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AN ESTIMATE OF THE PROTON SINGLET AXIAL CONSTANT

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In the last time the problem of calculation of the nucleon singlet axial constant has intensively been discussed in the literature. On the classical level, this constant determines the value of spin carried by the valence quarks inside the nonrelativistic nucleon. However, recent experimental data [1] lead one to the quantity which is much smaller than the naive theoretical value. The authors of ref. [2] (see also [3]) have firstly pointed out that it is the axial anomaly which suffers the naive interpretation at the quantum level. As a result, the apparent "spin crisis" was resolved [4].

In spite of this, the problem of calculation of the nucleon singlet axial constant is far from being solved successfully. Based on the detailed analysis [5]-[10], one can conclude that both perturbative and nonperturbative contributions are essential for this calculation.

In the present paper, we will try to estimate the value of the proton singlet axial constant taking into account characteristic peculiarities of the singlet axial channel.

Essentially, the problem is as follows: experimental measurements [1] have allowed one to calculate the integral over the Bjorken variable of the first structure function of polarized deep-inelastic lepton-nucleon scattering [1]. In accordance with the Ellis-Jaffe sum rule [11], this integral is related to the axial constants for the corresponding nucleon

$$\int_0^1 g_1^p(x, Q^2) dx = C^{NS}(1, \alpha_s(Q^2))[\frac{1}{12}g_A^{(3)} + \frac{1}{36}g_A^{(8)}] +$$
$$+ C^S(1, \alpha_s(Q^2))\frac{1}{9}\tilde{g}_A^{(0)}. \tag{1}$$

Here $C^{NS}(1, \alpha_s(Q^2))$ and $C^S(1, \alpha_s(Q^2))$ are the nonsinglet and singlet operator coefficient functions normalized to unity at the tree level. The perturbative expansion for the nonsinglet coefficient function is known at the next-next-to-leading approximation of perturbation theory [12] whenever it is known at the next-to-leading order for the singlet one [13]. The axial constants in eq. (1) are normalized as follows:

$$< P, S|A_\mu^{(3)}|P, S > = < P, S|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|P, S > = g_A^{(3)}S_\mu, \tag{2a}$$

$$< P, S|A_\mu^{(8)}|P, S > = < P, S|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s|P, S > = g_A^{(8)}S_\mu, \tag{2b}$$

$$< P, S|A_\mu^{(0)}|P, S > = < P, S|\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s|P, S > = g_A^{(0)}S_\mu, \tag{2c}$$
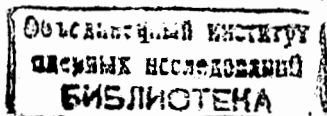
with S_μ being the proton spin four-vector . Here the state vectors are normalized in the Fock space in accordance with the following condition:

$$< P(k_1)|P(k_2) >= (2\pi)^3 2k_1^0 \delta^{(3)}(\vec{k}_2 - \vec{k}_1).$$

The renormalization group invariant quantity $\tilde{g}_A^{(0)}$ in eq. (1) can be written through the scale dependent axial constant $g_A^{(0)}$ [14]

$$\tilde{g}_A^{(0)} = e^{-\int_0^{\alpha_s(\mu)} \frac{\gamma(x)}{\beta(x)} dx} g_A^{(0)},$$

where $\gamma(x)$ is the singlet axial current anomalous dimension known at the three-loop approximation [15] and $\beta(x)$ is the QCD β-function determined at this order too [16]. Let us denote the first and second addendum in the r.h.s. of eq.(1) as a^{NS} and a^S, respectively. As a result, Ellis-Jaffe sum rule will take the form

$$\int_0^1 g_1^p(x, Q^2) dx = a^{NS}(1, \alpha_s(Q^2)) + a^S(1, \alpha_s(Q^2)). \qquad (3a)$$

In these notations singlet part of the Ellis-Jaffe sum rule in the leading order approximation can be written in the form

$$a^S = (1 - 0.3333(\frac{\alpha_s(Q^2)}{\pi}))\frac{1}{9}\tilde{g}_A^{(0)}. \qquad (3b)$$

Substituting the numerical values for the integral in the l.h.s., for the coefficient function $C^{NS}(Q^2)$ [12] and also for the nonsinglet axial constants [17], one obtains a surprisingly small value for the singlet part $a^S \simeq 0.02 \pm 0.01$. It is easy to see that the perturbative corrections presented in (3a) do not save the situation and the only possibility is to assume (in contrast with the naive expectation) that the proton singlet axial constant g_A^0 itself is a small quantity. In what follows we will try to estimate the magnitude for g_A^0.

Let us begin with the consideration of the anomaly equation for the flavor singlet axial current with three active flavors

$$\partial^\mu A_\mu^{(0)} = \sum_{q=u,d,s} 2im_q \bar{q}\gamma_5 q + \frac{\alpha_s N_f}{4\pi} G\tilde{G}. \qquad (4)$$

Brief mention should be made of an important role of the mass terms in this expression. At first glance it would seem that these terms are negligibly small due to the smallness of the corresponding quark masses. Mass terms are often dropped out in the chiral limit. However, it has been demonstrated in [18] [19] [20] that due

to the fermionic zero modes over the instanton background the matrix elements of the operators like $\bar{q}\gamma_5 q$ have singular behavior in the limit of zero quark mass. So, only the whole quantity $m_q\bar{q}\gamma_5 q$ has a strict sense in the chiral limit; moreover, this combination gives a nonzero contribution when the quark masses are set to be equal to zero after all calculations are performed.

On the other hand, one can assume that the mass term is absent in the Lagrangian from the very beginning. If so, the derivative of the axial current would be expressed through the gluon operator only. However, this is the case when the QCD Lagrangian is chiral invariant and consequently the corresponding generating functional could not be able to produce a nonzero value for the quark condensate (if the ordinary Feynman boundary conditions are assumed). The fulfillment of the axial Ward identities in this case becomes doubtful too (for the detailed discussion see ref. [20]). Another way of putting it is that one must calculate quasiaverages instead of simple averages [21] going to the limit $m_q \to 0$ after this [22]. Such a procedure allows one to take into account consequences of the spontaneous breaking of the chiral symmetry in the pure QCD. It has been demonstrated in [19] that the contributions coming from the mass terms in the anomaly equation lead one to the nonrenormalization of the θ-term in QCD. The mass terms play an important role in our calculation too.

Now we are in a position to consider the matrix element of the singlet axial current over the proton states

$$< P(k_1)|A_\mu^{(0)}|P(k_2) >= g_A^{(0)}(q^2)\bar{U}(k_1)\gamma_\mu\gamma_5 U(k_2) - iq_\mu g_P^{(0)}(q^2)\bar{U}(k_1)\gamma_5 U(k_2), \qquad (5)$$

where $q_\mu = k_{1\mu} - k_{2\mu}$, $U(k)$ is the proton spinor. It is well known that there are no massless excitations in the flavor singlet axial channel even in the chiral limit [23]. Consequently, the phenomenological expression for the pseudoscalar constant g_P^0 can be written in the form

$$g_P^0 = \sum_n \frac{B_n}{q^2 - m_n^2}, \qquad (6)$$

where m_n^2 are the mass squares of the corresponding pseudoscalar particles in the flavor singlet case. These quantities are not equal to zero even in the chiral limit. This is an important point where the singlet and nonsinglet constant calculations drastically differ from each other. An analogous expression for the nonsinglet pseudoscalar constant could be presented as a sum of terms containing poles in q^2 and the terms containing the multiplier of an order of $o(m_q^2)$; as a result, the mixing between the axial and the pseudoscalar constants will take place within the corresponding

sum rules. However, as we will see later, this is not true for the flavor singlet case. Reason is that there are no poles in eq. (6) even in the chiral limit. The coefficients B_n in this expression are determined by the proton-meson interaction vertices. Using now eq. (4), the matrix element (5) takes the form

$$< P(k_1)|\partial^\mu A_\mu^{(0)}|P(k_2) > = (-ig_A^{(0)}(q^2)2m_p + q^2 g_P^{(0)}(q^2))\bar{U}(k_1)\gamma_5 U(k_2), \qquad (7)$$

where m_p is the proton mass. Our aim will be to estimate the value for $g_A^{(0)}$ by means of the QCD sum rule method [24]. Following the ideology of this approach consider the three-point correlation function

$$T_\mu^{\alpha\beta}(k_1, k_2) = \int e^{ik_1 x - ik_2 y} < 0|T^*\eta^\alpha(x)A_\mu^{(0)}(0)\bar{\eta}^\beta(y)|0 > dx dy =$$

$$= \int e^{ipx + iqy} < 0|T^*\eta^\alpha(x/2)A_\mu^{(0)}(y)\bar{\eta}^\beta(-x/2)|0 > dx dy, \qquad (8)$$

where $p = (k_1 + k_2)/2$, and $\eta^\alpha(x)$ is an interpolating current for the proton state [25]

$$< 0|\eta^\alpha(x)|P(k_1) > = \lambda_p U^\alpha(k_1).$$

The phenomenological expression for this correlator can be written in the form

$$T_\mu^{\alpha\beta}(k_1, k_2) = \frac{< 0|\eta^\alpha|P(k_1) >< P(k_1)|A_\mu^{(0)}|P(k_2) >< P(k_2)|\bar{\eta}^\beta|0 >}{(k^2 - m_p^2)^2} + ... \qquad (9)$$

where $k^2 = k_1^2 = k_2^2 = p^2 + q^2/4$ and the kinematical condition (pq)=0 is assumed for simplicity. Dots denote here the contributions with the one-pole term and the terms without poles in $(k^2 - m_p^2)$. Multiplying expression (9) over iq^μ and using (7) we obtain

$$iq^\mu T_\mu^{\alpha\beta}(k_1, k_2) = \frac{i\lambda_p^2 g_A^0(q^2)2m_p^2\hat{q}\gamma_5}{(k^2 - m_p^2)^2} +$$

$$\frac{i\lambda_p^2 g_A^0(q^2)q^2[\hat{q}/4 + m_p]\gamma_5 + \lambda_p^2 q^2 g_P^0(q^2)(\hat{k}_1 - m_p)\gamma_5(\hat{k}_2 - m_p)}{(k^2 - m_p^2)^2} + ... \qquad (10)$$

Let us consider now the operator product expansion (OPE) for our three-point correlation function. (Hereafter, we will follow the definitions of [26]). The bilocal OPE for the correlator under investigation looks like

$$iq^\mu T_\mu^{\alpha\beta}(k_1, k_2)|_{|p^2|\to\infty} = \int e^{ipx + iqy} < 0|T^*\eta^\alpha(x/2)\partial^\mu A_\mu^{(0)}(y)\bar{\eta}^\beta(-x/2)|0 > dx dy|_{|p^2|\to\infty}$$

$$= \sum_n C_n(p)i \int e^{iqy} < 0|T^*\hat{O}_n(0)\partial^\mu A_\mu^{(0)}(y)|0 > dy + \sum_n R_n(p, q) < 0|\hat{O}_n(0)|0 > . \qquad (11)$$

The first sum in the r.h.s. of this expansion is determined by the vacuum expectation values of some bilocal operators and the second one gives the contributions of the local operator vacuum expectation values. The operators $\hat{O}_n(0)$ are generated within the OPE for the proton interpolating currents

$$i \int e^{ipx} T^* \eta^\alpha(x/2)\bar{\eta}^\beta(-x/2)dx = \sum_n C_n(p)\hat{O}_n(0)$$

It is worthwhile to point out here that the bilocal VEVs play the crucial role in this approach, in fact, they give the dominant contribution in the case of the sum rules for the proton pseudoscalar constants (for a detailed discussion see ref. [26]). However, we will demonstrate that the bilocal VEVs cancel each other in the singlet case and only the local part of the OPE determines the value for the proton singlet axial constant. Let us turn to the consideration of the correlators presented in the r.h.s. of eq. (11)

$$P_n(q^2) = i \int e^{iqy} < 0|T^*\hat{O}_n(0)\partial^\mu A_\mu^{(0)}(y)|0 > dy.$$

The dispersion relation for this quantity looks like

$$P_n(Q^2 = -q^2) = \frac{1}{\pi} \int \frac{ImP_n(s)ds}{s + Q^2} + subtractions.$$

Some remarks concerning the peculiarities of this expression are in order here. It is well known that the correlator for two singlet axial currents $< A_\mu^{(0)} A_\nu^{(0)} >$ contains the so-called Kogut-Susskind pole [27] beyond the intermediate physical particle contributions. This pole term is produced by the collective excitations over the complicated vacuum [28]. Within the momentum representation for $< A_\mu^{(0)} A_\nu^{(0)} >$ the Kogut-Susskind pole gives an additional term proportional to $(q_\mu q_\nu)/q^2$. Due to singularities in q^2, the existence of such a contribution makes it difficult to use the sum rule formalism. However, in our case, after multiplication of the corresponding correlator over q^μ the Kogut-Susskind contribution becomes the polynomial in momenta and, consequently, could be absorbed in the subtractional part of the dispersion relation. Then, using the Borel transformation this polynomial disappears from the sum rules under investigation. Later on, we shall follow this way.

On the other hand, the phenomenological expression for the imaginary part of the correlator under consideration looks like

$$ImP_n(s) = \pi\Delta(s) < 0|\hat{O}_n|0 >< 0| \sum_{q=u,d,s} 2im_q\bar{q}\gamma_5 q + \frac{\alpha_s N_f}{4\pi} G\tilde{G}|0 > +$$

$$+\pi \sum_k \delta(s - m_k^2) < 0|\hat{O}_n|P_k >< P_k| \sum_{q=u,d,s} 2im_q\bar{q}\gamma_5 q + \frac{\alpha_s N_f}{4\pi} G\tilde{G}|0 >,$$

where $\Delta(s)$ determines the complicated vacuum contribution. The second addendum in the r.h.s. of this relation gives the contribution only to the proton pseudoscalar constant g_P^0 and it is the first term in the r.h.s. which is responsible for g_A^0. However, following [18] [20] this last is equal to zero. Indeed, let us integrate over the fermionic fields in the path integral representation for $< 0|\bar{q}\gamma_5 q|0 >$ with the following calculation of the corresponding determinant in terms of the gluon fields. As an answer one can obtain [18] [20]

$$< 0|\bar{q}\gamma_5 q|0 > = -\frac{\alpha_s}{i8\pi}\frac{G\check{G}}{m_q}.$$

Consequently

$$< 0| \sum_{q=u,d,s} 2im_q\bar{q}\gamma_5 q + \frac{\alpha_s N_f}{4\pi} G\tilde{G}|0 > = 0.$$

We can conclude that the bilocal operator VEVs do not affect the sum rule for the proton singlet axial constant. It is important to point out that this statement is based on the assumption of absence of the massless excitations in the singlet channel in the chiral limit (relation with the $U(1)$ problem, see ref. [29]). Now it is evident that the proton singlet axial constant must be small in comparison with the nonsinglet one. The point is that the local part of the OPE determining the value for g_A^0 gives the contributions of the order α_s^2 only. Let us turn to the consideration of these terms. For practical calculations of the local part it is convenient to introduce the following quantity

$$D_\tau^{\alpha\beta} = [\frac{\partial}{\partial q^\tau}(iq^\mu T_\mu^{\alpha\beta})]|_{q=0} =$$

$$= \frac{i\lambda_p^2 g_A^0(0)2m_p^2\gamma_\tau\gamma_5}{(p^2 - m_p^2)^2} + \frac{C\gamma_\tau\gamma_5}{(p^2 - m_p^2)} + ..., \qquad (12)$$

where C is some unknown constant. The leading contributions to the sum rule for g_A^0 come from the three-loop perturbative diagram and the one-loop diagram with the four-fermion condensate. Our main assumption is that the perturbative three-loop contribution is suppressed by the loop-factor $1/(4\pi^2)^3$ and could be neglected in the case of rough estimation. The only thing we have done is the calculation of the four-fermion condensate contributions. In such an approximation, the corresponding sum rule takes the form

$$\frac{\lambda_p^2 g_A^0(0)2m_p^2}{(P^2 + m_p^2)^2} + \frac{iC}{(P^2 + m_p^2)} + ... = C_F^2(\frac{\alpha_s}{\pi})^2\frac{1}{2}\frac{< \bar{u}u >^2}{P^2}ln(P^2/\mu^2),$$

where C_F is the Casimir operator of the fundamental representation of $SU(3)_c$ group ($C_F = 4/3$) and $P^2 = -p^2$. Multiplying this expression over the quantity $P^2 + m_p^2$ and using then the Borel transformation, we obtain the following sum rule for the proton singlet axial constant

$$2\lambda_p^2 g_A^0(0)e^{-\frac{1}{\tau}} \simeq C_F^2(\frac{\alpha_s}{\pi})^2\frac{1}{2} < \bar{u}u >^2 (\tau^2 - \tau), \qquad (13)$$

where $\tau = M^2/m_p^2$ and M^2 is the Borel parameter. Substituting the known values for the quark condensate and all parameters in eq. (13) [25] we obtain the following result

$$g_A^0(0)|_{\mu=1GeV}e^{-\frac{1}{\tau}} \simeq 0.025(\tau^2 - \tau).$$

The relative stability in these sum rules is reached when $\tau \simeq 1.3 - 1.5$ and consequently

$$g_A^0(0)|_{\mu=1GeV} \simeq 0.02 - 0.03.$$

So our crude estimation leads to the value for the proton singlet axial constant which is sufficiently small in comparison with unity and the nonsinglet constants.

In conclusion, let us make some remarks concerning our consideration. First of all, it is necessary to point out that the result obtained here is qualitative in its essence and could be considered as some crude approximation only. For a more precise calculation of the value for the proton singlet axial constant it is necessary to take into account both the three-loop perturbative contribution and the dimension-seven quark-gluon operator contribution within the QCD sum rule approach. However, one could not expect that these contributions will lead to sufficient increasing in the value for the proton singlet axial constant. Missing contributions are of an order of α_s^2 only. We have demonstrated that correct treatment of the anomalous contributions leads to sufficient numerical suppression for the value of the proton singlet axial constant (in terms of sum rules the cancellation of the bilocal contributions takes place). This statement is based on the assumption that there are no massless excitations in the flavor singlet channel even in the chiral limit.

The second remark concerns the physical interpretation of the matrix element of the singlet axial current over the proton states. This constant does not determine the fraction of the spin carried by quarks inside the proton. There are two reasons of numerical deviation of g_A^0 from the value of spin carried by valence quarks. The first one is the existence of contributions comming from the axial anomaly and the second is the perturbative corrections which can be summed up in the leading logarithmic approximation and could produce substantial suppression at the low normalization points where the nonrelativistic treatment for g_A^0 as spin is valid.

Finally, let us compare the value for g_A^0 obtained here with the results of some other calculations. It is often assumed that the proton singlet axial constant is totally determined by the matrix element $< P|\frac{\alpha_s}{4\pi}G\tilde{G}|P >$. The analytical [30] and the lattice [31] calculations of this matrix element lead to the value for g_A^0 which is ten times higher than our result. This deviation is well understandable in terms of sum rules. The point is that in the case of pure gluonic definition for g_A^0 the cancellation of bilocal contributions does not take place. As a consequence, the dominant contribution in g_A^0 is due to the vacuum expectation values of bilocal operators. All local contributions give just small corrections to this quantity. As we have shown, correct treatment of the mass terms leads to the cancellation of large bilocal contributions.

So, we have estimated the value for the proton singlet axial constant. This value is sufficiently suppressed in comparison with the naively expected one.

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Габададзе Г.Т.   E2-94-518
Оценка синглетной аксиальной константы протона

Дана оценка величины синглетной аксиальной константы протона. Показано, что аксиальная аномалия играет ключевую роль для данного вычисления.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова ОИЯИ.

Gabadadze G.T.   E2-94-518
An Estimate of the Proton Singlet Axial Constant

The value of the proton singlet axial constant is estimated. It has been shown that the axial anomaly plays a crucial role in this calculation.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.