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MASS SPLITTING OF THE PSEUDOSCALAR AND VECTOR MESONS INDUCED BY THE HOMOGENEOUS VACUUM GLUON FIELD[1]

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Расщепление масс псевдоскалярных и векторных мезонов, порождаемое однородным вакуумным глюонным полем

Массы псевдоскалярных и векторных мезонов вычислены в рамках обобщенной модели Намбу — Иона-Лазинио с однородным вакуумным глюонным полем. Наличие вакуумного поля приводит к аналитическому конфайнменту кварков. Бесцветные моды определяются конфайнмированными глюонами и описываются нелокальными кварковыми токами с соответствующими радиальными и орбитальными квантовыми числами. В рамках рассматриваемой модели естественно возникает взаимодействие спина кварка с вакуумным глюонным полем. Показано, что это взаимодействие ведет к расщеплению масс псевдоскалярных и векторных мезонов с одинаковой кварковой структурой ($\rho-\pi$, $K-K^*$ и т.п.). В отличие от стандартной модели Намбу — Иона-Лазинио это позволяет использовать общую для обоих нонетов константу четырехфермионного взаимодействия.

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Mass Splitting of the Pseudoscalar and Vector Mesons Induced by the Homogeneous Vacuum Gluon Field

Masses of the pseudoscalar and vector mesons are calculated within the generalized Nambu — Jona-Lasinio model taking into account homogeneous vacuum gluon field. This vacuum provides an analytical quark confinement. Colorless modes are determined by the confined gluons and are described by the nonlocal quark currents with appropriate radial and angular quantum numbers. An interaction of the quark spin with the vacuum gluon field arises naturally within the model under consideration. It is shown that this spin-field interaction leads to mass splitting between vector and pseudoscalar mesons with identical quark structure ($\rho-\pi$, $K-K^*$ and so on). In contrast to the standard NJL-model, this allows to use the four-fermion coupling constant being common for both nonets.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

Vacuum self-dual gluon fields like instanton solutions [1]-[4], stochastic fields [5] or the fields with a constant strength [6, 7, 8] are widely used to explain various features of the low-energy hadron physics. In particular, H. Leutwyler has shown that (anti)self-dual homogeneous gluon field provides an analytical quark confinement [6]. In contrast to pure chromomagnetic or chromoelectric configurations, self- or antiself-dual fields are stable in a sense that the effective potential for these fields is a real function. Unfortunately, numerous attempts to estimate the field strength, minimizing the effective potential, have not given definite result. A general reason for this is quite clear: phase transitions in quantum field systems accompanied by appearance of nonzero vacuum fields occur out of perturbation region and their successful investigation is damped by a lack of nonperturbative methods. At present time, a task to prove an existance of the vacuum field and to estimate its strength, starting with the first principles, seems to be quite complicated.

We shall follow another, in some sense phenomenological, point of view. Namely, we suppose, that self- or antiself-dual homogeneous gluon field realizes the QCD vacuum at low energies, and find out the points in hadron physics where this vacuum field can play an important role. Our consideration is based on the bosonization procedure of the standard NJL-model [9, 10]. At the same time, taking into account the background field both in the quark and gluon propagators requires essential modification of this procedure.

Starting with the Euclidean generating functional of QCD with the background gluon field [2, 11] we construct the color singlet bilocal quark currents. Confined gluon fields ensure a natural expansion of the bilocal quark currents over the nonlocal ones with appropriate radial and orbital quantum numbers. An idea of such expansion was discussed in general form in [12]. Realization of this idea implies an existance of a set of orthonormalized functions. Particular form of these functions reflects specific physical peculiarities of the system. We show that the homogeneous (anti)self-dual vacuum field determines quite definite set – generalized Laguerre polynomials. As a result of the expansion, an interaction of quarks is realized by the current-current terms in the effective Lagrangian, the currents are nonlocal and carry radial and orbital quantum numbers. In contrast to nonrenormalizable local NJL model, our generalization leads to the effective four-fermion theory, that is superrenormalizable due to a nonlocality of the currents.

By means of the standard NJL bosonization we get a representation of generating functional in terms of the local meson fields, interacting with the nonlocal quark currents. These meson fields have a complete set of quantum numbers including radial n and orbital ℓ ones. Effective meson theory is ultraviolet finite due to nonlocal meson interactions. It should be noted that we are based on the representation for generating functional of QCD, that implies an averaging over some parameters of background field (for detailes see [2, 11]). In the case of the homogeneous field we have to average over self- and antiself-dual configurations and over directions of the field. Due to this averaging all amplitudes at hadron level are invariant under space rotations and parity transformation.

Parametrization of the generalized model is quite natural. The quark masses, the four-fermion coupling costant and the background field strength are the free parameters.

In presence of the background field under consideration the quark propagator and the meson-quark vertices are nonlocal. This nonlocality has an interesting consequence –

the spectra of radial and orbital meson excitations are equidistant [13]. In other words, the homogeneous self-dual vacuum field provides Regge spectra of the meson excitations.

In this paper we pay attention to another point where the vacuum gluon field seems to be quite important. From the quantum mechanical point of view, difference between the masses of the pseudoscalar and vector mesons with identical quark content relates to the contact spin-spin interaction of quarks. In relativistic field theory an interaction of this kind should be realized *via* vector gauge field.

An interaction of the quark spin with the vacuum gluon field arises naturally within the generalized NJL-model. It is taken into account in the quark propagator through the term $\gamma_\mu B_{\mu\nu}\gamma_\nu$, where γ_μ are the Dirac matrices and $B_{\mu\nu}$ is the background field strength tensor. It is shown that this spin-field interaction leads to mass splitting between the vector and pseudoscalar mesons with identical quark structure (ρ-π, K-K^* and so on). Accounting of this dynamical reason for the mass splitting reduces number of free parameters of the model in comparison with the local NJL-model, where successful description of the meson masses requires four-fermion coupling constants to be different for the pseudoscalar and vector nonets [14].

The paper is organized as follows. In Sect. 2,3 we introduce all definitions and discuss the quark and gluon dynamics in the background field. The collective modes are considered in Sect. 4. The masses of the pseudoscalar and vector mesons are calculated in Sect. 5.

2 Euclidean generating functional

After integration over the quantum gluon fields A^a_μ the generating functional for QCD with the background gluon field B^a_μ in the Euclidean metrics can be represented in the form [2, 11]:

$$Z = N\int d\sigma_{\rm vac} DqD\bar q\,\exp\left\{\int d^4x\sum_f^{N_f}\bar q_f(x)(i\gamma_\mu\hat\nabla_\mu - m_f)q_f(x) + \sum_{n=2}^{\infty}L_n\right\}, \quad (2.1)$$

where

$$L_n = \frac{g^n}{n!}\int d^4y_1 ... \int d^4y_n j^{a_1}_{\mu_1}(y_1)\cdot ... \cdot j^{a_n}_{\mu_n}(y_n) G^{a_1...a_n}_{\mu_1...\mu_n}(y_1, ..., y_n \mid B),$$
$$j^a_\mu(y) = \bar q_f(x)\gamma_\mu t^a q_f(x).$$

The representation (2.1) implies that the vacuum field B^a_μ is characterized by a set of parameters $\{\sigma_{\rm vac}\}$ and the vacuum state is degenerated respecting to change of these parameters. Integration over $d\sigma_{\rm vac}$ means that all amplitudes have to be averaged over $\{\sigma_{\rm vac}\}$. The function $G^{a_1...a_n}_{\mu_1...\mu_n}$ is the exact (up to the quark loops) n-point gluon Green function in the external field B^a_μ. Here and below we use the notation:

$$\hat\nabla_\mu = \partial_\mu - ig\hat B_\mu\,,\quad \hat B_\mu = B^a_\mu t^a\,,\quad \check\nabla_\mu = \partial_\mu - ig\check B_\mu\,,\quad \check B_\mu = B^a_\mu C^a\,,$$

where the matrices t^a and C^a are the generators of the color group $SU_c(3)$ in the fundamental and adjoint representations.

We will be interested in $(q\bar q)$-collective modes and consider the form of Eq.(2.1) truncated up to the term L_2

$$Z = N\int d\sigma_{\rm vac} DqD\bar q\exp\left\{\int d^4x\sum_f^{N_f}\bar q_f(x)(i\gamma_\mu\hat\nabla_\mu - m_f)q_f(x) + L_2\right\}, \quad (2.2)$$
$$L_2 = \frac{g^2}{2}\int d^4x\int d^4y j^a_\mu(x) G^{ab}_{\mu\nu}(x, y \mid B) j^b_\nu(y).$$

Representations (2.1) and (2.2) are manifestly invariant under the background gauge transformations [15]:

$$q_f(x)\to e^{-i\hat\omega(x)}q_f(x)\,,\quad \hat A_\mu \to e^{-i\hat\omega(x)}\hat A_\mu e^{i\hat\omega(x)},$$
$$\hat B_\mu \to e^{-i\hat\omega(x)}\hat B_\mu e^{i\hat\omega(x)} + \frac{i}{g}e^{-i\hat\omega(x)}\partial_\mu e^{i\hat\omega(x)}. \quad (2.3)$$

The standard NJL-model corresponds to the generating functional (2.2) with

$$B^a_\mu = 0 \text{ and } G^{ab}_{\mu\nu}(x,y) = \delta^{ab}\delta_{\mu\nu}\delta(x-y).$$

We will approximate the two-point Green function in Eq. (2.2) by the gluon propagator in the external field. In other words, generalization of the NJL-model consists in taking into account an influence of the background field both on the gluon and quark propagators.

3 Quarks and gluons in the homogeneous background field

The homogeneous (anti)self-dual gluon field has the following form (e.g., see [6, 7, 16, 17])

$$B^a_\mu(x) = B^a_{\mu\nu}x_\nu = n^a B_{\mu\nu}x_\nu\,,\quad n^2 = 1\,,$$

where the vector n defines a direction in the color space. The constant tensor $B_{\mu\nu}$ satisfies the following conditions:

$$B_{\mu\nu} = -B_{\nu\mu}\,,\quad B_{\mu\rho}B_{\rho\nu} = -B^2\delta_{\mu\nu}\,,\quad \tilde B_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}B_{\alpha\beta} = \pm B_{\mu\nu}\,. \quad (3.1)$$

The gauge invariant quantity B is a tension of the background field.

Any matrix $\hat n = n^a t^a \in SU_c(3)$ can be reduced to the general form

$$\hat n = t^3\cos\xi + t^8\sin\xi\,,\quad 0 \le \xi < 2\pi$$

by appropriate global gauge transformation. As soon as the chromomagnetic $\vec H$ and chromoelectric $\vec E$ fields relates to each other like $\vec H = \pm\vec E$ for the (anti)self-dual configuration, one has only two spherical angles (θ, φ) defining a direction of the fields in the x-space. Now we can write down the explicit form of the measure $d\sigma_{\rm vac}$:

$$\int d\sigma_{\rm vac} = \frac{1}{(4\pi)^2}\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\varphi\int_0^{2\pi}d\xi\sum_\pm\,, \quad (3.2)$$

where the sign $\sum_{\pm}$ denotes averaging over the self- and antiself-dual configurations, which are assumed to be equiprobable.

The averaging over ξ can be included into the formalizm described below. However, in order to simplify further calculations and clarify a contribution of the background field under consideration into forming the bound quark systems we will omit the integral over ξ in Eq.(3.3) and fix particular vector $n^a = \delta^{a8}$, so that:

$$\hat{n} = t^8 = \mathrm{diag}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right), \quad \hat{B}_{\mu\rho}\hat{B}_{\rho\nu} = -(t^8)^2 B^2 \delta_{\mu\nu} = -v^2 \Lambda^4 \delta_{\mu\nu} \,,$$

where we use notation:

$$\Lambda^2 = \sqrt{3}B \,, \quad v = \mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{2}{3}\right) ,$$

that is convenient for further calculations.

In the adjoint representation one has

$$\breve{n} = C^8 = \frac{\sqrt{3}}{2}K \,, \quad \breve{B}_{\mu\rho}\breve{B}_{\rho\nu} = -\frac{3}{4}K^2 B^2 \delta_{\mu\nu} \,,$$
$$K_{54} = -K_{45} = K_{76} = -K_{67} = i \,, \quad K^2 = \mathrm{diag}(0,0,0,1,1,1,1,0) \,.$$

The rest of elements of the matrix K are equal to zero.

The quark and gluon propagators satisfy the equations ($gB^a_\mu \equiv B^a_\mu$):

$$(i\gamma_\mu \hat{\nabla}_\mu - m_f) S_f(x, y \mid B) = -\delta(x-y) \,, \tag{3.3}$$
$$(\breve{\nabla}^2 \delta_{\mu\nu} + 4i\breve{B}_{\mu\nu}) G_{\nu\rho}(x, y \mid B) = -\delta_{\mu\rho}\delta(x-y) \,. \tag{3.4}$$

The equations (3.4) and (3.5) can be solved using the Schwinger proper time technique (e.g., see [11]). The quark propagator takes the following form:

$$S_f(x, y \mid B) = \frac{i\hat{\nabla}_\mu \gamma_\mu + m_f}{\hat{\nabla}^2 - m_f^2 + \sigma_{\mu\nu}\hat{B}_{\mu\nu}} \delta(x-y) = \mathrm{e}^{\frac{i}{2}(x\hat{B}y)} H_f(x-y \mid B) \mathrm{e}^{\frac{i}{2}(x\hat{B}y)} \,, \tag{3.5}$$

$$\tilde{H}_f(p \mid B) = \frac{1}{v\Lambda} \int_0^1 \mathrm{d}s\, \mathrm{e}^{-\frac{p^2}{2v\Lambda^2}s} \left(\frac{1-s}{1+s}\right)^{\frac{\alpha_f^2}{4v}} \left[\alpha_f + \frac{1}{\Lambda} p_\mu \gamma_\mu - is\frac{1}{\Lambda}(\gamma f p)\right]$$
$$\times \left[P_\pm + P_\mp \frac{1+s^2}{1-s^2} - \frac{i}{2}(\gamma f \gamma)\frac{s}{1-s^2}\right] , \tag{3.6}$$

where the following notation are introduced (see also Eq.(3.1)):

$$P_\pm = \frac{1}{2}(1 \pm \gamma_5) \,, \quad \alpha_f = \frac{m_f}{\Lambda} \,, \quad (xBy) = x_\mu B_{\mu\nu} y_\nu \,,$$
$$(pf\gamma) = p_\mu f_{\mu\nu} \gamma_\nu \,, \quad f_{\mu\nu} = \frac{t^8}{v\Lambda^2} B_{\mu\nu} \,, \quad f_{\mu\rho} f_{\rho\nu} = -\delta_{\mu\nu} \,.$$

The upper (lower) sign in the matrix P corresponds to the self-dual (antiself-dual) field.

Let us introduce the variable $\mu = p_\nu \gamma_\nu$. The function $\tilde{H}_f(\mu \mid B)$ is an entire analytical function in the complex μ-plane. It means that there is no pole corresponding to quark

mass. In other words, the so-called analytical quark confinement is manifested. The parameter Λ (tension B) determines a characteristic region of the variation of the function $\tilde{H}_f(p|B)$, i.e., it defines the scale of confinement.

The term $\sigma_{\mu\nu}\hat{B}_{\mu\nu}$ in the quark propagator (3.6) can be interpreted as interaction of quark spin with the vacuum field. The second line in Eq.(3.7) relates just to the term $\sigma_{\mu\nu}\hat{B}_{\mu\nu}$ in Eq. (3.6). We shall discuss an influence of this spin-field interaction on meson masses in Sect. 5.

Gluon propagator can be represented in the form (see Eq.(3.5)) :

$$G_{\mu\nu}(x, y \mid B) = \mathrm{e}^{\frac{i}{2}(x\breve{B}y)} D_{\mu\nu}(x-y \mid B) \mathrm{e}^{\frac{i}{2}(x\breve{B}y)} \,, \tag{3.7}$$
$$D^{ab}_{\mu\nu}(z \mid B) = \delta_{\mu\nu}[K^2]^{ab} D(z \mid \Lambda^2) + R^{ab}_{\mu\nu}(z),$$
$$R^{ab}_{\mu\nu}(z \mid B) = \delta_{\mu\nu}[(1-K^2)D(z \mid 0) + K^2 D_0(z \mid \Lambda^2)]]^{ab} + 2if_{\mu\nu}K^{ab}D_1(z \mid \Lambda^2) \,;$$
$$D(z|\Lambda^2) = \frac{1}{(2\pi)^2 z^2} \exp\left\{-\frac{\Lambda^2 z^2}{4}\right\} . \tag{3.8}$$

The Fourier transform of the function $D(z \mid \Lambda^2)$ is an entire analytical function in the momentum space. It describes a propagation of confined modes of the gluon fields. We suppose that these confined modes play crucial role in generation of hadron bound states. Other terms $R^{ab}_{\mu\nu}(z)$ do not take any significant part in forming the bound states and will be omitted below.

4 Colorless collective modes

According to Eq.(3.8) the term L_2 in Eq.(2.2) looks:

$$L_2 = \frac{g^2}{2} \iint \mathrm{d}^4x \mathrm{d}^4y \{\bar{q}_f(x)\gamma_\mu t^a \left[\mathrm{e}^{\frac{i}{2}(x\breve{B}y)}\right]^{aa'} q_f(x)\} D^{a'b'}_{\mu\nu}(x-y \mid B)$$
$$\times \{\bar{q}_{f'}(y)\gamma_\nu \left[\mathrm{e}^{\frac{i}{2}(x\breve{B}y)}\right]^{b'b} t^b q_{f'}(y)\} \,.$$

Using the identity

$$t^a_{kj}\left[\mathrm{e}^{-i\breve{\omega}}\right]^{aa'} = \left[\mathrm{e}^{-i\hat{\omega}} t^{a'} \mathrm{e}^{i\hat{\omega}}\right]_{kj} , \quad (\breve{\omega} = \omega^a C^a \quad \hat{\omega} = \omega^a t^a \,) ,$$

one can get

$$L_2 = \frac{g^2}{2} \iint \mathrm{d}^4x \mathrm{d}^4y J^a_\mu(x,y) D^{ab}_{\mu\nu}(x-y \mid B) J^b_\mu(y,x) \,,$$
$$J^a_\mu(x,y) = \bar{Q}_f(x,y)\gamma_\mu t^a Q_f(x,y) \,,$$

where $Q_f(x,y) = \mathrm{e}^{-\frac{i}{2}(x\hat{B}y)} q_f(x)$, $\bar{Q}_f(x,y) = \bar{q}_f(x) \mathrm{e}^{\frac{i}{2}(x\hat{B}y)}$.

Then the Fierz transformation of the color, flavor (we consider $N_f = 3$) and Dirac matrices is performed. Keeping only the scalar J^{aS}, pseudoscalar J^{aP}, vector J^{aV} and axial vector J^{aA} colorless currents we obtain:

$$L_2 = \frac{g^2}{2}\sum_{aJ} C_J \iint d^4x d^4y D(x-y \mid \Lambda^2) J^+_{aJ}(x,y) J_{aJ}(x,y) , \tag{4.1}$$

$$J_{aJ}(x,y) = \bar{q}_f(y) M^a_{ff'} \Gamma^J e^{i(y\hat{B}x)} q_{f'}(x) , \tag{4.2}$$

$$\Gamma^S = 1 , \quad \Gamma^P = i\gamma_5 , \quad \Gamma^V = \gamma_\mu , \quad \Gamma^A = \gamma_5\gamma_\mu ,$$

$$C_S = C_P = \frac{1}{9} , \quad C_V = C_A = \frac{1}{18} ,$$

M^a–flavor mixing matrices, being equal to the matrices λ^a or their linear combinations $(a = 0, ..., 8)$. The currents (4.2) are the scalars under color group transformations (2.3).

Under transformation $(x \to x + \frac{1}{2}y , \quad y \to x - \frac{1}{2}y)$ representation (4.1) takes the form:

$$L_2 = g^2 \sum_{aJ} C_J \iint d^4x d^4y D(y \mid \Lambda^2) J^+_{aJ}(x,y) J_{aJ}(x,y) ,$$

$$J_{aJ}(x,y) = \bar{q}(x) M^a \Gamma^J e^{-\frac{1}{2}y\overleftrightarrow{\nabla}(x)} q(x) ,$$

$$\overleftrightarrow{\nabla}_\mu (x) = \overrightarrow{\nabla}_\mu (x) - \overleftarrow{\nabla}_\mu (x) , \quad \overrightarrow{\nabla}_\mu (x) = \overrightarrow{\partial}_\mu (x) - i\hat{B}_\mu(x) ,$$

$$\overleftarrow{\nabla}_\mu (x) = \overleftarrow{\partial}_\mu (x) + i\hat{B}_\mu(x) , \quad \partial_\mu(x) = \frac{\partial}{\partial x_\mu} .$$

From a quantum mechanical viewpoint, description of the radial and orbital excitations is provided by an appropriate orthogonal set of polynomials. It means that a decomposition of the bilocal currents has the form:

$$J_{aJ}(x,y) = \sum_{n\ell} (y^2)^{\ell/2} f^{n\ell}_{\mu_1...\mu_\ell}(y) J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) , \tag{4.3}$$

$$f^{\ell n}_{\mu_1...\mu_\ell}(y) = L_{n\ell}(y^2) T^{(\ell)}_{\mu_1...\mu_\ell}(n_y) \quad (n_y = y/\sqrt{y^2}) .$$

The irreducible tensors of the four-dimensional rotational group $T^{(\ell)}_{\mu_1...\mu_\ell}$ are orthogonal:

$$\int_\Omega \frac{d\omega}{2\pi^2} T^{(\ell)}_{\mu_1...\mu_\ell}(n_y) T^{(k)}_{\nu_1...\nu_k}(n_y) = \frac{1}{2^\ell(\ell+1)} \delta^{\ell k} \delta_{\mu_1\nu_1} ... \delta_{\mu_\ell\nu_\ell} , \tag{4.4}$$

and are subjected to the conditions:

$$T^{(\ell)}_{\mu_1...\mu...\nu...\mu_\ell}(n_y) = T^{(\ell)}_{\mu_1...\nu...\mu...\mu_\ell}(n_y) , \quad T^{(\ell)}_{\mu...\mu...\mu_\ell}(n_y) = 0,$$

$$T^{(\ell)}_{\mu_1...\mu_\ell}(n_y) T^{(\ell)}_{\mu_1...\mu_\ell}(n_{y'}) = \frac{1}{2^\ell} C^{(1)}_\ell(n_y n_{y'}) . \tag{4.5}$$

The measure $d\omega$ in Eq.(4.4) relates to integration over the spherical angles of the vector n_y and $C^{(1)}_\ell$ in Eq.(4.5) are the Gegenbauer (ultraspherical) polynomials. The polynomials $L_{n\ell}(u)$ obey the condition:

$$\int_0^\infty du \rho_\ell(u) L_{n\ell}(u) L_{n'\ell}(u) = \delta_{nn'} .$$

The function $D(y|\Lambda^2)$ (see Eq.(3.8)) leads to the weight function

$$\rho_\ell(u) = u^\ell e^{-u} ,$$

that corresponds to the generalized Laguerre's polynomials $L_{n\ell}(u)$. Other terms of the gluon propagator do not provide any possibility for decomposition like (4.3). Such a situation suggests that only confined gluons (contributing to $D(y|\Lambda^2)$) provide the generation of bound states. At the same time, the function D accumulates the nonabelian nature of the gluons – their self-interaction. This is the point, where QCD differs from quantum electrodynamics actually. As a result we have a mechanism of the generation of bound states, that is different from the potential picture.

The decomposition leads to the representation for L_2 [13]:

$$L_2 = \sum_{aJ\ell n} \frac{1}{2\Lambda^2} G^2_{J\ell n} \int d^4x \left[J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) \right]^2 , \tag{4.6}$$

$$G^2_{J\ell n} = C_J\, g^2 \frac{(\ell+1)}{2^\ell n! (\ell+n)!} , \quad J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) = \bar{q}(x) V^{aJ\ell n}_{\mu_1...\mu_\ell}(x) q(x) , \tag{4.7}$$

$$V^{aJ\ell n}_{\mu_1...\mu_\ell}(x) \equiv V^{aJ\ell n}_{\mu_1...\mu_\ell}\left(\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right)$$

$$= M^a \Gamma^J \left\{\!\!\left\{ F_{n\ell}\left(\frac{\overleftrightarrow{\nabla}^2(x)}{\Lambda^2}\right) T^{(\ell)}_{\mu_1...\mu_\ell}\left(\frac{1}{i}\frac{\overleftrightarrow{\nabla}(x)}{\Lambda}\right) \right\}\!\!\right\}, \tag{4.8}$$

$$F_{n\ell}(s) = \left(\frac{s}{4}\right)^n \int_0^1 dt\, t^{\ell+n} \exp\left\{\frac{1}{4}st\right\} . \tag{4.9}$$

The doubled brackets in Eq.(4.8) mean that the covariant derivatives commute inside this brackets. The functions $F_{n\ell}(s)$ are entire analytical functions in the complex s-plane, they obey confinement condition.

The vertex functions have the following asymptotic behavior for large Euclidean momentum:

$$\lim_{p^2\to\infty} F_{n\ell}[p^2] T^{(\ell)}_{\mu_1...\mu_\ell}[p] \sim \frac{1}{(p^2)^{1+\ell/2}} , \tag{4.10}$$

Therefore, there is single divergent diagram given in Fig.1. This divergency can be removed by the counter-term of the form $-2J(x)\mathrm{Tr}VS$, which can be inserted into the interaction Lagrangian (4.6). After this renormalization we arrive at the expression:

$$Z = \int d\sigma_{vac} Dq D\bar{q} \exp\left\{ \iint d^4x d^4y \bar{q}(x) S^{-1}(x,y|B) q(y) \right.$$
$$\left. + \sum_{aJ\ell n} \frac{1}{2\Lambda^2} G^2_{J\ell n} \int d^4x \left[J^{aJ\ell n}_{\mu_1...\mu_\ell}(x) - \mathrm{Tr} V^{aJ\ell n}_{\mu_1...\mu_\ell} S \right]^2 \right\} . \tag{4.11}$$

The sign $d\sigma_{vac}$ means an averaging over self- and antiself-dual configurations and directions of the background field.

Representation ((4.11),(4.7)–(4.9)) manifests the following properties. The quarks are confined due to background field. The nonlocal quark currents (4.7) are invariant under transformations (2.3), i.e., they are color singlets. These currents have a suitable set of quantum numbers including the radial n and orbital ℓ ones. The effective theory determined by Eqs.((4.11),(4.7)–(4.9)) is UV-finite due to a nonlocality of the vertex (4.8).

The quark masses m_f, four-fermion coupling constant g and scale Λ are the parameters of the effective theory, defined in Eqs.((4.11),(4.7)–(4.9)).

Starting with the representation (4.11) one can introduce the local meson-like fields by means of the bosonization procedure like in the standard NJL-model [9, 10, 14] and get the following representation for the generating functional in terms of the meson fields $\Phi^{a\kappa}_{\mu}$ ($\kappa \equiv (J\ell n)$, $\mu \equiv (\mu_1 ... \mu_\ell)$):

$$Z = N \int \prod_{a\kappa} D\Phi^{a\kappa}_{\mu} \exp\Big\{ \frac{1}{2} \int\!\!\int \mathrm{d}^4x \mathrm{d}^4y \Phi^{a\kappa}_{\mu}(x) \Big[(\Box - M^2_{a\kappa}) \delta(x-y) - h^2_{a\kappa} \Pi^R_{a\kappa}(x-y) \Big] \Phi^{a\kappa}_{\mu}(y) + I_{int}[\Phi] \Big\}, \quad (4.12)$$

$$I_{int} = -\frac{1}{2} \int\!\!\int \mathrm{d}^4x \mathrm{d}^4y h_{a\kappa} h_{a'\kappa'} \times \Phi^{a\kappa}_{\mu}(x) \Big[\Pi^{a\kappa,a'\kappa'}_{\mu\mu'}(x-y) - \delta^{a\kappa,a'\kappa'}_{\mu\mu'} \Pi(x-y) \Big] \Phi^{a'\kappa'}_{\mu'}(y) - \sum_{m=3} \frac{1}{m} \int \mathrm{d}^4x_1 ... \int \mathrm{d}^4x_m \prod_{k=1}^{m} h_{a_k\kappa_k} \Phi^{a_k\kappa_k}_{\mu_k}(x_k) \, \Gamma^{a_1\kappa_1,...,a_m\kappa_m}_{\mu_1,...,\mu_m}(x_1,...,x_m) , \quad (4.13)$$

$$\Gamma^{a_1\kappa_1,...,a_m\kappa_m}_{\mu_1,...,\mu_m}(x_1,...,x_m) = \int \mathrm{d}\sigma_{\mathrm{vac}} \mathrm{Tr} \Big\{ V^{a_1\kappa_1}_{\mu_1}(x_1) S(x_1, x_2 \mid B) \cdot ... \cdot V^{a_m\kappa_m}_{\mu_m}(x_m) S(x_m, x_1 \mid B) \Big\} . \quad (4.14)$$

Below we restrict ourselves by the one-loop approximation. The integral over $\mathrm{d}\sigma_{\mathrm{vac}}$ in Eq.(4.12) has been transferred to the exponential, that is accurate at the one-loop level and suitable for our nearest purposes.

Note that terms linear in fields $\Phi^{a\kappa}_{\mu}$ have not appeared in the action (4.13). They are subtracted by the counterterms in Eq.(4.11) related to the diagram in Fig.1.

Meson masses $M_{a\kappa}$ are defined by the equation

$$\Lambda^2 + G^2_{\kappa} \tilde{\Pi}_{a\kappa}(-M^2_{a\kappa}) = 0 , \quad (4.15)$$

where $\tilde{\Pi}_{a\kappa}(-M^2_{a\kappa})$ is the diagonal part of the polarization operator

$$\Pi^{a\kappa,a'\kappa'}_{\mu,\mu'}(x-y) = \int \mathrm{d}\sigma_{\mathrm{vac}} \mathrm{Tr} \Big\{ V^{a\kappa}_{\mu}(x) S(x, y \mid B) V^{a'\kappa'}_{\mu'}(y) S(y, x \mid B) \Big\} . \quad (4.16)$$

Due to the asymptotics (4.10) the polarization operator (4.16) is UV-finite.

The constants

$$h_{a\kappa} = 1/\sqrt{\tilde{\Pi}'_{a\kappa}(-M^2_{a\kappa})} \quad (4.17)$$

play a role of the effective coupling constants of the meson-quark interaction. Relation (4.17) agrees with the compositiness condition in quantum field theory [18].

Generating functional (4.12) satisfies all demands of the nonlocal QFT [19]. Particularly, this functional leads to the unitary S-matrix. According to representation (4.12)-(4.14) meson-meson interaction is described by the vertex functions Γ given in Eq.(4.14). They are UV-finite and can be computed.

Free parameters defining the effective meson theory (4.12)–(4.14) have a clear physical meaning. They are the quark masses m_f, the confinement scale Λ (strength of the background field) and the four-fermion coupling constant g. The last one enters only through bozon masses, which are calculated by means of Eq.(4.15).

5 The Masses of Pseudoscalar and Vector Mesons

In this section we apply the model (4.12)-(4.15) for calculation of the masses of light pseudoscalar and vector mesons. There are many papers [20]-[23] devoted to calculation of the light meson spectra within the standard (local) NJL-model. Their successful description of the mass splitting between the pseudoscalar and vector mesons is achieved by using independent four-fermion coupling constants $g_P \neq g_V$ for the pseudoscalar and vector mesons [14]. From our point of view, it would be more reasonable to consider the coupling constant to be common for both nonets $g_P \equiv g_V = g$. The parameter g relates to the four-fermion interaction L_2 in Eq.(2.2) and there are no intrinsic reason for difference between g_P and g_V. One has to appeal to some dynamical mechanism in order to explain mass splitting between the pseudoscalar and vector mesons with identical quark structure. In the nonrelativistic quantum mechanics this dynamics arises from the spin-spin interaction of quarks. In other words, one has to take into account the quark spin degrees of freedom. The point where spin of quarks enters into the model (4.12)-(4.15) is the quark propagator (3.6). As it has been already mentioned, the term $\sigma_{\mu\nu}B_{\mu\nu}$ can be interpreted as an interaction of the quark spin with the vacuum field. Now we have to investigate how this spin-field interaction contribute to the mass splitting.

Determination of the parameters. We determine the values of parameters using the observable masses of π, ρ, K and K^* mesons (we suppose that $m_u = m_d$) and then calculate the masses of ϕ, η and η'. We have to solve Eqs.(4.15) rewritten in more convenient notation:

$$1 + C_J g^2 \tilde{\Pi}_{aJ}\Big(\frac{m_q}{\Lambda}, \frac{M_{aJ}}{\Lambda}\Big) = 0 \qquad \Big(\begin{matrix} J = P, V \\ q = u, s \end{matrix} \Big) , \quad (5.1)$$

where M_{aJ} is a mass of meson and index 'a' corresponds to the pseudoscalar (π, η, η', K) and vector (ρ, ϕ, K^*) particles. The polarization operator $\tilde{\Pi}_{aJ}$ is given by Eq.(4.16) with the quark propagator (3.6) including the spin-field interaction term. The details of calculation of $\tilde{\Pi}_{aJ}$ can be found in Appendix. Equations (5.1) describe dependence of the masses M_{aJ} on the parametres of the model: the quark masses m_u, m_s, the strength of the background field Λ and the four-fermion coupling constant g.

We put the masses of pseudoscalar π and K mesons equal to their experimental values 140 MeV and 496 MeV and find the region in the parameter space corresponding to these values. The rest of ambiguity in the values of m_u, m_s, Λ and g is used to optimize description of the vector ρ and K^* mesons. The best description is provided by the choice:

$$m_u = 300\text{MeV}, m_s = 452\text{MeV}, \Lambda = 425\text{MeV}, g = 4.8. \qquad (5.2)$$

The values of the parameters and their variations providing an accuracy of description of meson masses less than 10% are summarized in Table 1.

Table 1:

Parameters of the model.
Δ – variation of the parameters providing the description of π, K, ρ and K^* mesons with accuracy less than 10%.

parametres	*the best fit*	Δ
m_u, MeV	300	50
m_s, MeV	452	100
Λ, MeV	425	75
$\alpha_s = \frac{g^2}{4\pi}$	1.84	1

The masses of ϕ, η and η' mesons. The values (5.2) can be used to calculate the masses of ϕ, η and η' mesons. It is known that the physical particles η and η' are the mixture of the singlet η_1 and octet η_8 states:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_8 \end{pmatrix}.$$

Experimental data from decays $\eta(\eta') \to \gamma\gamma$, $\psi \to \eta(\eta')\gamma$ indicate that $\theta = -20^0$, while tensor-decay data suggest that $\theta = -10^0$ [14]. The semiemperical value obtained from the quadratic Gell-Mann-Okubo mass formula is $\theta = -10^0$, while that from the linear Gell-Mann-Okubo formula is $\theta = -23^0$ [14].

Besides that, it is accepted that the state η_1 contains an admixture of c-quark [24]:

$$\eta_1 = \sqrt{\frac{1}{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right)\cos\alpha + c\bar{c}\,\sin\alpha\,,$$

where α is a mixing angle. The semiempirical value α obtained from the mass sum rule is $\cos^2\alpha = 1/5$ [24].

Using the values of parameters from Table 1 one can find that the best description of the masses of η and η' mesons is achieved when $\theta = -10^0$. The mass of c-quark m_c is taken to be 1500-1800 MeV. Although the admixture of c-quark itself is very important, the variation of m_c in these limits does not change the masses of η and η' mesons essentially ($< 1\%$).

The results of calculation of meson masses M_{aJ} and the constants of meson-quark interaction h_{aJ} for both nonets are summarized in the Table 2. The constants h_{aJ} are calculated by means of Eq.(4.17).

Table 2.

Masses M_{aJ} of the pseudoscalar and vector light mesons (MeV)
and the constants h_{aJ} of meson-quark interaction.
The experimental values are given in brackets.

	π	K	η	η'	ρ	K^*	ω	ϕ
M_{aJ}	140 (140)	496 (496)	578 (550)	923 (960)	757 (770)	921 (890)	757 (783)	1087 (1020)
h_{aJ}	3.12	3.29	3.31	2.88	1.59	1.78	1.59	1.96

One can see that description of masses is qualitatively right and quantitatively acceptable if we use the coupling constant g common for both nonets and take into account spin-field interaction in the quark propagator. At the same time, elimination of the term $\sigma_{\mu\nu}B_{\mu\nu}$ in Eq.(3.6), i.e., turning off the quark spin interaction, destroys this picture – errors in meson masses becomes unsatisfactory large (more than 20%).

Appendix

Calculation of the polarization operator. Using the expressions for the quark propagator (3.6) and the vertex (4.8) one can rewrite the polarization operator (4.16) as:

$$\Pi^{aJ,aJ}(x-y) = \frac{1}{(2\pi)^8}\int d\sigma_{\text{vac}} \iint d^4p\, d^4q\, \mathrm{Tr}\Big\{M^a_{ff'}\Gamma^J \tilde{H}_{f'}(p \mid B) M^a_{f'f}\Gamma^J \tilde{H}_f(q \mid B)\Big\} R(x-y;p,q), \qquad (A.1)$$

$$R(x-y;p,q) = F_{00}\Big(\frac{\overleftrightarrow{\nabla}^2(x)}{\Lambda^2}\Big)\exp\Big\{ix\hat{B}y + ip(x-y)\Big\} \times F_{00}\Big(\frac{\overleftrightarrow{\nabla}^2(y)}{\Lambda^2}\Big)\exp\Big\{iy\hat{B}x + iq(y-x)\Big\}, \qquad (A.2)$$

where F_{00} is given by (4.9), $\Gamma^P = i\gamma^5$, $\Gamma^V = \gamma_\mu$. Substitution of F_{00} into Eq.(A.2) gives:

$$R(x-y;p,q) = \int_0^1\!\!\int_0^1 dt_1 dt_2 \exp\Big\{\frac{t_1}{4\Lambda^2}\overleftrightarrow{\nabla}^2(x)\Big\} \times \exp\Big\{ix\hat{B}y + ip(x-y)\Big\}\exp\Big\{\frac{t_2}{4\Lambda^2}\overleftrightarrow{\nabla}^2(y)\Big\}\exp\Big\{iy\hat{B}x + iq(y-x)\Big\}. \qquad (A.3)$$

Using the representation:

$$\exp\Big\{\alpha\overleftrightarrow{\nabla}^2(x)\Big\} = \int d\sigma_a \exp\Big\{2\sqrt{\alpha}\int_0^1 d\beta\, a_\mu(\beta)\overleftrightarrow{\nabla}_\mu(x)\Big\},$$

$$d\sigma_a = \delta a(\beta)\exp\Big\{-\int\limits_0^1 d\beta a^2(\beta)\Big\},$$

one can transform (A.3) to the form:

$$\begin{aligned}
&R(x-y;p,q)\\
&=\int\limits_0^1\!\!\int\limits_0^1 dt_1dt_2\int d\sigma_a\!\int d\sigma_b \exp\left\{\frac{2i\sqrt{t_1}}{\Lambda}\int\limits_0^1 d\beta a_\mu(\beta)\hat{B}_\mu(x)+\frac{2i\sqrt{t_2}}{\Lambda}\int\limits_0^1 d\beta b_\mu(\beta)\hat{B}_\mu(y)\right.\\
&\quad +ip\left(x-y-\frac{\sqrt{t_1}}{\Lambda}\int\limits_0^1 d\beta a(\beta)-\frac{\sqrt{t_2}}{\Lambda}\int\limits_0^1 d\beta b(\beta)\right)\\
&\quad +iq\left(y-x-\frac{\sqrt{t_1}}{\Lambda}\int\limits_0^1 d\beta a(\beta)-\frac{\sqrt{t_2}}{\Lambda}\int\limits_0^1 d\beta b(\beta)\right)\\
&\quad +i\left(x-\frac{\sqrt{t_1}}{\Lambda}\int\limits_0^1 d\beta a(\beta)\right)\hat{B}\left(y+\frac{\sqrt{t_2}}{\Lambda}\int\limits_0^1 d\beta b(\beta)\right)\\
&\quad \left.+i\left(y-\frac{\sqrt{t_2}}{\Lambda}\int\limits_0^1 d\beta b(\beta)\right)\hat{B}\left(x+\frac{\sqrt{t_1}}{\Lambda}\int\limits_0^1 d\beta a(\beta)\right)\right\}. \qquad (A.4)
\end{aligned}$$

Only the constant part of $a(\beta)$ and $b(\beta)$ provides nonzero contribution to the integrals in (A.4), so that

$$\begin{aligned}
R(x-y;p,q)&=\int\limits_0^1\!\!\int\limits_0^1 dt_1dt_2\exp\Big\{i(x-y)(p-q)\Big\}\\
&\times\int da\exp\left\{-a^2+\frac{2i\sqrt{t_1}}{\Lambda}a_\mu\hat{B}_\mu(x-y)-\frac{i\sqrt{t_1}}{\Lambda}(p+q)_\mu a_\mu\right\}\\
&\times\int db\exp\left\{-b^2+\frac{2i\sqrt{t_2}}{\Lambda}b_\mu\hat{B}_\mu(y-x)-\frac{i\sqrt{t_2}}{\Lambda}(p+q)_\mu b_\mu\right\}.
\end{aligned}$$

Integrating over a and b and taking into account that

$$\hat{B}_\mu(x-y)\hat{B}_\mu(x-y)=v^2\Lambda^4(x-y)^2$$

we obtain:

$$\begin{aligned}
R(x-y;p,q)=\int\limits_0^1\!\!\int\limits_0^1 dt_1dt_2\exp&\left\{i(x-y)[p-q-i\frac{t_2-t_1}{\Lambda^2}\hat{B}(p+q)]\right.\\
&\left.-(t_1+t_2)v^2\Lambda^2(x-y)^2-i\frac{t_2+t_1}{4\Lambda^2}(p+q)^2\right\}.
\end{aligned}$$

Fourier transformed $R(x-y;p,q)$ is:

$$\begin{aligned}
\tilde{R}(k;p,q)=\frac{\pi^2}{v^4\Lambda^4}\int\limits_0^1\!\!\int\limits_0^1\frac{dt_1dt_2}{(t_1+t_2)^2}\exp&\left\{-(p^2+q^2)\left(\frac{1+4v^2t_1t_2}{4v^2\Lambda^2(t_1+t_2)}\right)\right.\\
&+p\left(q\frac{1-4v^2t_1t_2}{2v^2\Lambda^2(t_1+t_2)}+\frac{k}{2v^2\Lambda^2(t_1+t_2)}-\frac{i(t_1-t_2)}{v^2\Lambda^4(t_1+t_2)}\hat{B}\left(q+\frac{k}{2}\right)\right)\\
&\left.-q\left(\frac{k}{2v^2\Lambda^2(t_1+t_2)}+\frac{i(t_1-t_2)}{2v^2\Lambda^4(t_1+t_2)}\hat{B}(k)\right)-\frac{k^2}{4v^2\Lambda^2s_1}\right\}, \qquad (A.5)
\end{aligned}$$

where k is an external momentum. After insertion of Eq.(A.5) into Eq.(A.1) one has to calculate the traces, integrate over p and q and average with the measure $d\sigma_{vac}$. These simple but cumbersome calculations has been done by means of analytic computer methods. As a result we get the following expression for diagonal part of the polarization operator:

$$\begin{aligned}
\tilde{\Pi}^{aJ}(k^2)=&\frac{1}{256\pi^2v^6}M^a_{ff'}M^a_{f'f}\int\limits_0^1\!\int\limits_0^1\frac{dt_1dt_2}{(t_1+t_2)^2}\int\limits_0^1 ds_1\frac{(1-s_1)^{\beta_{f'}}}{(1+s_1)^{1+\beta_{f'}}}\int\limits_0^1 ds_2\frac{(1-s_2)^{\beta_f}}{(1+s_2)^{1+\beta_f}}\\
&\times F^J_{f'f}(t_1,t_2,s_1,s_2;k^2)\exp\left\{-\frac{2s_1s_2+v(t_1+t_2)(s_1+s_2)}{2v(t_1+t_2)(1+s_1s_2)+(s_1+s_2)(1+4v^2t_1t_2)}\frac{k^2}{4v\Lambda^2}\right\},
\end{aligned}$$

where $\beta_f=\alpha_f^2/4v-1$, $\alpha_f=m_f/\Lambda$ and the function $F^J_{f'f}(t_1,t_2,s_1,s_2;k^2)$ looks :

$$F^J_{f'f}=\frac{1}{A^3\,B^4}\left[F^J_1(t_1,t_2,s_1,s_2)+\alpha_{f'}\alpha_f F^J_2(t_1,t_2,s_1,s_2)+F^J_3(t_1,t_2,s_1,s_2)\frac{k^2}{\Lambda^2}\right].$$

For pseudoscalar mesons ($J=P$) the functions F^J_1, F^J_2, F^J_3 have the form :

$$\begin{aligned}
F^P_1&=\frac{2v(t_1-t_2)\,E_1+(1-4v^2t_1t_2)\,E_2}{v^2(t_1+t_2)\,(1+s_1)\,(1+s_2)}\,B\,,\\
F^P_2&=-4\,\frac{1+2s_1s_2+s_1^2s_2^2}{(1+s_1)\,(1+s_2)}\,A\,B^2\,,\\
F^P_3&=\frac{B\,[\,v(t_1-t_2)\,C-D\,]-v(t_1-t_2)\,(D^2-C^2)}{4v^2(t_1+t_2)\,(1+s_1)\,(1+s_2)}\,E_1\\
&-\frac{2\,B\,[\,v(t_1-t_2)\,D-C\,]+(1-4v^2t_1t_2)\,(D^2-C^2)}{8v^2(t_1+t_2)\,(1+s_1)\,(1+s_2)}\,E_2,
\end{aligned}$$

where the following notation are used :

$$\begin{aligned}
E_1&=(s_2-s_1)\,[\,(1+s_1^2)\,(1-s_2^2)+(1+s_2^2)\,(1-s_1^2)\,]\\
&\quad+2\,(s_1s_2-1)\,[\,s_1(1-s_2^2)-s_2(1-s_1^2)\,]\,,\\
E_2&=(s_1s_2-1)\,[\,(1+s_1^2)\,(1-s_2^2)+(1+s_2^2)\,(1-s_1^2)\,]\\
&\quad+2\,(s_2-s_1)\,[\,s_1(1-s_2^2)-s_2(1-s_1^2)\,]\,,\\
A&=\frac{s_1}{2\,v}+\frac{1+4v^2t_1t_2}{4v^2(t_1+t_2)}\,, \qquad (A.6)
\end{aligned}$$

$$
\begin{aligned}
B &= \frac{4v^2(t_1^2+t_2^2) - 16v^4 t_1^2 t_2^2 - 1}{16\ A\ v^4 (t_1+t_2)^2} + \frac{s_2}{2\ v} + \frac{1+4v^2 t_1 t_2}{4v^2(t_1+t_2)}, \\
C &= \frac{1-2v^2(t_1^2+t_2^2)}{8\ v^4\ A\ (t_1+t_2)^2} - \frac{1}{2v^2(t_1+t_2)}, \\
D &= -\frac{(t_1-t_2)\ (1+4v^2 t_1 t_2)}{8\ v^3\ A\ (t_1+t_2)^2} + \frac{(t_1-t_2)}{2\ v\ (t_1+t_2)}.
\end{aligned}
$$

For the vector mesons (J = V) we have :

$$
\begin{aligned}
F_1^V &= \frac{2v(t_1-t_2)\ E_3 + (1-4v^2 t_1 t_2)\ E_4}{v^2(t_1+t_2)\ (1+s_1)\ (1+s_2)}\ B, \\
F_2^V &= -4\,\frac{1-s_1^2 s_2^2}{(1+s_1)\ (1+s_2)}\ A\ B^2, \\
F_3^V &= \frac{v(t_1-t_2)\ B\ D + 2\ (1-4v^2 t_1 t_2)\ D^2}{24v^2(t_1+t_2)\ (1+s_1)\ (1+s_2)}\ E_5 + \frac{(t_1-t_2)\ (4\ C^2 + B\ C)}{24v(t_1+t_2)\ (1+s_1)\ (1+s_2)}\ E_6 \\
&+ \frac{B\ C + 2\ (1-4v^2 t_1 t_2)\ C^2}{24v^2(t_1+t_2)\ (1+s_1)\ (1+s_2)}\ E_7 + \frac{4v(t_1-t_2)\ D^2 + B\ D}{24v^2(t_1+t_2)\ (1+s_1)\ (1+s_2)}\ E_8 \\
&- \frac{E_9\ (1-4v^2 t_1 t_2) + 2v(t_1-t_2)\ E_{10}}{6v^2(t_1+t_2)\ (1+s_1)\ (1+s_2)}\ C\ D,
\end{aligned}
$$

where

$$
\begin{aligned}
E_3 &= (s_1 s_2 - 1)\ [\ 1 + s_1^2 s_2^2 - 2 s_1 s_2\ ] - (s_2 - s_1)\ [\ s_2\ (1+s_1^2) - s_1\ (1+s_2^2)\ ], \\
E_4 &= (s_2 - s_1)\ [\ 1 + s_1^2 s_2^2 - 2 s_1 s_2\ ] - (s_1 s_2 - 1)\ [\ s_2\ (1+s_1^2) - s_1\ (1+s_2^2)\ ], \\
E_5 &= (1 - 3 s_1 s_2)\ (1 + s_1^2 s_2^2) + 2 s_1 s_2\ (s_1 s_2 - 3) + s_2\ (1+s_1^2)\ (s_2 - 3 s_1) \\
&\quad + s_1\ (1+s_2^2)\ (s_1 - 3 s_2), \\
E_6 &= (s_2 - 3 s_1)\ (1 + s_1^2 s_2^2) + 2 s_1 s_2\ (s_1 - 3 s_2) + s_2\ (1+s_1^2)\ (1 - 3 s_1 s_2) \\
&\quad + s_1\ (1+s_2^2)\ (s_1 s_2 - 3), \\
E_7 &= (s_1 s_2 - 3)\ (1 + s_1^2 s_2^2) + 2 s_1 s_2\ (1 - 3 s_1 s_2) + s_2\ (1+s_1^2)\ (s_1 - 3 s_2) \\
&\quad + s_1\ (1+s_2^2)\ (s_2 - 3 s_1), \\
E_8 &= (s_1 - 3 s_2)\ (1 + s_1^2 s_2^2) + 2 s_1 s_2\ (s_2 - 3 s_1) + s_2\ (1+s_1^2)\ (s_1 s_2 - 3) \\
&\quad + s_1\ (1+s_2^2)\ (1 - 3 s_1 s_2), \\
E_9 &= (s_1 + s_2)\ (1 + 2 s_1 s_2 + s_1^2 s_2^2) + (1 + s_1 s_2)\ [\ s_2\ (1+s_1^2) + s_1\ (1+s_2^2)\ ], \\
E_{10} &= (1 + s_1 s_2)\ (1 + 2 s_1 s_2 + s_1^2 s_2^2) + (s_1 + s_2)\ [\ s_2\ (1+s_1^2) + s_1\ (1+s_2^2)\ ],
\end{aligned}
$$

and the functions A, B, C, D are given by Eq.(A.6) .

Averaging over space directions of the background field. The generating formula can be chosen as:

$$
\langle \exp(i f_{\mu\nu} J_{\mu\nu}) \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \exp(i f_{\mu\nu} J_{\mu\nu}), \tag{A.7}
$$

where φ, θ are the spherical angles which determine space direction of the background field and $J_{\mu\nu}$ is an antisymmetric tensor. Integration in (A.7) over the angles gives:

$$
\langle \exp(i f_{\mu\nu} J_{\mu\nu}) \rangle = \frac{\sin\sqrt{2(J_{\mu\nu} J_{\mu\nu} \pm \tilde{J}_{\mu\nu} J_{\mu\nu})}}{\sqrt{2(J_{\mu\nu} J_{\mu\nu} \pm \tilde{J}_{\mu\nu} J_{\mu\nu})}}, \tag{A.8}
$$

where $\tilde{J}_{\mu\nu}$ is a dual tensor. Differentiation of (A.8) at $J_{\mu\nu} = 0$ leads to the formulas:

$$
\begin{aligned}
\langle f_{\mu\nu} \rangle &= 0, \\
\langle f_{\mu\nu} f_{\alpha\beta} \rangle &= \frac{1}{3}(\delta_{\alpha\mu}\delta_{\beta\nu} - \delta_{\alpha\nu}\delta_{\beta\mu} \pm \varepsilon_{\alpha\beta\mu\nu}),
\end{aligned}
$$

where the sign $\pm$ relates to the self- and antiself-dual configurations.

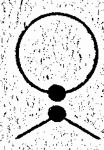

Fig.1 The divergent bubble diagram.

References

[1] E.V.Shuryak, Phys.Rep.**115** (1984) 151

[2] D.I.Dyakonov, V.Yu.Petrov, Nucl.Phys.**B245** (1984) 259

[3] C.G.Callan, R.Dashen, D.J.Gross, Phys.Rev.**D17** (1978) 2717

[4] M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl.Phys.**B147** (1979) 385

[5] J.Ambjorn, N.K.Nilsen, P.Olesen, Nucl.Phys. **B152** (1979) 75

[6] H.Leutwyler, Nucl.Phys.**B179** (1981) 129; *ibid*, Phys.Lett.**96B** (1980) 154

[7] S.G.Matinyan, G.K.Savvidy, Nucl.Phys.**B134** (1978) 539

[8] G.Preparata, Phys.Lett. **B201** (1988) 139; preprint LNF-86/2(P); preprint MITH 89/1 (1989) Milano

[9] Y.Nambu, G.Jona-Lasinio, Phys.Rev.**122** (1961) 345

[10] T.Eguchi,Phys.Rev.**D14**, (1976)2755;
T.Goldman, R.W.Haymaker, Phys.Rev.**D24** (1981) 724

[11] G.V.Efimov, M.A.Ivanov, "The Quark Confinement Model of Hadrons", IOP, Bristol and Philadelfia, 1993

[12] C.D.Roberts, R.T.Cahill, Aust.J.Phys. **40** (1987) 499

[13] G.V. Efimov, S.N. Nedelko, Heidelberg Univ. preprint No. 693 (1992), to be published elsewhere

[14] S.P.Klevansky Rev.Mod.Phys.**Vol.64** (1992) 649

[15] L.F.Abbott, Nucl.Phys.**B185** (1981) 189

[16] J.Finjord, Nucl.Phys.**B194** (1982) 77

[17] E.Elizalde, Nucl.Phys.**B243** (1984) 398;
E.Elizalde, J.Soto, Nucl.Phys.**B260** (1985) 136

[18] K.Hayashi *et al*, Fortsch.Phys. **15** (1967) 625

[19] G.V.Efimov, "Nonlocal Interactions of Quantized Fields", Nauka, Moscow, 1977

[20] S.Klimt, M.Lutz, W.Weise, Phys.Lett.**B249** (1990) 386

[21] M.Takizawa, K.Tsushina, Nucl.Phys.**A507** (1989) 511

[22] M.Takizawa, K.Kubolera, Phys.Lett.**B261** (1991) 221

[23] U.Vogl, W.Weise, Prog.Part.Nucl. **27** (1991) 195

[24] F.E.Close, "An Introduction to Quarks and Partons", Acad. Press, London,1979

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