(n_{y'}) = \frac{1}{2^\ell} C^{(1)}_\ell(n_y n_{y'}) \,. \tag{4.6}$$

The measure $\mathrm{d}\omega$ in Eq.(4.5) relates to integration over the spherical angles of the vector n_y and $C^{(1)}_\ell$ in Eq.(4.6) are the Gegenbauer (ultraspherical) polynomials. The polynomials $L_{n\ell}(u)$ obey the condition:

$$\int_0^\infty \mathrm{d}u \rho_\ell(u) L_{n\ell}(u) L_{n'\ell}(u) = \delta_{nn'} \,.$$

The function $D(y|\Lambda^2)$ leads to the weight function

$$\rho_\ell(u) = u^\ell \mathrm{e}^{-u} \,,$$

that corresponds to the generalized Laguerre's polynomials $L_{n\ell}(u)$. Other terms of the gluon propagator do not provide any possibility for decomposition like (4.4). Such a situation suggests that only confined gluons (contributing to $D(y|\Lambda^2)$) provide the generation of bound states. At the same time, the function D accumulates the nonabelian nature of the gluons – their self-interaction. This is the point, where QCD differs from quantum electrodynamics actually. As a result we have a mechanism of the generation of bound states, that is different from the potential picture.

The decomposition leads to the representation for L_2 [13]:

$$L_2 = \sum_{aJ\ell n} \frac{1}{2\Lambda^2} G^2_{J\ell n} \int \mathrm{d}^4x \left[J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) \right]^2 \,, \tag{4.7}$$

$$G^2_{J\ell n} = C_J\, g^2 \frac{(\ell+1)}{2^\ell n!(\ell+n)!} \,, \quad J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) = \bar{q}(x) V^{aJ\ell n}_{\mu_1...\mu_\ell}(x) q(x) \,, \tag{4.8}$$

$$V^{aJ\ell n}_{\mu_1...\mu_\ell}(x) \equiv V^{aJ\ell n}_{\mu_1...\mu_\ell}\left(\frac{\overleftrightarrow{\nabla}(x)}{\Lambda} \right)$$

$$= M^a \Gamma^J \left\{\!\!\left\{ F_{n\ell}\left(\frac{\overleftrightarrow{\nabla}^2(x)}{\Lambda^2} \right) T^{(\ell)}_{\mu_1...\mu_\ell}\left(\frac{1}{i} \frac{\overleftrightarrow{\nabla}(x)}{\Lambda} \right) \right\}\!\!\right\}, \tag{4.9}$$

$$F_{n\ell}(s) = \left(\frac{s}{4} \right)^n \int_0^1 \mathrm{d}t\, t^{\ell+n} \exp\left\{ \frac{1}{4} st \right\} . \tag{4.10}$$

The doubled brackets in Eq.(4.9) mean that the covariant derivatives commute inside these brackets. The functions $F_{n\ell}(s)$ are entire analytical functions in the complex s-plane, they obey confinement condition.

The vertex functions have the following asymptotic behavior for large Euclidean momentum

$$\lim_{p^2\to\infty} F_{n\ell}[p^2] T^{(\ell)}_{\mu_1...\mu_\ell}[p] \sim \frac{1}{(p^2)^{1+\ell/2}} \,, \tag{4.11}$$

Therefore, there is single divergent diagram given in Fig.1. This divergency can be removed by the counter-term of the form $-2J(x)\mathrm{Tr}VS$, which can be inserted into the interaction Lagrangian (4.7). After this renormalization we arrive at the expression:

$$Z = \int \mathrm{d}\sigma_{\mathrm{vac}} Dq D\bar{q} \exp\left\{ \iint \mathrm{d}^4x \mathrm{d}^4y \bar{q}(x) S^{-1}(x,y|B) q(y) \right.$$

$$\left. + \sum_{aJ\ell n} \frac{1}{2\Lambda^2} G^2_{J\ell n} \int \mathrm{d}^4x \left[J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) - \mathrm{Tr} V^{aJ\ell n}_{\mu_1...\mu_\ell} S \right]^2 \right\} . \tag{4.12}$$

The sign $\mathrm{d}\sigma_{\mathrm{vac}}$ means an averaging over self- and antiself-dual configurations and directions of the background field.

Representation ((4.12),(4.8)-(4.10)) manifests the following properties. The quarks are confined due to background field. The nonlocal quark currents (4.8) are invariant under transformations (2.3), i.e., they are color singlets. These currents have a suitable set of quantum numbers including the radial n and orbital ℓ ones. The effective theory determined by Eqs.((4.12),(4.8)–(4.10)) is UV-finite due to a nonlocality of the vertex (4.9).

The quark masses m_f, four-fermion coupling constant g and scale Λ are the parameters of the effective theory, defined in Eqs.((4.12),(4.8)–(4.10)).

Starting with the representation (4.12) one can introduce the local meson-like fields by means of the bosonization procedure like in the standard NJL-model [9, 10, 14] and get the following representation for the generating functional in terms of the meson fields $\Phi^{a\kappa}_{\mu}$ $(\kappa \equiv (J\ell n),\ \mu \equiv (\mu_1...\mu_\ell))$:

$$Z = N\int\prod_{a\kappa} D\Phi^{a\kappa}_{\mu}\exp\left\{\frac{1}{2}\iint \mathrm{d}^4x\mathrm{d}^4y\Phi^{a\kappa}_{\mu}(x)\left[(\Box - M^2_{a\kappa})\delta(x-y)\right.\right.$$
$$\left.\left.-h^2_{a\kappa}\Pi^R_{a\kappa}(x-y)\right]\Phi^{a\kappa}_{\mu}(y) + I_{int}[\Phi]\right\}, \qquad (4.13)$$

$$I_{int} = -\frac{1}{2}\iint \mathrm{d}^4x\mathrm{d}^4y h_{a\kappa}h_{a'\kappa'}$$
$$\times\Phi^{a\kappa}_{\mu}(x)\left[\Pi^{a\kappa,a'\kappa'}_{\mu\mu'}(x-y) - \delta^{a\kappa,a'\kappa'}_{\mu\mu'}\Pi(x-y)\right]\Phi^{a'\kappa'}_{\mu'}(y)$$
$$-\sum_{m=3}\frac{1}{m}\int \mathrm{d}^4x_1...\int \mathrm{d}^4x_m\prod_{k=1}^{m} h_{a_k\kappa_k}\Phi^{a_k\kappa_k}_{\mu_k}(x_k)$$
$$\times\Gamma^{a_1\kappa_1,...,a_m\kappa_m}_{\mu_1,...,\mu_m}(x_1,...,x_m)\ , \qquad (4.14)$$

$$\Gamma^{a_1\kappa_1,...,a_m\kappa_m}_{\mu_1,...,\mu_m}(x_1,...,x_m)$$
$$= \int \mathrm{d}\sigma_{\mathrm{vac}}\mathrm{Tr}\left\{V^{a_1\kappa_1}_{\mu_1}(x_1)S(x_1,x_2\mid B)\cdot...\cdot V^{a_m\kappa_m}_{\mu_m}(x_m)S(x_m,x_1\mid B)\right\} \quad (4.15)$$

Below we restrict ourselves by the one-loop approximation. The integral over $\mathrm{d}\sigma_{\mathrm{vac}}$ in Eq.(4.13) has been transferred to the exponential, that is accurate at the one-loop level and suitable for our nearest purposes.

Note that terms linear in fields $\Phi^{a\kappa}_{\mu}$ have not appeared in the action (4.14). They are subtracted by the counterterms in Eq.(4.12) related to the diagram in Fig.1.

Meson masses $M_{a\kappa}$ are defined by the equation

$$\Lambda^2 + G^2_{\kappa}\tilde{\Pi}_{a\kappa}(-M^2_{a\kappa}) = 0\ , \qquad (4.16)$$

where $\tilde{\Pi}_{a\kappa}(-M^2_{a\kappa})$ is the diagonal part of the polarization operator

$$\Pi^{a\kappa,a'\kappa'}_{\mu,\mu'}(x-y) = \int \mathrm{d}\sigma_{\mathrm{vac}}\mathrm{Tr}\left\{V^{a\kappa}_{\mu}(x)S(x,y\mid B)V^{a'\kappa'}_{\mu'}(y)S(y,x\mid B)\right\}. \qquad (4.17)$$

Due to the asymptotics (4.11) the polarization operator (4.17) is UV-finite. The constants

$$h_{a\kappa} = 1/\sqrt{\tilde{\Pi}'_{a\kappa}(-M^2_{a\kappa})} \qquad (4.18)$$

play a role of the effective coupling constants of the meson-quark interaction. Relation (4.18) agrees with the compositeness condition in quantum field theory [18].

Generating functional (4.13) satisfies all demands of the nonlocal QFT [19]. Particularly, this functional leads to the unitary S-matrix.. According to representation (4.13)-(4.15) meson-meson interaction is described by the vertex functions Γ given in Eq.(4.15). They are UV-finite and can be computed.

Free parameters defining the effective meson theory (4.13)–(4.15) have a clear physical meaning. They are the quark masses m_f, the confinement scale Λ (strength of the background field) and the four-fermion coupling constant g. The last one enters only through meson masses, which are calculated by means of Eq.(4.16).

5 Excited States of Mesons

As a rule, meson excitations are described within nonrelativistic potential models [20, 21], based on the Schrödinger equation. To provide equidistant character of the spectra in these models it is necessary to introduce the term $r^{2/3}$ into the bound potential [20]. Since these models are nonrelativistic, region of their validity is limited by the systems containing the heavy quarks. Excited states of light mesons are considered mostly within the so-called relativized Schrödinger equation [21], that is a modification of the potential picture of the bound state generation. The effective meson theory (4.13)–(4.15) realizes a quantum field theoretical, i.e., intrinsically relativistic, description of the meson masses, including the radial and orbital excitations. We have to study does such approach reproduce main features of the meson spectra? In order to answer this question we should calculate the polarization operator (4.17). Then we shall define the parameters $m_u = m_d$, m_s, Λ and g, providing the best description of the masses of π, K, ρ and K^* mesons and calculate the spectra of the radial and orbital excitations.

5.1 Polarization Operator

As soon as one has to solve Eq.(4.16) for the case of excited states of mesons, when the mass $M_{a\kappa}$ exceeds the confinement scale Λ and, hence, the leading terms in the polarization operator (4.17) relate to behavior of the quark propagator S_f and the vertex $V^{a\kappa}_{\mu}$ in region of large Minkowsky momenta, it would be quite reasonable to omit the phase factors (like $\exp\{i(x\hat{B}y)\}$) and use the following approximation (see Eqs.(3.7),(4.9)):

$$S_f(x,y\mid B) \sim H_f(x-y), \qquad (5.1)$$
$$V^{aJ\ell n}_{\mu_1...\mu_\ell}\left(\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right) \sim V^{aJ\ell n}_{\mu_1...\mu_\ell}\left(\frac{\overleftrightarrow{\partial}(x)}{\Lambda}\right) = M^a F_{n\ell}\left(\frac{\overleftrightarrow{\partial}^2(x)}{\Lambda^2}\right)T^{(\ell)}_{\mu_1...\mu_\ell}\left(\frac{1}{i}\frac{\overleftrightarrow{\partial}(x)}{\Lambda}\right).$$

This approximation simplifies the calculations, preserving at the same time the basic property of the model – the character of momentum dependence of the propagator and vertices.

It is convenient to use the representations:

$$T^{(\ell)}_{\mu_1...\mu_\ell}\left(\frac{1}{i}\frac{\overleftrightarrow{\partial}}{\Lambda}\right) = T^{(\ell)}_{\mu_1...\mu_\ell}\left(\frac{1}{i}\frac{\partial}{\partial\xi}\right)\exp\left\{\frac{\overleftrightarrow{\partial}}{\Lambda}\xi\right\}_{\xi=0},$$

$$F_{n\ell}\left(\frac{\overleftrightarrow{\partial}^2}{\Lambda^2}\right) = \int_0^1 dt t^{\ell+n}\left[\frac{d}{dt}\right]^n \int d\sigma_p \exp\left\{\sqrt{t}\frac{\overleftrightarrow{\partial}}{\Lambda}p\right\}, \tag{5.2}$$

$$d\sigma_p = \frac{d^4p}{\pi^2}\exp\left\{-p^2\right\}.$$

Substituting Eqs.(5.1) and (5.2) in the expression for the polarization operator (4.17) one obtains:

$$\Pi^{aJ\ell n}_{\mu_1\ldots\mu_\ell,\nu_1\ldots\nu_\ell}(x-y) = \mathrm{Tr}\int d\sigma_{\rm vac}\int_0^1 dt_1\int_0^1 dt_2 (t_1t_2)^{\ell+n}\iint d\sigma_p d\sigma_q\left(\frac{d}{dt_1}\frac{d}{dt_2}\right)^n$$
$$\times T^{(\ell)}_{\mu_1\ldots\mu_\ell}\left(\frac{\partial}{i\partial\xi}\right)T^{(\ell)}_{\nu_1\ldots\nu_\ell}\left(\frac{\partial}{i\partial\eta}\right)M^a\Gamma^J H(x-y-(\xi+\eta+\sqrt{t_1}p+\sqrt{t_2}q)/\Lambda)$$
$$\times M^a\Gamma^J H(y-x-(\xi+\eta+\sqrt{t_1}p+\sqrt{t_2}q)/\Lambda)\,|_{\xi=\eta=0}.$$

The sign Tr means trace of the Dirac matrices and summation over all elements of the diagonal matrix v. Taking into account Eqs.(5.2), we integrate over the variables p and q, then perform the Fourier transformation and get in the momentum representation:

$$\tilde{\Pi}^{aJ\ell n}_{\mu_1\ldots\mu_\ell,\nu_1\ldots\nu_\ell}(k) = \Lambda^2\mathrm{Tr}\int d\sigma_{\rm vac}\int_0^1 dt_1\int_0^1 dt_2 (t_1t_2)^{\ell+n}\int\frac{d^4q}{(2\pi)^4}$$
$$\times T^{\ell}_{\mu_1\ldots\mu_\ell}\left(\frac{\partial}{i\partial\xi}\right)T^{\ell}_{\nu_1\ldots\nu_\ell}\left(\frac{\partial}{i\partial\eta}\right)\left[\frac{d}{dt_1}\frac{d}{dt_2}\right]^n$$
$$\times M^a\Gamma^J\tilde{H}\left(q+\frac{k}{2}\right)M^a\Gamma^J\tilde{H}\left(q-\frac{k}{2}\right)\exp\left\{-(t_1+t_2)q^2+2iq(\xi+\eta)\right\}|_{\xi=\eta=0}.$$

All variables in the integrand were made dimensionless by rescaling with Λ. Differentiation with respect to ξ, η, t_1 and t_2 leads to the expression:

$$\tilde{\Pi}^{aJ\ell n}_{\mu_1\ldots\mu_\ell,\nu_1\ldots\nu_\ell}(k) = \Lambda^2\mathrm{Tr}\int d\sigma_{\rm vac}\int_0^1 dt_1\int_0^1 dt_2 (t_1t_2)^{\ell+n}\int\frac{d^4q}{(2\pi)^4}$$
$$\times T^{(\ell)}_{\mu_1\ldots\mu_\ell}(2q)T^{(\ell)}_{\nu_1\ldots\nu_\ell}(2q)(q^2)^{2n}$$
$$\times M^a\Gamma^J\tilde{H}\left(q+\frac{k}{2}\right)M^a\Gamma^J\tilde{H}\left(q-\frac{k}{2}\right)\exp\left\{-(t_1+t_2)q^2\right\}. \tag{5.3}$$

Further calculation implies an averaging over space directions of the background field. The generating formula can be taken in the form:

$$\langle\exp(if_{\mu\nu}J_{\mu\nu})\rangle = \frac{1}{4\pi}\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\sin\theta\exp(if_{\mu\nu}J_{\mu\nu}), \tag{5.4}$$

where φ and θ are the spherical angles and $J_{\mu\nu}$ is an antisymmetric tensor. After integration over the angles in Eq.(5.4) one obtains:

$$\langle\exp(if_{\mu\nu}J_{\mu\nu})\rangle = \frac{\sin\sqrt{2(J_{\mu\nu}J_{\mu\nu}\pm\tilde{J}_{\mu\nu}J_{\mu\nu})}}{\sqrt{2(J_{\mu\nu}J_{\mu\nu}\pm\tilde{J}_{\mu\nu}J_{\mu\nu})}}, \tag{5.5}$$

where $\tilde{J}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}J_{\alpha\beta}$. Differentiation of Eq.(5.5) at $J_{\mu\nu}=0$ gives the formulae:

$$\langle f_{\mu\nu}\rangle = 0,$$
$$\langle f_{\mu\nu}f_{\alpha\beta}\rangle = \frac{1}{3}(\delta_{\alpha\mu}\delta_{\beta\nu}-\delta_{\alpha\nu}\delta_{\beta\mu}\pm\varepsilon_{\alpha\beta\mu\nu}),$$

where the sign $\pm$ corresponds to the self-dual and antiself-dual configurations.

After calculation of the trace, averaging and integration over the loop momentum q one arrives at the expression for $\tilde{\Pi}$, which should be substituted into Eq.(4.16). For the case $n=\ell=0$ the final form of the diagonal part of the polarization operator is given in the Appendix. General form of $\tilde{\Pi}$ for $n\neq 0$ and $\ell\neq 0$ is quite cumbersome and we do not display it here.

5.2 Definition of the Model Parameters

Let us determine the values of parameters corresponding to observable masses of π, K, ρ and K^* mesons, i.e., for $n=0$ and $\ell=0$. Supposing $m_u = m_d$ we start with the equations:

$$\Lambda^2 + C_J g^2\tilde{\Pi}_{aJ00}\left(\frac{m_q}{\Lambda},\frac{M_{aJ00}}{\Lambda}\right) = 0, \tag{5.6}$$

where J=P,V; q=u,s. Index 'a' corresponds to the pseudoscalar (π, K) and vector (ρ,ϕ,K^*) particles and M_{aJ} are their masses. Equations (5.6) describe dependence of the masses M_{aJ} on the parameters of the model: the quark masses m_u, m_s, the strength of the background field Λ and the four-fermion coupling constant g.

Putting the masses of π, K, ρ and K^* mesons equal to their experimental values one finds the set of parameters summarized in Table 1.

Table 1. Values of parameters. Δ is the variation of parametres providing description of the spectrum with accuracy less than 10%.

parameter	the best fit	Δ
m_u, MeV	300	15
m_s, MeV	400	25
Λ, MeV	572	37
g	3.16	0.1

These values can be used to calculate the mass of ϕ meson, which is turned out to be equal

$$M_\phi = 1024\text{MeV}\quad(M_\phi^{\rm exp} = 1020\text{MeV}).$$

We see that the mass splitting between the pseudoscalar and vector mesons with identical quark content can be obtained, if one uses the coupling constant g common for pseudoscalar and vector mesons and takes into account the spin-field interaction $(\sigma_{\mu\nu}B_{\mu\nu})$ in the quark propagator. At the same time, elimination of the term $\sigma_{\mu\nu}B_{\mu\nu}$ in Eq.(3.6), i.e., turning off the quark spin degree of freedom, destroys this picture – mass splitting becomes unsatisfactory small. In the standard local NJL model successful description of the mass splitting requires to introduce two independent coupling constants $g_P \neq g_V$ for vector and pseudoscalar particles [14].

5.3 Regge Behavior

First of all, let us consider the asymptotic solutions of Eq.(4.16), subjected to the condition:

$$M^2_{aJ\ell n} \gg \Lambda^2\,, \quad \text{if } n \gg \ell \text{ (or } \ell \gg n) \,. \tag{5.7}$$

After some calculations, based on the Laplace method, one can get from Eq.(5.3) the following asymptotic expression for $\tilde{\Pi}_{aJ\ell n}$

$$\tilde{\Pi}^{aJ\ell n}(-M^2_{aJ\ell n}) \sim -\Lambda^2 \frac{(2n+\ell)!2^\ell}{5^{2n+\ell}} \exp\left\{\frac{3}{4}\frac{M^2_{aJ\ell n}}{\Lambda^2}\right\}\,, \tag{5.8}$$

Only factorial and exponential over n and ℓ factors are written in Eq.(5.8), since only they determine the asymptotic behavior of the spectrum. The formula (5.8) becomes quite obvious, if one notices that both quark propagator (3.7) and the vertex (4.9) has an exponential behavior in the Minkowsky region. Taking into account Eqs.(5.8), (4.8) and (4.16), we obtain the equation

$$\frac{(2n+\ell)!}{n!(n+\ell)!5^{2n+\ell}} \exp\left\{\frac{3}{4}\frac{M^2_{aJ\ell n}}{\Lambda^2}\right\} \sim 1\,. \tag{5.9}$$

Let us consider two limits.

- $n \gg \ell$ Eq.(5.9) takes the form

$$\exp\left\{\frac{3}{4}\frac{M^2_{aJ\ell n}}{\Lambda^2}\right\} \sim \frac{(n!)^2}{(2n)!}5^{2n} \approx \left(\frac{5}{2}\right)^{2n}\,,$$

or

$$M^2_{aJ\ell n} = \frac{8}{3}\ln\left(\frac{5}{2}\right)\cdot\Lambda^2\cdot n + O(\ln n)\,. \tag{5.10}$$

- $\ell \gg n$ The equation (5.9) becomes:

$$\exp\left\{\frac{3}{4}\frac{M^2_{aJ\ell n}}{\Lambda^2}\right\} \sim 5^\ell\,, \quad M^2_{aJ\ell n} = \frac{4}{3}\ln 5\cdot\Lambda^2\cdot\ell + O(\ln \ell)\,. \tag{5.11}$$

One can see that Eqs.(5.10) and (5.11) manifest equidistant character of the meson spectrum both for large n and ℓ. We would like to stress that Regge behavior in the model is a consequence of nonlocality of the quark propagator (3.7) and the vertex function (4.9) as well as the form of the coupling constant $G_{J\ell n}$ (4.8), that is conditioned completely by the vacuum gluon field under consideration.

Now, using the parameters m_u, m_s, g and Λ given in the Table 1, one can find solutions of Eq.(4.16) for the lowest excitations of mesons. There are rich experimental data for orbital excitations of π, ρ and K^* mesons [21]. The experimental and calculated by means of Eq.(4.16) masses are summarized in the Table 2. One can see that for real values of ℓ the model describes experimental data quantitatively (with an accuracy about ten percent).

Table 2. The masses of the excited meson states.

Meson	ℓ	$M_{aJ\ell 0}$, MeV	$M^{\rm exp}_{aJ\ell 0}$, MeV
π	0	140	140
b_1	1	1206	1235
π_2	2	1599	1670
ρ	0	770	770
a_2	1	1248	1320
ρ_3	2	1514	1690
K^*	0	892	892
K^*_2	1	1367	1430
K^*_3	2	1623	1780

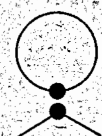

Fig.1 The divergent bubble diagram.

Appendix

The diagonal part of the polarization operator has the form ($\ell = 0,\ n = 0$)

$$\tilde{\Pi}^{aJ}(k^2) = \frac{v^2}{\pi^2} M^a_{ff'} M^a_{f'f} \int\limits_0^1\int\limits_0^1 dt_1 dt_2 \int\limits_0^1\int\limits_0^1 \frac{ds_1 ds_2}{(s_1+s_2+2v(t_1+t_2))^4} \frac{(1-s_1)^{\beta_{f'}}}{(1+s_1)^{1+\beta_{f'}}}$$
$$\times \frac{(1-s_2)^{\beta_f}}{(1+s_2)^{1+\beta_f}} K^J_{f'f}(s_1,s_2,t_1,t_2;k^2) \exp\left\{-\frac{2s_1s_2+v(t_1+t_2)(s_1+s_2)}{2v(t_1+t_2)+s_1+s_2}\frac{k^2}{4v\Lambda^2}\right\},$$

where $\beta_f = \alpha_f^2/4v - 1,\ \alpha_f = m_f/\Lambda$ and function $K^J_{f'f}(s_1,s_2,t_1,t_2,k^2)$ has the form:

$$K^J_{f'f}(s_1,s_2,t_1,t_2;k^2) = F^J_1 + \alpha_{f'}\alpha_f F^J_2 + F^J_3\frac{k^2}{\Lambda^2}.$$

For pseudoscalar mesons ($J = P$) the functions F^J_1, F^J_2, F^J_3 look as

$$F^P_1 = 8(t_1+t_2+\frac{s_1+s_2}{2v})\,\frac{(1+s_1s_2)(1-s_1^2-s_2^2+s_1^2s_2^2)}{(1+s_1)(1+s_2)},$$
$$F^P_2 = -4(t_1+t_2+\frac{s_1+s_2}{2v})^2\,\frac{(1+s_1s_2)^2}{(1+s_1)(1+s_2)},$$
$$F^P_3 = \frac{(1-s_1s_2)^2(s_1-s_2)^2+(s_1-s_2)^3(s_1(1-s_2^2)-s_2(1-s_2^2))}{4v^2(1+s_1)(1+s_2)}$$
$$+\frac{1}{8}(t_1+t_2+\frac{s_1+s_2}{2v})F^P_1.$$

For vector mesons ($J = V$) one obtains:

$$F^V_1 = -4(t_1+t_2+\frac{s_1+s_2}{2v})\,\frac{(1-s_1s_2)^2+(s_1-s_2)(s_1(1+s_2^2)-s_2(1+s_1^2))}{(1+s_1)(1+s_2)},$$
$$F^V_2 = -4(t_1+t_2+\frac{s_1+s_2}{2v})^2\,\frac{1-s_1^2s_2^2}{(1+s_1)(1+s_2)},$$
$$F^V_3 = \frac{(s_1-s_2)}{12v^2(1+s_1)(1+s_2)}\,[\,(3-s_1s_2)(1+s_1^2s_2^2)+2s_1s_2(3s_1s_2-1)$$
$$+s_1(1+s_2^2)(3s_1-s_2)+s_2(1+s_1^2)(3s_2-s_1)\,]$$
$$+\frac{(2v(t_1+t_2)+s_1+s_2)^2}{12v^2(1+s_1)(1+s_2)}\,[\,(3-s_1s_2)(1+s_1^2s_2^2)+(1+s_1^2)(s_1-s_2)s_2$$
$$+(1+s_2^2)(s_1-s_2)s_1+2s_1s_2(3s_1s_2-1)\,].$$

References

[1] E.V.Shuryak, Phys.Rep.**115** (1984) 151

[2] D.I.Dyakonov, V.Yu.Petrov, Nucl.Phys.**B245** (1984) 259

[3] C.G.Callan, R.Dashen, D.J.Gross, Phys.Rev.**D17** (1978) 2717

[4] M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl.Phys.**B147** (1979) 385

[5] J.Ambjorn, N.K.Nilsen, P.Olesen, Nucl.Phys. **B152** (1979) 75

[6] H.Leutwyler, Nucl.Phys.**B179** (1981) 129; *ibid*, Phys.Lett.**96B** (1980) 154

[7] S.G.Matinyan, G.K.Savvidy, Nucl.Phys.**B134** (1978) 539

[8] G.Preparata, Phys.Lett. **B201** (1988) 139; preprint LNF-86/2(P); preprint MITH 89/1 (1989) Milano

[9] Y.Nambu, G.Jona-Lasinio, Phys.Rev.**122** (1961) 345

[10] T.Eguchi,Phys.Rev.**D14**, (1976)2755;
T.Goldman, R.W.Haymaker, Phys.Rev.**D24** (1981) 724

[11] G.V.Efimov, M.A.Ivanov, "The Quark Confinement Model of Hadrons", IOP, Bristol and Philadelfia, 1993

[12] C.D.Roberts, R.T.Cahill, Aust.J.Phys. **40** (1987) 499

[13] G.V. Efimov, S.N. Nedelko, Heidelberg Univ. preprint No. 693 (1992), to be published elsewhere

[14] S.P.Klevansky Rev.Mod.Phys.**Vol.64** (1992) 649

[15] L.F.Abbott, Nucl.Phys.**B185** (1981) 189

[16] J.Finjord, Nucl.Phys.**B194** (1982) 77

[17] E.Elizalde, Nucl.Phys.**B243** (1984) 398;
E.Elizalde, J.Soto, Nucl.Phys.**B260** (1985) 136

[18] K.Hayashi *et al*, Fortsch.Phys. **15** (1967) 625

[19] G.V.Efimov, "Nonlocal Interactions of Quantized Fields", Nauka, Moscow, 1977

[20] F.Paccanoni, S.S.Stepanov, R.S.Tutik, Mod.Phys.Lett. **A8** (1993) 549

[21] W.Lucha, F.Schöberl, D.Gromes, Phys. Rep. **Vol.200** (1991) 127

[22] F.E.Close, "An Introduction to Quarks and Partons", Acad. Press, London, 1979

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