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HARD e^+e^--PAIR BREMSSTRAHLUNG AS A LEPTON POLARIMETER

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Жесткое тормозное излучение e^+e^--пар как поляриметр лептона

Мы предлагаем применять концепцию handedness для измерения поляризации конечного лептона в реакции $\vec{A}B \to \vec{l}^- l^+ X$, с помощью которой можно изучать поляризованное глюонное распределение в нуклоне.

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Hard e^+e^--Pair Bremsstrahlung as a Lepton Polarimeter

We suggest to apply a handedness concept to measure the polarization of final lepton in the reaction $\vec{A}B \to \vec{l}^- l^+ X$ which is capable of probing the size of polarized gluon distribution in the nucleon.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1 Introduction

The SLAC and EMC measurcments of the proton spin dependent structure function $g_1^p(x)$ have showed our misunderstanding of the nucleon helicity content. It was found that the valence quark contribution to the proton spin is abnormally small [1]. This results are in a conflict with our cld knowledge from the constituent quark picture where the nucleon spin projection was the sum of the quarks ones. Various ways were proposed for to resolve the above problem: large gluon polarization contribution in DIS via axial anomaly or abnormal sea quark contribution due to instantons or an unusual orbital angular momentum contribution in Skyrm model. Each of the proposal needs in a further experimental check. In particular a number of processes were proposed capable to probe the size of polarized gluon distribution. As a rule such processes involve the polarized beam as well as polarized target, like the direct γ-asymmetry, what seems difficult from the point of view of the experimental facilities. The polarized target only with unpolarized beam seems more available. Then to test the proton spin content a final state polarization must be observed. Such a possibility have been proposed in paper [2] where spin asymmetry is studied in the process $\vec{A}B \to \vec{l}^- l^+ X$ (A,B are hadrons) with a measurement of the final lepton polarization. Here we investigate an application of the handedness concept for to measure the polarization of final lepton in this reaction using the e^+e^--pair bremsstrahlung.

2 Handedness

In this section we just outline an idea of the handedness concept for particle decay, more detailed information can be found in the original works [3] (see also section 2 of ref.[4]). This notion relies on a possibility to measure polarization of particle from the investigation of decay products. The minimal number of final state particles depends on the interaction process.

The parity conservation in electromagnetic and strong-interaction processes implies a rigorous constraints on the form of possible invariant amplitudes. It follows that in electromagnetic two-body decay it is impossible to get information about polarization of the decaying state from the observation of the momentum distribution of the decay products. Therefore one is forced to consider at least three particles in the final state for 'o construct a pseudovector V_μ from their momenta ($V_\mu = \epsilon_{\mu\nu\rho\sigma} k_1^\nu k_2^\rho k_3^\sigma$) and for to contract it with a polarization pseudovector to form a parity conserving term in the amplitude. The longitudinal handedness is defined in term of left- and right-handed events. The event is called right-handed or left-handed if $n_l < 0$ or

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$n_l > 0$ respectively (it reminds the usual definition of particle helicity), where

$$n_l = \frac{(\vec{n}\vec{p})}{|\vec{p}|} \tag{1}$$

The normalized vector n_μ is

$$n_\mu = \frac{V_\mu}{\sqrt{-V^\nu V_\nu}} \tag{2}$$

and

$$p = k_1 + k_2 + k_3 \equiv (E, \vec{p}) \tag{3}$$

The handedness value for this case is equal

$$H = \frac{L-R}{L+R} \tag{4}$$

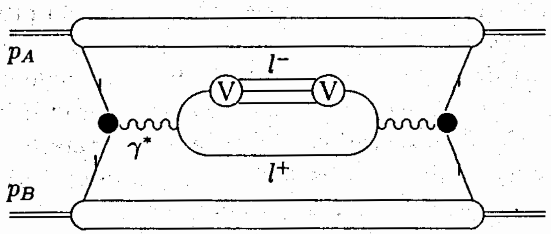
where L, R are the number of left- and right-handed events respectively. The value of handedness is proportional to the polarization of decaying state:

$$H = \alpha s \tag{5}$$

For strong-interaction process the analyzing power α cannot be found consistently theoretically because it is determined by nonpertubative strong-interaction dynamics. But in QED due to small coupling constant the analyzing power can be calculated by use of perturbation theory only.

3 Calculation of the Handedness

In this section we consider fragmentation subprocess that helps us to determine the polarization of final state lepton produced in any hard collision



(Drell-Yan process, for example). Hard process with subsequent fragmentation is described by unitary diagram, where the up and down big blobs

represent the structure functions of incoming particles, the small black blobs denote the dominated hard parton subprocesses and V-blob is a vertex function of virtual lepton (l^-) fragmenting into $l^- e^+ e^-$.

To proceed further we have to factorize fragmentation process of final lepton (later muon, $l = \mu$) from hard production process. This can be done in the squared amplitude by (1) use of Fiertz transformation for the numerators of the muon propagators:

$$\begin{aligned}
&(\hat{p} + m_\mu)_{\alpha\alpha'}(\hat{p} + m_\mu)_{\beta\beta'} \\
&= \tfrac{p^2+m_\mu^2}{4p^2}\left(p^2 I_{\alpha\beta} I_{\beta'\alpha'} + \hat{p}_{\alpha\beta}\hat{p}_{\beta'\alpha'} + p^2(\hat{s}\gamma_5)_{\alpha\beta}(\hat{s}\gamma_5)_{\beta'\alpha'} + (\hat{s}\hat{p}\gamma_5)_{\alpha\beta}(\hat{s}\hat{p}\gamma_5)_{\beta'\alpha'}\right) \\
&\quad + \tfrac{m_\mu}{2}\left(\hat{p}_{\alpha\beta} I_{\beta'\alpha'} + I_{\alpha\beta}\hat{p}_{\beta'\alpha'} - (\hat{s}\hat{p}\gamma_5)_{\alpha\beta}(\hat{s}\gamma_5)_{\beta'\alpha'} - (\hat{s}\gamma_5)_{\alpha\beta}(\hat{s}\hat{p}\gamma_5)_{\beta'\alpha'}\right)
\end{aligned} \tag{6}$$

where s_μ is a polarization pseudovector of muon and p_μ its momentum four-vector ($s^2 = -1$, $sp = 0$); (2) and some transformation in the formula for total cross section:

$$\begin{aligned}
d\sigma^{total} &= \tfrac{1}{2s}\textstyle\sum_X \left|M_X^{total}\right|^2 dF \\
&= \tfrac{1}{2s}\int d^4p\,\delta^{(4)}(p - p_1 - p_2 - k)\textstyle\sum_X \left|M_X^{total}\right|^2 dF \\
&\equiv d\sigma^{(\mu^+\mu^- pair\ creation)} dw^{(fragm)}
\end{aligned} \tag{7}$$

where

$$d\sigma^{(\mu^+\mu^- pair\ creation)} \equiv \tfrac{1}{2s}\textstyle\sum_X \left|M_X^{\mu^+\mu^-}\right|^2 (2\pi)^4\delta^{(4)}(p_A + p_B - p - p' - p_X)\tfrac{d^3p'}{(2\pi)^3 2p'_0}\tfrac{d^3p}{(2\pi)^3 2p_0}\tfrac{dp^2}{p^2} \tag{8}$$

and

$$dw^{(fragm)} \equiv \left|M^{\mu^- e^+ e^-}\right|^2 d\Phi_{(2)} \tag{9}$$

where

$$d\Phi_{(2)} = \textstyle\prod_{i=1}^{2} \tfrac{d^3p_i}{(2\pi)^3 2p_i^0}\,\delta\left(1 - \tfrac{(p_1+p_2+k)^2}{p^2}\right) \tag{10}$$

$d\sigma^{(\mu^+\mu^- pair\ creation)}$ describes cross section of lepton pair creation with a polarized virtual lepton and $dw^{(fragm)}$ characterizes fragmentation process. The summation over γ-matrix structures in the last line of expression (7) is assumed.

The observable studied in this paper is the asymmetry defined by the following relation:

$$H(\phi) = \frac{d(\Delta w^{(fragm)})}{dw^{(fragm)}} \tag{11}$$

where ϕ is an angle between the momenta of electron and positron in the c.m.s. of decay products ,$dw^{(fragm)}$ is the unpolarized cross section and

$$d(\Delta w^{(fragm)}) = dw_{++}^{(fragm)} - dw_{+-}^{(fragm)} \tag{12}$$

The quantities $dw_{++}^{(fragm)}$ and $dw_{+-}^{(fragm)}$ in this relation are the probabilities of the positive polarized virtual lepton radiation of the right- and left-handed e^+e^--pair respectively.

The analytic formulae for $dw^{(fragm)}$ and $d(\Delta w^{(fragm)})$ are given by the following relation:

$$dw^{(fragm)} = \sum_{spins} |M^{tree}|^2 \, d\Phi_{(2)} \tag{13}$$

$$d(\Delta w^{(fragm)}) = 2Im\left(\sum_{spins} M^{tree*}(M_{(1)}^{loop} + M_{(2)}^{loop} + M_{(3)}^{loop})\right) d\Phi_{(2)} \tag{14}$$

where amplitudes M^{tree} and $M_{(1,2,3)}^{loop}$ are given by the Feynman diagrams

a b c d

Figure 1

on Figs.1 a,b,c and d respectively. As it can be easily seen the asymmetry appears only as a result of interference of the Born amplitude and one-loop ones because only the latter have the imaginary part. From the point of view of gauge invariance we have to add other one-loop diagrams but all of them give zero contribution to the asymmetry due to the vanishing of Dirac traces. In the c.m.s. of decay particles the asymmetry is proportional to the $sin\phi$, so it reaches the biggest value for $\phi = \frac{\pi}{2}$. In this region e^+e^--invariant mass $M_{e^+e^-}^2 = (p_1 + p_2)^2$ is of the same order of magnitude as the $\mu e^\pm$-invariant masses $M_{\mu e^\pm}^2 = (k + p_{2,1})^2$. So the evaluation of the Feynman diagrams was made in this kinematical region for highly virtual muon ($p^2 \gg m_\mu^2$).

Now we make some remarks about the calculations. As can be easily seen from dimensional counting the loop diagrams do not possess the infrared divergences because one of the external line is off-shell. But all of them possess mass singularities due to the vanishing of electron or muon masses for box and vertex graphs respectively. To eliminate such singularities we have to add according to Lee and Nauenberg's theorem tree diagrams with hard

collinear photon radiation. But the Born diagrams do not have the imaginary part and so to leave the asymmetry nonsingular the above mass singularities cannot give contribution to the imaginary part of the amplitudes. This was checked by the straightforward calculations which give the following results:

$$dw^{(fragm)} = \alpha_{em}^2 \frac{4(4\pi)^2}{p^4 M_{e^+e^-}^4} \left(2M_{\mu e^+}^2 M_{\mu e^-}^2 + M_{e^+e^-}^2(M_{\mu e^+}^2 + M_{\mu e^-}^2)\right) d\Phi_{(2)}$$

$$d(\Delta w^{(fragm)})$$

$$= \alpha_{em}^3 \epsilon_{k12s} \frac{8(4\pi)^2}{p^2\sqrt{p^2} M_{e^+e^-}^4} \left(\frac{M_{\mu e^+}^2}{M_{\mu e^+}^2 + M_{e^+e^-}^2} ln(\frac{M_{\mu e^-}^2}{p^2}) + \frac{M_{\mu e^-}^2}{M_{\mu e^-}^2 + M_{e^+e^-}^2} ln(\frac{M_{\mu e^+}^2}{p^2}) + \frac{M_{\mu e^+}^2 - M_{\mu e^-}^2}{M_{\mu e^+}^2 + M_{\mu e^-}^2}\left(1 + \frac{M_{e^+e^-}^2}{M_{\mu e^+}^2 + M_{\mu e^-}^2} ln(\frac{M_{e^+e^-}^2}{p^2})\right)\right) d\Phi_{(2)} \tag{15}$$

Dividing the last expression by the first one one gets the result of interest:

$$H = \alpha_{em}\epsilon_{k12s} \frac{2\sqrt{p^2}}{\left(2M_{\mu e^+}^2 M_{\mu e^-}^2 + M_{e^+e^-}^2(M_{\mu e^+}^2 + M_{\mu e^-}^2)\right)} \left(\frac{M_{\mu e^+}^2}{M_{\mu e^+}^2 + M_{e^+e^-}^2} ln(\frac{M_{\mu e^-}^2}{p^2}) + \frac{M_{\mu e^-}^2}{M_{\mu e^-}^2 + M_{e^+e^-}^2} ln(\frac{M_{\mu e^+}^2}{p^2}) + \frac{M_{\mu e^+}^2 - M_{\mu e^-}^2}{M_{\mu e^+}^2 + M_{\mu e^-}^2}\left(1 + \frac{M_{e^+e^-}^2}{M_{\mu e^+}^2 + M_{\mu e^-}^2} ln(\frac{M_{e^+e^-}^2}{p^2})\right)\right) \tag{16}$$

To measure reliably this asymmetry beyond 10% accuracy the relative statistical error should be smaller then $\frac{\Delta H}{H} \leq 0.1$. This puts sever constraints on the lower limit of the number of $\mu^+\mu^-$- creation events:

$$tan\left(\frac{\phi}{2}\right) \leq 0.1\alpha_{em}\sqrt{N_{e^+e^-}} \left| ln\left(cos\frac{\phi}{2}\right)\right| \tag{17}$$

$N_{e^+e^-}$ is a number of e^+e^--pair bremsstrahlung events.The number of $\mu^+\mu^-$-events is approximately α_{em}^{-2} times bigger due to the fragmentation process which is of order α_{em}^2. The biggest value of the asymmetry — about half of a percent — is achieved for the $\phi = \frac{\pi}{2}$, but the required amount of $\mu^+\mu^-$-events is enormous in this region ($N_{\mu^+\mu^-} \geq 10^{11}$). For smaller angles ϕ fragmentation cross section becomes constant (for p^2 =const),but the asymmetry decreases further and we cannot diminish the statistical error for the same (or smaller) number of $\mu^+\mu^-$-events. On the contrary the last one increases drastically to keep the same precision of measurements.

4 Conclusion

In summary,we have proposed a way to determine the polarization of final lepton produced in hard collision using recently introduced measurable quantity — handedness. It was shown that the possible realization of this experiment is obviously very difficult (or even impossible at the present stage) because of the required amount of $\mu^+\mu^-$-creation events. But we think that one can decrease the latter considering near on-shell initial muon (formally this can happen due to the pole in the muon propagator, but this effect can be suppressed by vanishing of the phase space volume).

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