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A.G.Galperin, V.V.Uzhinskii

THE CALCULATION OF NUCLEUS-NUCLEUS INTERACTION CROSS-SECTIONS AT HIGH ENERGY IN THE GLAUBER APPROACH

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1. Introduction

General characters of hadron-nucleus(hA) and AA interactions at HE and SHE such as total,inelastic and elastic cross–sections are necessary to calculate radiation damage of the matter upon action of cosmical rays,broad atmosphere showers, radiation protection of accelerators, physical background in some experiments etc. There are detailed compilations of experimental data on hh collisions at accelerator energies[1,2].

At SHE cross–sections of hadron-hadron (hh) collisions are calculated in the reggeon theory [3-5].

Data on hA interactions can be obtained from [6,7].

In an energy region where experimental data on AA collisions are not existing up to now, analytical approximations [7], the "soft sphere" model [8] or Glauber's type approaches [9] are used.

The expression of approximate measured values of inelastic cross–sections given in [10,11] (see [12] too)

$$\sigma_{in}\,(A,B) \;=\; \pi r_0^2 \cdot (A^{1/3} + B^{1/3} - c)^2, \tag{1}$$

where $r_0 \simeq 1.5 fm$, $c \simeq 1.3$ and A and B are mass numbers of colliding nuclei, has been used often enough. Experimental data are described with expression (1) quite reasonably, although r and c are slightly different at various energies. The marked discrepancy between experimental data and their description by expression (1) takes place both with most light and with most heavy nuclei.

A better compliance is achieved [13] when the expression

$$\sigma_{in}\,(A,B) = \;\pi r_0^2 \left[A^{1/3} + B^{1/3} - c\,(\frac{1}{A^{1/3}} + \frac{1}{B^{1/3}})\,\right]^2, \tag{2}$$

is used to describe data. Here r = 1.2–1.4 fm, c = 0.7–1 and varies slightly with energy.

A more universal and precise approximation of AA inelastic cross–sections has been proposed in [14] and does for low and high energy. While fitting elastic cross-sections by means of expression (1) or (2) the conformity to data is worse than applying of these expressions to inelastic cross–section calculations.

The main goal of present paper is to produce calculations of total, inelastic and elastic cross–sections of AA interactions in Glauber's approach at HE and SHE.

2. The Glauber theory of AA interactions

The amplitude of scattering of nucleus A on nucleus B, when each of them transforms from the initial state $|i\rangle$ into the final states $|f\rangle$, is given in Glauber's approach [9,15-18] by expression

$$F_{A,B}(\vec{q}) = \frac{ip_A}{2\pi}\int d^2b\, e^{i\vec{q}\vec{b}} \langle f_A, f_B|1 - \prod_{j=1}^{A}\prod_{k=1}^{B}(1-\gamma(\vec{b}-\vec{s}_j+\vec{r}_k))|i_B, i_A\rangle, \tag{3}$$

where p_A is the momentum of the projectile nucleus A, $\vec{q}$ is the transferred transversal momentum, $\vec{b}$ is an impact parameter, γ is an amplitude of elastic NN scattering in the impact parameter representation, $\{\vec{s}_j\}$, $j = 1,2,...,A$ and $\{\vec{r}_k\}$, $k = 1,2,...,B$ are coordinates of nucleons within, respectively, A and B nucleus on the impact parameter plane. These coordinates are measured from the center of mass of each nucleus respectively, too.

Starting from eq.(3), it is possible to find AA elastic scattering amplitude

$$F^{el}_{AB}(\vec{q}) = \frac{i p_A}{2\pi} \int d^2b\, e^{i\vec{q}\vec{b}} \{1 - \prod_{j=1}^{A}\prod_{k=1}^{B}(1-\gamma(\vec{b}-\vec{s}_j+\vec{\tau}_k))\} \cdot$$

$$|\psi_A(\vec{r}_1,\ldots,\vec{r}_A)|^2 \prod_{j=1}^{A} d^3r_j \left|\psi_B(\vec{t}_1,\ldots,\vec{t}_B)\right|^2 \prod_{k=1}^{B} d^3t_k, \tag{4}$$

differential cross–section

$$\frac{d\sigma^{el}}{d\Omega} = \left|F^{el}_{AB}\right|^2, \tag{5}$$

and total cross–section

$$\sigma^{tot}_{AB} = \frac{4\pi}{p_A} Im F^{el}_{AB}(0). \tag{6}$$

The cross–section of quasi–elastic scattering of nucleus A when it is conserved, but other nucleus B undergoes all excitations including destruction too ($A + B \to A + X$) is given by expression

$$\sigma(A+B\to A+X) = \int d^2b \{1 - \prod_{i=1}^{A}\prod_{k=1}^{B}(1-\gamma(\vec{b}-\vec{s}_i+\vec{\tau}_k))\} \cdot$$

$$\{1 - \prod_{j=1}^{A}\prod_{k=1}^{B}(1-\gamma^*(\vec{b}-\vec{s}_i{}'+\vec{\tau}_j))\} \cdot$$

$$|\psi_A(\vec{r}_1,\ldots,\vec{r}_A)|^2 \left|\psi_A(\vec{r}_1{}',\ldots,\vec{r}_A{}')\right|^2 \left|\psi_B(\vec{t}_1,\ldots,\vec{t}_B)\right|^2 \cdot$$

$$\prod_{i=1}^{A} d^3r_i \prod_{i=1}^{A} d^3r_i{}' \prod_{i=1}^{B} d^3t_i - \sigma^{el}_{AB}. \tag{7}$$

Finally, the cross–section of production of new particles may be defined as

$$\sigma^{prod}_{AB} = \int d^2b \{1 - \prod_{i=1}^{A}\prod_{k=1}^{B}(1-p(\vec{b}-\vec{s}_i+\vec{\tau}_k))\} \cdot$$

$$|\psi_A(\vec{r}_1,\ldots,\vec{r}_A)|^2 \prod_{i=1}^{A} d^3r_i \left|\psi_B(\vec{t}_1,\ldots,\vec{t}_B)\right|^2 \prod_{i=1}^{B} d^3t_i. \tag{8}$$

$$p(\vec{b}) = \gamma(\vec{b}) + \gamma^*(\vec{b}) - \gamma(\vec{b})\gamma^*(\vec{b}).$$

Eq.(8) may be rewritten in some form where each of terms would be interpreted as a probability of some process

$$\sigma^{prod}_{AB} = \int d^2b \{\prod_{i=1}^{A}\prod_{j=1}^{B} \frac{p(\vec{b}-\vec{s}_i+\vec{\tau}_j)}{1-p(\vec{b}-\vec{s}_i+\vec{\tau}_j)} \cdot$$

$$\prod_{k=1}^{A}\prod_{l=1}^{B}(1-p(\vec{b}-\vec{s}_k+\vec{\tau}_l)) \cdot$$

$$+\frac{1}{2} \cdot \prod_{\substack{i=1,j=1\\ i\neq j}}^{A} \prod_{k=1}^{B} \frac{p(\vec{b}-\vec{s}_i+\vec{\tau}_k)}{1-p(\vec{b}-\vec{s}_i+\vec{\tau}_k)} \cdot$$

$$\frac{p(\vec{b}-\vec{s}_j+\vec{\tau}_k)}{1-p(\vec{b}-\vec{s}_j+\vec{\tau}_k)} \prod_{l=1}^{A}\prod_{m=1}^{B}(1-p(\vec{b}-\vec{s}_l+\vec{\tau}_m)) + \ldots\} \cdot$$

$$|\psi_A(\vec{r}_1,\ldots,\vec{r}_A)|^2 \prod_{i=1}^{A} d^3r_i \left|\psi_B(\vec{t}_1,\ldots,\vec{t}_B)\right|^2 \prod_{i=1}^{B} d^3t_i. \tag{9}$$

Here the first term in the first braces is interpreted as a probability that the only one inelastic collision between i–th nucleon from nucleus A and j–th nucleon from nucleus B takes place when all nucleons coordinates are fixed.The second term describes a probability of inelastic collision of the k–th nucleon from nucleus B with i–th and j–th nucleons in A nucleus, etc. Nucleons involved in collisions were named "wounded" but others were named "spectators".

3. The method of calculation

To calculate all cross–sections discussed above, it is necessary to give a function $\gamma(\vec{b})$ and square of modulus of ground state wave function $|\psi|^2$ of A and B nuclei.

The approximation

$$\gamma(\vec{b}) = \sigma^{tot}_{NN} \cdot \frac{1-i\alpha}{4\pi B} e^{-\vec{b}^2/2B} \tag{10}$$

is often used at $E > 1 GeV/nucl$. Here σ_{NN} is total cross–section of NN interaction, α is ratio of real part to imaginary part of elastic scattering amplitude at zero momentum transfer, B is the slope parameter of differential cross–section of elastic NN scattering. Expression (10) corresponds to the following parameterization of the elastic NN scattering in the momentum representation

$$f_{NN}(\vec{q}) = \frac{ip_A}{4\pi} \cdot \sigma^{tot}_{NN}(1-i\alpha)e^{-\vec{q}^2/2B}.$$

Sets of values $\sigma^{tot}_{NN}, \alpha$ and B at different energies are presented in a number of compilations (see [1,2,6]). Function $|\psi_A|^2$ is often given as

$$|\psi_A|^2 = \prod_{i=1}^{A} \rho_A(\vec{r}_i), \tag{11}$$

where ρ represents one-particle density of nuclei.In this case the aggregate of nucleon coordinates does not meets self–evident demands

$$\sum_{i=1}^{A} \vec{r}_i = 0. \tag{12}$$

Taking into account of this condition is named " an account of center of mass correlation".

If projectile is a nucleon it is necessary to set a density ρ_A as a delta-function $\rho_A(\vec{r}) = \delta(\vec{r})$ in this case.

The parameterization from [19] is used **for deuterons**

$$|\psi_d|^2 = \sum_{i=1}^{3} c_i e^{-\vec{r}^2/4\gamma_i}. \tag{13}$$

$$\gamma_1 = 225(GeV/c)^2, \quad c_1 = 0.178/(4\pi\gamma_1)^{3/2}.$$
$$\gamma_2 = 45(GeV/c)^2, \quad c_1 = 0.287/(4\pi\gamma_2)^{3/2}.$$
$$\gamma_3 = 25(GeV/c)^2, \quad c_1 = 0.535/(4\pi\gamma_3)^{3/2}.$$

Here r is a distance between a proton and a neutron within a deuteron.

For ^{3}H, ^{3}He and ^{4}He nuclei $|\psi_A|^2$ has been chosen as

$$|\psi_A|^2 = \delta(\frac{1}{A}\sum_{i=1}^{A}\vec{r}_i)\prod_{i=1}^{A}\frac{1}{(\pi R_A^2)^{3/2}}e^{-r_i^2/R_A^2}. \tag{14}$$

where $R_{^3\mathrm{H}} = R_{^3\mathrm{He}} = 1.81fm$, $R_{^4\mathrm{He}} = 1.37fm$. To all other nuclei ($A \geq 6$) a one-particle density has been defined as

$$\rho_A(r) = const\ /\ (1 + e^{\frac{r-R_A}{c}}), \tag{15}$$

with $R_A = 1.07 \cdot A^{1/3}fm$, $c = 0.545fm$. The center mass correlation has been taken as in paper [20].

The calculation of cross-sections is performed in accordance with the method discussed in [21], where the statement

$$|\psi_A|^2 \prod_{i=1}^{A} d^3r_i |\psi_B|^2 \prod_{j=1}^{B} d^3t_j$$

is treated as a probability measure to find out different sets of nucleon coordinates in A or B nucleus. In this case, the cross-sections are searched as mean values over various sets of nucleon coordinates. Thus it can be written in the form

$$\sigma_{AB}^{tot} = \frac{1}{N_{stat}} \cdot \sum_{i=1}^{N_{stat}} \int d^2b E_{AB}^{tot}(\vec{b}, \{\vec{r_A}\}, \{\vec{t_B}\}) \ , \tag{16}$$

where

$$E_{AB}^{tot} = 1 - \prod_{i=1}^{A}\prod_{j=1}^{B}(1 - \gamma(\vec{b} - \vec{s_i} + \vec{\tau_j}).$$

and N_{stat} denotes a number of various sets of nucleon coordinates. Therefore an accuracy of the calculation depends on the value of N_{stat}.

Expressions analogous to (16) may be written to all other cross-sections.

The next function is named as distribution over impact parameter for inelastic AA interactions

$$p(\vec{b}) = \frac{1}{N_{stat}} \cdot \sum_{i=1}^{N_{stat}} E_{AB}^{prod}(\vec{b}, \{\vec{r_A}\}, \{\vec{t_B}\}), \tag{17}$$

where

$$E_{AB}^{prod} = 1 - \prod_{i=1}^{A}\prod_{j=1}^{B}(1 - p(\vec{b} - \vec{s_i} + \vec{\tau_j}).$$

In accordance with expression (9) the algorithm to simulation of inelastic collisions is described this way.

1. The function $p(\vec{b})$ is calculated and tabulated.
2. Impact parameter $\vec{b}$ is generated in accordance with $p(\vec{b})$ distribution.
3. Coordinates of nucleons within nuclei are sampled according to $|\psi_{A,B}|^2$ distribution functions.

4. Pairs of nucleons interacting inelastically are detected and stored. For this purposes $A \cdot B$ random numbers ξ_{ij} ($i = 1, \ldots, A$, $j = 1, \ldots, B$), uniformly distributed in the [0,1.] interval, are chosen. If $\xi_{ij} < p(\vec{b} - \vec{s_i} + \vec{\tau_j})$ then it is considered that inelastic collision has taken place between i-th nucleon of A nucleus and j-th nucleon of B nucleus.

The first step is performed only once with given nuclei A and B. But steps 2-4 are repeated as many times, as it is necessary.

4. Program user guide

The method described above was realized as a set of routines. These routines and their assignments are presented in the table.

Table

N	name	Routines' function	Subject
1	TOTAL	Calc. total cross–sect.	σ_{AB}^{tot}
2	ELASTIC	Calc. elastic cross–sect.	σ_{el}
3	AB_TO_AX	Calc. cross–sect. of reaction	$A+B \to A+X$
4	AB_TO_XB	Calc. cross–sect. of reaction	$A+B \to X+B$
5	AB_TO_X	Calc. cross–sect. of reaction without process of production of particles	$A+B \to X$
6	PRODUCT	Calc. cross–sect. of multi–particle production	σ_{AB}^{prod}
7	G_X_SECT	Calc. all above cross–sect.	
8	GL_STAR	Simulates the inelastic events	

Program operates interactively. A calling sequence is analogous to each routine. While starting, the prompt "Type output file name" is displayed. Naturally, final results are written on this file. But if there is another file with the name you are choosing, the existing file will be lost (of course, after that you will push CR pad) if you don't change your opinion. In any case a new file with the name you are defining will be created (of course, after you will push CR pad, too).

Don't use symbols " and ' in a user defined file names, please.

Don't worry, program shall put on a few questions.

If it prompts you about a mass and charge number of the projectile or target nucleus, type as your answer the necessary numbers in integer form. For example, if you are interesting in O+Cu interaction you answer 32 16 on the question about the projectile nucleus and 64 32 about the target one.

Program shall prompt you *"Enter NN—interaction characters: total cross-sect.(mb)"* . This value ought be determined as $\sigma_{NN}^{tot} = \frac{1}{4}\sigma_{pp}^{tot} + \frac{1}{2}\sigma_{np}^{tot} + \frac{1}{4}\sigma_{nn}^{tot}$.

It is equal approximately to $42mb$ at the energy 3-3.5$GeV/nucl$ and you are to type 42. The value of the slope parameter of differential cross-section of elastic scattering (the next prompt) at mentioned upper energy is close to $B = 7.6$ and you type it as a floating point number 7.6.[2]

[2] See comp. [1,2] to various energies.

Type your answer in this format too, when you are prompted about a value of a ratio of real part to imaginary part of elastic NN scattering amplitude at zero momentum transfer,so-called α. At the interval of referred energy $\alpha = 0.23$.

The prompt *"Enter statistic"*does mean an integer number of events (in terms of a set of nucleon coordinates in colliding nuclei) you suppose be reasonable to achieve a desirable accuracy in assessing averages. If statistics is of order of 10 you have got the evaluated values with a poor accuracy, but when you increase the statistics a time of calculation becomes larger, too. We would recommend statistics between 50 and 100 to light and middle nuclei.

All results of the calculations you would do, will be displayed and will be written on the user defined output file.

There is a little remark about the routine GL_STAR. Here you will be prompted *"Enter number of stars"*.Try to answer 100, please. In running time it will be displayed to each event: the event number, the value of impact parameter (in fm), the quantity of NN interactions within colliding nuclei, the multiplicity of " wounded " protons and neutrons to both of colliding nuclei. These values would be useful to you.

Distributing of program

The copies of programs can be taken through the e.mail: uzhin@lcta9.jinr.dubna.su

You are welcome with notes and wishes.

Good luck.

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Гальперин А.Г., Ужинский В.В. E2-94-505
Вычисление сечений ядро-ядерных взаимодействий при высоких энергиях в глауберовском приближении

Полные, неупругие и упругие сечения ядро-ядерных взаимодействий при высоких и сверхвысоких энергиях вычисляются в глауберовском подходе. Вычислительная схема реализована в виде набора программ. В процессе вычислений используется метод статистического усреднения. Программы работают в интерактивном режиме. Пользователь запрашивается о зарядовых и массовых числах взаимодействующих ядер, а также характеристикаx *NN*-взаимодействия при интересующей его энергии, значениях полного сечения, параметра наклона дифференциального сечения упругого рассеяния и отношения реальной и мнимой частей амплитуды упругого рассеяния при нулевой передаче поперечного импульса. Необходимые данные могут быть извлечены из соответствующих компиляций.

Результаты вычислений выводятся на экран и записываются на указанный пользователем (в ответ на запрос программы) файл.

Программы работают на *PC*.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Galperin A.G., Uzhinskii V.V. E2-94-505
The Calculation of Nucleus-Nucleus Interaction Cross-Sections at High Energy in the Glauber Approach

Total, inelastic and elastic cross-sections of nucleus-nucleus (*AA*)-interactions at *HE*[1] are calculated on the base of Glauber approach. The calculation scheme is realized as a set of routines. The statistical average method is used in calculations. Program runs in an interactive regime. User is prompted about charge and mass numbers of nuclei and *NN*-interaction characters at the energy he is interested in: total cross-section, the slope parameter of differential cross-section of elastic scattering and ratio of real part to imaginary part of elastic scattering amplitude at zero momentum transfer. These data can be extracted from proper compilations.

Results of calculations are displayed and are written on user defined output file.

The program runs on *PC*.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

[1] *SHE*/*HE* means (super) high energy

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