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BOUND STATES OF «DRESSED» PARTICLES

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Связанные состояния «одетых» частиц

Предложен новый подход к проблеме связанных состояний в релятивистских квантовых теориях поля. Он использует операторы рождения-уничтожения «одетых» частиц, которые определяются процедурой «одевания» Фаддеева (1963). Обсуждаются отличия от известных подходов: Бете — Солпитера, Логунова — Тавхелидзе, Кадышевского, Тамма — Данкова.

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Bound States of «Dressed» Particles

A new approach to the problem of bound states in relativistic quantum field theories is suggested. It uses the creation-destruction operators of «dressed» particles which have been granted by Faddeev's (1963) «dressing» formalism. Peculiarities of the proposed approach as compared to the known ones are discussed.

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1. INTRODUCTION

We know many approaches to the problem of determining such bound states as hydrogen atom (pe^-), positronium (e^+e^-), hadrons, etc. Examples are non-relativistic Breit's approach [1,2]; Tamm — Dankoff approach [3]; Bethe — Salpeter equation [4]; three-dimensional quasipotential approaches, see refs. [5,6] and references therein. One more approach is suggested here. It is based on the «dressing» formalism, given by L.Faddeev [7], which introduces Schroedinger creation-destruction operator of «dressed» particles instead of the «bare» ones. The main properties of the former are: a) the state without «dressed» particles Ω must coincide with the physical vacuum (lowest energy eigenstate of the total Hamiltonian H); b) «dressed» one-particle states of the kind $\alpha_p^+\Omega$ also must be H eigenstates. The Hamiltonian H expression in terms of the «dressed» operators allows one to define bound states as eigenstates of a part of H which contains besides the usual «free» part also come potential-like particle-particle interactions, see below sect.4.

2. TOTAL HAMILTONIAN IN TERMS OF «DRESSED» OPERATORS

«Dressed» particle creation α_p^+ and destruction α_p operators are determined as functions of «bare» ones a_p^+ and a_p, so that $\alpha_p = W a_p W^+$, where W is an isometric transformation $W^+W = WW^+ = 1$. This allows one to find $H(a)$ as a function of α:

$$H(a) = H(W^+\alpha W) = W^+H(\alpha)\,W \equiv K(\alpha). \qquad (1)$$

Here $K(\alpha)$ is a different function of its argument as compared to $H(a)$, but K is the same total Hamiltonian.

I take here as an example the theory of fermions and mesons with the Yukawa interaction $\lambda\bar{\psi}\gamma_5\psi\varphi$, see [8]. Let a_p, b_q, g_k be «bare» destruction operators of fermions, antifermions and mesons, respectively. Let α_p, β_q, γ_k be the respective «dressed» operators. Interaction terms of the kind a^+ag, abg, $a^+b^+g^+$,... enter into $H(a, b, g)$. Due to them the «bare» no-paricle state Ω_0 ($a_p\Omega_0 = 0$, etc.) and one-particle states $a_p^+\Omega_0$; etc., are not H eigenvectors. W must be such that $K(\alpha, \beta, \gamma)$, see eq.(1), does not contain the above trilinear terms as well as other «bad» terms, i.e., those preventing Ω and $\alpha_p^+\Omega$ from being K eigenstates.

The needed W has been determined for this model in [8] under the assumption $\mu < 2m$, μ and m being the meson and fermion physical masses. As a result, K becomes an infinite series of the form

$$K = K_0 + K_2 + K_3 + K_4 + \ldots, \qquad K_n \sim \lambda^n, \qquad (2)$$

$$K_0 = \int_p \varepsilon_p \alpha_p^+\alpha_p + \int_q \varepsilon_q \beta_q^+\beta_q + \int d^3k\,\omega_k \gamma_k^+\gamma_k,$$
$$\varepsilon_p \equiv \sqrt{p^2 + m^2}, \qquad \omega_k \equiv \sqrt{k^2 + \mu^2}. \qquad (3)$$

Here k is the meson three-momentum, p includes besides the fermion momentum also its spin projection, so that $\int_p$ is the integration and summation over all p values. K_2 contains tetralinear terms, see below eq.(4); K_3 contains pentalinear terms of the kind $\alpha^+\beta^+\alpha\beta\gamma$; K_4 contains not only sextalinear terms but also tetralinear ones of the order λ^4.

3. PARTICLE-PARTICLE EFFECTIVE POTENTIALS

Now let us discuss K_2

$$K_2 = F(p_1, p_2; p_1', p_2')\,\alpha_{p_1}^+\alpha_{p_2}^+\alpha_{p_1'}\alpha_{p_2'} + \tilde{F}(q_1, q_2; q_1', q_2')\,\beta_{p_1}^+\beta_{p_2}^+\beta_{p_1'}\beta_{p_2'} +$$
$$+ C(p, k; p', k')\,\alpha_p^+\gamma_k^+\alpha_{p'}\gamma_{k'} + \tilde{C}(q, k; q', k')\,\beta_q^+\gamma_k^+\beta_{q'}\gamma_{k'} +$$
$$+ A(p, q, k_1, k_2)\,\alpha_p^+\beta_q^+\gamma_{k_1}\gamma_{k_2} + A^*(p, q, k_1, k_2)\,\gamma_{k_1}^+\gamma_{k_2}^+\beta_q\alpha_p +$$
$$V(p, q; p', q')\,\alpha_p^+\beta_q^+\beta_{q'}\alpha_{p'}\,. \qquad (4)$$

The integration-summation is implied over all values of p, q, k. The term $F\alpha^+\alpha^+\alpha\alpha$ leads to fermion-fermion scattering in the first order of the usual «old» perturbation theory [8]. In the coordinate representation $F(p_1, p_2; p_1', p_2')$ may be a nonlocal fermion-fermion potential and, moreover, may depend on derivatives. So F will be called the quasipotential as well as C and V. Due to these quasipotentials, two-particle bound states can arise: fermion-fermion, meson-fermion, etc.

4. EQUATIONS FOR BOUND STATES

The perturbation theory uses usually eigenstates of the «free» part of the total Hamiltonian as the zero approximation. One can use instead the eigenstates of K_0+K_2 under the important condition that one is able to find them nonperturbatively (see [3], ch.40.3). Let us define the bound state as the K_0+K_2 eigenstate Φ_E, $(K_0+K_2)\,\Phi_E=E\Phi_E$ corresponding to a discrete value of the state mass (i.e., its energy in the c.m.s.).

Consider the case of a meson-fermion bound state. The operator K_0+K_2 transforms $\alpha^+_{p'}\gamma^+_{k'}\Omega$ to states of the same kind $\alpha^+_p\gamma^+_k\Omega$ only. This suggests that there exists such an eigenstate Φ^{mf}_E of K_0+K_2 which can be expanded in states $\alpha^+_p\gamma^+_k\Omega$, $\forall p$, $\forall k$ only without an admixture of other two-particle states $\alpha^+\beta^+\Omega$, $\gamma^+\gamma^+\Omega$, etc.,

$$\Phi^{mf}_E=\int_{p'}\int_{k'}\Phi_E(p',k')\,\alpha^+_{p'}\gamma^+_{k'}\Omega\,. \tag{5}$$

Only the term $C\alpha^+\gamma^+\alpha\gamma$ from K_2 contributes to the equation $(K_0+K_2)\,\Phi^{mf}_E=E\Phi^{mf}_E$. Taking the scalar products of both parts of this equation with $\langle\alpha^+_p\gamma^+_k\Omega|$, one gets the integral equation for the coefficients $\Phi_E(p,k)$

$$(\varepsilon_p+\omega_k-E)\,\Phi_E(p,k)=\int_{p'}\int_{k'}C(p,k;p',k')\,\Phi_E(p',k'). \tag{6}$$

The quasipotential $C(p,k;p',k')$ has been written in ref. [8], eqs. (20) and (A.7). Another variant for C (corresponding to a different choice of «dressed» operators) has been described in sect.4 of ref. [8].

The two-fermion bound state can be treated in an analogous manner.

A different situation arises in the case of the bound state Φ^0_E such that $(N_f-N_{\bar f})\,\Phi^0_E=0$, $N_f=\int_p\alpha^+_p\alpha_p$, $N_{\bar f}=\int_q\beta^+_q\beta_q$. The operator K_2 transforms $f\bar f$ states $\alpha^+_p\beta^+_q\Omega$ not only to themselves but also to two-meson $\gamma^+_{k_1}\gamma^+_{k_2}\Omega$, see terms $A\alpha^+\beta^+\gamma\gamma$ in eq. (4). So one may suppose that

$$\Phi^0_E=\int_{p'}\int_{q'}P(p',q')\,\alpha^+_{p'}\beta^+_{q'}\Omega+\int_{k_1}\int_{k_2}M(k_1,k_2)\,\gamma^+_{k_1}\gamma^+_{k_2}\Omega\,. \tag{7}$$

Taking the scalar products of $(K_0+K_2)\,\Phi^0_E=E\Phi^0_E$ with $\langle\alpha^+_p\beta^+_q\Omega|$ and $\langle\gamma^+_{K_1}\gamma^+_{K_2}\Omega|$, one gets a system of two coupled equations for P and M. So Φ^0_E must not be called the $f\bar f$ bound state. We have no separate $f\bar f$ and meson-meson bound states but only one state Φ^0_E, see eq. (7).

5. COMPARISON WITH OTHER APPROACHES

The proposed approach differs in many respects from the four-dimensional Bethe — Salpeter approach [4]. Not only the relative energy or time but also any time variables are absent. Our bound state vectors Φ_E or their components, e.g., $\Phi^{mf}_E(p,k)$, have the usual probability interpretation.

In distinction to the three-dimensional approaches by Logunov and Tavkhelidze et al. (see e.g. [5,6] and references therein) our equation is obtained without the intervention of the Bethe — Salpeter equation BSE. Our quasipotentials are hermitian and only a stable bound state can be considered.

Kadyshevsky's equation, see, e.g., eq. (2.28) in ref. [6], is derived without using the BSE, but its quasipotential has been determined by means of a modified technique of Feynman's diagrams. Meanwhile our quasipotentials follow from the «dressing» procedure.

Unlike the Tamm — Dankoff approach TDA [3], the «dressed» states are used here instead of «bare» ones and, moreover, bound states are defined as eigenvectors of a part K_0+K_2 of the total Hamiltonian and not as eigenvectors of the latter as in the TDA. Due to these circumstances one needs not neglect Fock's amplitudes with increasing numbers of particles: the TDA becomes exact in our case.

6. GENERALIZATION

Of course, using K_0+K_2 instead of K in our bound state equation is an approximation which is reminescent of the ladder approximation in the BSE.

If one adds to K_0+K_2 tetralinear terms which are present in $K_4\sim\lambda^4$, see eq. (2), then no additional difficulties arise.

If pentalinear terms of $K_3\sim\lambda^3$ of the kind $\alpha^+\alpha^+\gamma^+\alpha\alpha$ are accounted for, then all Fock's amplitudes (corresponding to an infinitely increasing number of particles) would enter into the equation $(K_0+K_2+K_3)\,\Phi_E=E\Phi_E$ and one would need approximations of the TDA. But one can show that the ensuring contributions to the quasipotentials would be of the order λ^6.

When considering three-particle bound states, one can take into account sextalinear terms of K_4 of the kind $\alpha^+\alpha^+\alpha^+\alpha\alpha\alpha$. They describe three-particle interactions irreducible to the two-particle ones.

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REFERENCES

1. Akhiezer A.I., Berestetskii V.B. — Quantum Electrodynamics, Interscience, New York, 1965.
2. Lucha W. et al. — Physics Rep., 1991, 200, No.4, p.128.
3. Bethe H.A., F. de Hoffmann. — Mesons and Fields, v.11, ch.40, Row, Peterson, New York, 1955.
4. Itzykson C., Zuber J.—B. — Quantum Field Theory, ch.10, McGraw-Hill, New York, 1980.
5. Klein A., Lee T.H. — Phys. Rev. D, 1974, 10, p.4308.
6. Kadyshevsky V.G., Mir-Kasimov R.M., Skachkov N.B. — Soviet Journal of Particles and Nuclei, 1972, 2, No.3, p.635.
7. Faddeev L.D. — Dokl. Akad. Nauk USSR, 1963, 152, p.573.
8. Shirokov M.I., Visinescu M.M. — Revue Roumaine Phys., 1974, 19, p.461.