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PARTICLE DENSITIES AND INCLUSIVE DISTRIBUTIONS IN $pp/p\bar{p}$ COLLISIONS AT HIGH ENERGIES[2]

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In the study of single particle inclusive distributions new methods of the phenomenological descriptions have been proposed. One of them, based on unified approach to the production of hadrons with large transverse momenta and the cumulative type (subthreshold) particles has been developed in [1]. Common feature for both types of processes is the enhanced A-dependence and especially the evidence for the local character of the interaction resulting from numerous experimental and theoretical investigations.

Though the production of particles with large transverse momenta is connected mostly with the 'hard' physics and the processes in which secondaries are emitted in the longitudinal direction with extremely large momenta (e.g. cumulative particles) are predominantly 'soft', it is just the locality of the interaction which is common for both of them. The fact that the interaction is local naturally leads to the conclusion about the scale-invariance of the hadron interactions cross sections [2].

The phenomenological approach proposed in [1] and later generalized in [3, 4] is based just on the scale-invariant properties of hadron/nuclear interactions at high energies. It assumes that the invariant cross section of the inclusive reaction

$$M_1 + M_2 \to m_1 + X \tag{1}$$

depends on the scale-invariant fractions x_1 and x_2 of the incoming 4-momenta P_1 and P_2, and/or on the minimum of the combination

$$s_x^{1/2} = \sqrt{(x_1 P_1 + x_2 P_2)^2} \tag{2}$$

which represents the energy of the colliding constituents that is necessary for the production of the inclusive particle. According to this scheme, the gross features of the single particle distributions for the reaction (1) can be described in terms of the corresponding kinematical characteristics of the exclusive parton subprocess

$$(x_1 M_1) + (x_2 M_2) \rightarrow m_1 + (x_1 M_1 + x_2 M_2 + m_2). \quad (3)$$

The parameter m_2 is a minimal mass introduced in connection with internal conservation laws. The minimal energy of colliding constituents $s_{min}^{1/2}$ or the threshold for the production of the particle m_1 with 4-momentum q is determined by the minimalization of (2), under the additional constraint

$$(x_1 P_1 + x_2 P_2 - q)^2 = (x_1 M_1 + x_2 M_2 + m_2)^2 \quad (4)$$

on the energy-momentum conservation of the subprocess (3).

It was demonstrated (see,e.g. [3, 4] and references cited therein) that in the incoming lab. energy range of some tens GeV/u the invariant differential cross sections for the production of various types of particles, at different angles, at different production energies and for the entire diversity of reactions can be represented as a universal simple scaling law in dependence on the kinematical variable $s_{min}^{1/2}$.

At higher energies, however, significant deviations from the universality have been observed [1]. Over the ISR and the CERN proton-antiproton Collider energy range, $pp/p\bar{p}$ interactions are characterized by strong increase of the inclusive distributions of the particles with large $\vec{q}_\perp$. Consequently, the cross sections at large $s_{min}^{1/2}$ exceed the cross sections for the production of particles into fragmentation region up to several orders of magnitude, and the universality is lost.

In pp interactions the problems of the method proposed in [1] arise even at lower energies. Namely, the minimalization of $s_x^{1/2}$ under the condition (4) gives the unphysical values of $x_2 > 1$ for large q. As a demonstration, we use the data on π^+ backward production in pp interactions at 8.9 GeV lab. energy [2]. For the π^+ meson momenta larger than $\sim$ 280 MeV/c (the variable x_2 here saturates) we fix $x_2 = 1$ from the physical reasons and calculate the value of x_1 according to (4). Constructing thus the

variable $s_x^{1/2}$, we present data [2] (+ symbols) in Fig.1. The open circles are the data taken from [5] for $q_{||}$=0.6 GeV/c (cms.) at initial lab. proton energy 12.4 GeV. Here we see that the universality of the description is broken at the end of the spectrum considerably.

As concerns the determination of $s_{min}^{1/2}$ the additional condition (4) is essential. The energy-momentum conservation of the parton subprocess (3) reflects the situation when the subsystem does not interact with the rest of the system at all. In reality, this is certainly not the case. Therefore, we leave out this requirement in our approach and make the fractions x_1 and x_2 independent of each other. Instead of (4) we admit the off-shell behaviour of the subprocess (3) and write:

$$(x_1 P_1 + x_2 P_2 - q)^2 = (x_1 M_1 + x_2 M_2 + m_2)^2 + \Delta_q(x_1, x_2). \quad (5)$$

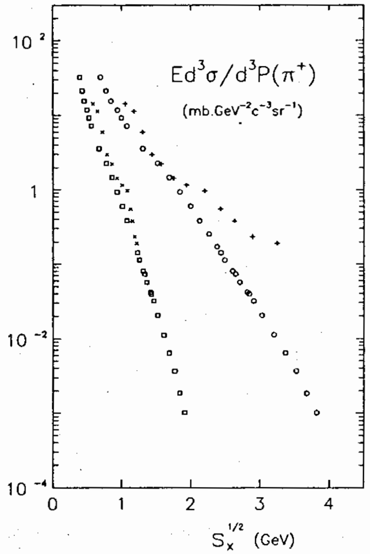
The function Δ_q can acquire various values. Now, we determine the fractions in the way to minimize the value of Δ_q, simultaneously fulfilling the symmetry requirement of the problem, i.e. $A_1 x_1 = A_2 x_2$ for the inclusive particle detected at 90^0 in the corresponding NN centre-of-mass system. This gives

$$x_1 \equiv \frac{\bar{x}_1}{A_1} = \frac{(P_2 q) + M_2 m_2}{(P_1 P_2) - M_1 M_2}, \qquad x_2 \equiv \frac{\bar{x}_2}{A_2} = \frac{(P_1 q) + M_1 m_2}{(P_1 P_2) - M_1 M_2}, \quad (6)$$

where A_1 and A_2 are mass numbers and $\bar{x}_1$ and $\bar{x}_2$ are the fractions of the colliding nuclei expressed in units of the nucleon mass. Note, that x_1 and x_2 are therefore less than unity for all values of q. The minimal value of Δ_q corresponds to the subprocess (3) with the minimal released energy to the away side direction. The missing energy in the energy balance forms a binding of the subprocess to the rest of the system and it might by consumed on the local creation of the associate multiplicity.

From the fractions (6) determined by the off-shell method we construct the energy $s_x^{1/2}$ according to (2) and represent the above mentioned pp data in dependence on this variable. The result is shown in Fig.1 with open squares and ($\times$) symbols. It is seen from the figure immediately, that the discrepancy at the end of the spectrum has disappeared. The problem with the increasing energy remains, however, still unsolved.

Fig.1. Invariant differential cross section for the production of π^+ in pp interactions as a function of $s_x^{1/2}$. Data [5] with fixed $q_{\|}$=0.6 GeV/c at 12.4 GeV lab. incoming energy (o) and data [2] for 180° at 8.9 GeV lab. incoming energy (+) in dependence on the variable $s_x^{1/2}$ determined using the condition (4) are presented here. The same data (open squares) and (×) are shown when the off-shell method for the determination of $s_x^{1/2}$ has been used.



In the elaboration of our approach to the description of inclusive production of particles in $pp/p\bar{p}$ collisions (in the next we use $A_1 = A_2 = 1$ and $M_1 = M_2 \equiv M$ for the nucleon mass) at high energies we shall search for a solution which, similarly as in [1], depends on one variable only. Let's denote such a variable as z and write

$$\frac{d\sigma}{dz} \equiv \psi(z), \tag{7}$$

where ψ has to be a scaling function. The invariant differential cross section for the production of the inclusive particle m_1 depends on two variables, say $q_\perp$ and $q_{\|}$, through $z = z(x_1(q_\perp, q_{\|}), x_2(q_\perp, q_{\|}))$ in the following way:

$$E_q \frac{d\sigma}{d\vec{q}} = -\frac{1}{s\pi}\left(\frac{d\psi(z)}{dz}\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2} + \psi(z)\frac{\partial^2 z}{\partial x_1 \partial x_2}\right). \tag{8}$$

This can be easy shown by partially differentiating using the approximation to the Jacobian of the transformation (6) which at high energies tends to the value $-2q_\perp/(sE_q)$. Now, the problem is to find and choose z as a physically meaningful variable which could reflect the self-similarity (scale-invariance) as a general pattern of the hadron production. In the first step we use the simple choice

$$z = \frac{s_x^{1/2}}{Q} \rightarrow \frac{\sqrt{x_1 x_2}\sqrt{s}}{Q}, \tag{9}$$

with Q as a scale which in a first approximation doesn't depend on x_1 and x_2. Then the expression (8) at asymptotically high energies reads:

$$Q^2 E_q \frac{d\sigma}{d\vec{q}}(q_\perp, q_{\|}) = 4H\left(\frac{s_x^{1/2}}{Q}\right); \quad H(z) \doteq -\frac{1}{16\pi}\left(\frac{d\psi(z)}{dz} + \frac{\psi(z)}{z}\right). \tag{10}$$

Next, in analogy with KNO 'ansatz' [6]

$$< n > P(n) = \psi_{KNO}\left(\frac{n}{< n >}\right) \tag{11}$$

we introduce a new dynamical scaling hypothesis for the inclusive particle production in $pp/p\bar{p}$ interactions at high energies. We determine the scale Q to be proportional to the dynamical quantity - average multiplicity density $dN(0)/d\eta$ - produced in the central region of the collision at a given energy. We write

$$z = \frac{s_x^{1/2}}{\Delta M \cdot dN(0)/d\eta}, \tag{12}$$

where the coefficient ΔM has the dimension of energy and we determine it as 'the reaction energy of the inclusive reaction' or, in other words, as the kinetic energy transmitted from the initial channel to the final channel of the subprocess (3). From the total energy conservation we have

$$\Delta M = 2M - m_1 - (x_1 M + x_2 M + m_2) = T_f^x - (T_i - E_R) \equiv T_f^x - T_i^x, \tag{13}$$

where T_i, T_f^x and E_R are the initial kinetic energy, the kinetic energy in the final state of the subprocess (3) and the energy consumed on creation of the associate multiplicity, respectively. Inserting (12) into (8) we obtain the expression

$$E_q \frac{d\sigma}{d\vec{q}} = -\frac{1}{16\pi(dN(0)/d\eta)^2 M^2}\left(\frac{d\psi(z)}{dz}h_1(x_1, x_2) + \frac{\psi(z)}{z}h_2(x_1, x_2)\right), \tag{14}$$

where the functions h_1 and h_2 are proportional to the partial derivatives in (8). For the ISR energy region and beyond, we neglect the terms which die out with increasing energy and get the expressions (with 2% accuracy according to the exact calculations):

$$h_1 = 4\frac{\delta^2 - (x_1 - x_2)^2}{(\delta - x_1 - x_2)^4} \qquad h_2 = 4\frac{\delta^2 - (x_1 - x_2)^2 + 4x_1 x_2}{(\delta - x_1 - x_2)^4} \tag{15}$$

with $\delta \equiv 2 - (m_1 + m_2)M^{-1}$.

At low $\sqrt{s}$ (< 15 GeV) the spectra are described reasonably well by thermodynamic models in which the cross sections fall exponentially. At CERN ISR and Collider energies significant departure is seen from this behaviour and the inclusive spectra exhibit a clear power-law tail characteristic for the QCD processes. We have confronted our approach with data [9, 10, 11] on inclusive transverse-momentum spectra for charged particles produced in $pp/p\bar{p}$ collisions at high energies. As detected here hadrons are predominantly pions, protons and kaons, the mass m_1 in determination of $E_q = \sqrt{q^2 + m_1^2}$ was left as a free parameter. The fit in the soft part of the spectrum prefers the value m_1=0.31 GeV which is something like the mass of the lightest leading constituent quark. And so, in determination of the minimal mass m_2 we make a simple additional assumption that (anti)quarks are produced in pairs and fix it on the value m_2=0.31 GeV, too. From (6) and (15) we see, that for $\sqrt{s} > 15$ GeV we can neglect the difference between h_1 and h_2, allowing for the inaccuracy of 20-30% at the end of the spectra. Therefore, according to (14), (15) and (10), we obtain the relation

$$H(z) = \frac{(\delta - x_1 - x_2)^4 M^2 (dN(0)/d\eta)^2}{4(\delta^2 - (x_1 - x_2)^2)} \cdot E_q \frac{d\sigma}{d\vec{q}} \tag{16}$$

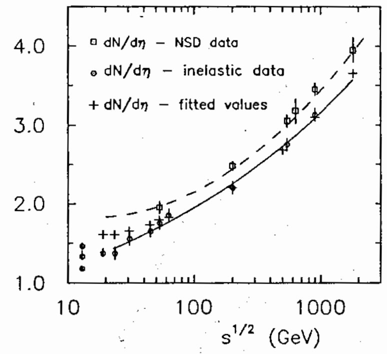
in the high energy region. The expression on the right-hand side of (16) was fitted to the scaling function

$$H(z) = \frac{A}{z^2(1+z)^\alpha} \tag{17}$$

for the data sets at $\sqrt{s}$=45, 53, 63, 200, 500 and 900 GeV. The magnitudes of $dN(0)/d\eta$, A and α were left as free parameters. The MINUIT program has been used for fitting with the result A=11.14 mb, $\alpha = 10.5$ and $\chi^2 = 265$ per 163 N_{DF}.

In Fig.2 we plot the values of the fitted parameters $dN(0)/d\eta$ (crosses) together with the relevant experimental data. Good agreement in the the energy dependence between the parameters and the data is seen with the exception of a small enhancement for $\sqrt{s} < 30$ GeV. At this energy the proposed scaling evidently sets on. The resultant distributions presented in the form (16) are shown in Fig.3.

Fig.2. $dN/d\eta$ at $\eta = 0$ as a function of $\sqrt{s}$. The full line is the fit $dN(0)/d\eta = 0.74 s^{0.105}$ to the inelastic data taken from [8]. The non single-diffractive data (NSD) and the parameterization $dN(0)/d\eta = 0.023 \ln^2(s) - 0.25 \ln(s) + 2.5$ are from [7]. The values of the free parameters $dN(0)/d\eta$ resulting from the requirement of the scaling (16) are shown with crosses.



Further, we have compared data [10, 12] on π^- meson production (here we have set m_1=0.14 GeV) for various angles at the energies $\sqrt{s}$=45, 53 and 63 GeV (see Fig.4). As we don't know the densities of π^- meson production in the central region, we have used the corresponding densities for the charged particles. The H(z) functions for π^- mesons have a scaling behaviour also, but by reason of this differ from H(z) in Fig.3.

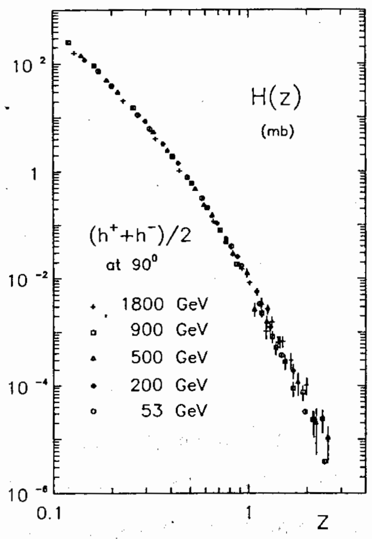
In conclusion, we have modified the kinematical construction proposed in [1] and introduced a new dynamical scaling hypothesis for inclusive particle production in $pp/p\bar{p}$ interactions at high energies. The invariant cross section for the production of the hadron m_1 in collision of composite objects can be written at sufficiently high energies as a convolution

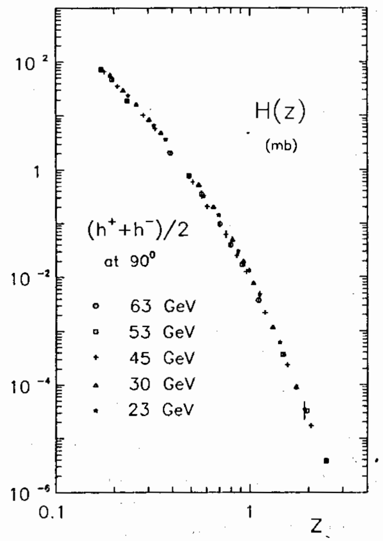
$$E_q \frac{d\sigma}{d\vec{q}} = P(x) \otimes \sigma_x \otimes D(y) \tag{18}$$

of three terms describing overlap of the initial structures, characteristics of the elementary subprocess and hadronization properties. According to intuitive arguments we write similarly

$$s_x^{1/2} = (dN(0)/d\eta) \cdot \Delta M \cdot z. \tag{19}$$

Particle density in the central region of the collision is determined by the initial





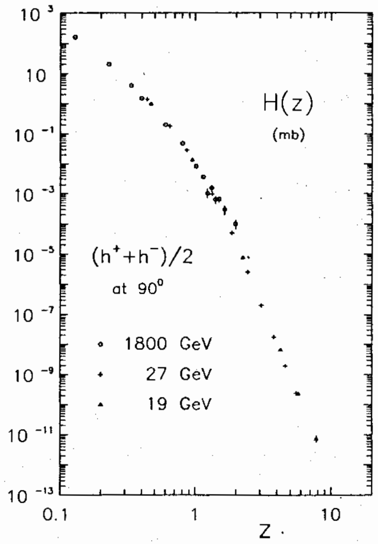
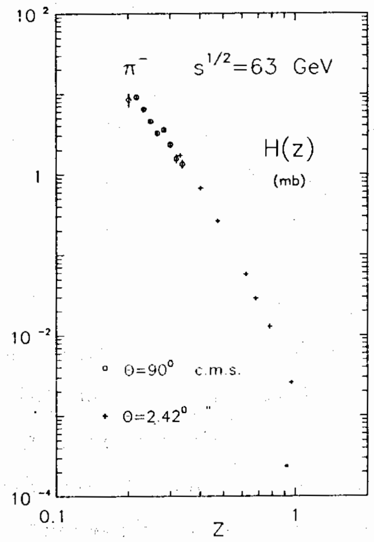
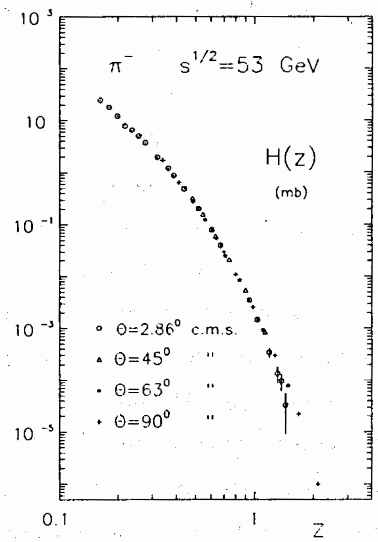


Fig.3. H(z) determined according to (16). Data on inclusive differential cross sections for charged hadrons produced in $pp/p\bar{p}$ interactions at 90° cms. are taken from [9, 10, 11].

energy density [7] in the overlap region and it is an analog of $P(x)$. The 'reaction energy of the inclusive reaction' ΔM reflects the fact of what amount of the energy was transmitted to the underlying subprocess and it depends on the characteristics of the latter one.





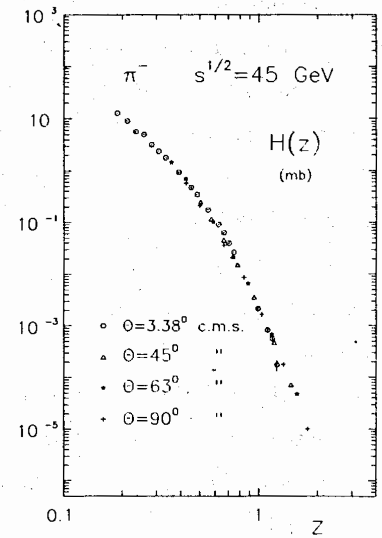


Fig.4. H(z) determined according to (16). Data on inclusive differential cross sections for π^- meson production (see comment in text) in pp interactions at $\sqrt{s}$=63, 53, and 45 GeV cms. energy are taken from [10, 12].

Apart from the fact that (18) is the probability and (19) is the energetic equation, the 'yield' of the inclusive particles is an analog of the amount of energy (expressed by its threshold) which the inclusive particle has obtained from the system. On the basis of this analogy the variable z can be interpreted as a hadronization parameter. The scaling function $H(z)$ reflects local properties of the hadronization.

To be more specific, let's interpret the variable z in terms of parton-parton collision with the subsequent formation of a string stretched by the leading quark out of which the inclusive particle is formed. The minimal energy of the colliding constituents $s_x^{1/2}$ is just the energy of the string which connects the two objects in the final state of the subprocess (3). The off-shell behaviour of the subprocess corresponds to a scenario in which the string has the maximal possible space-like virtuality.

According to general ideas, the string evolves further, splits into peaces decreasing so its virtuality. The resultant number of the string peaces is proportional to number/density of the final hadrons measured in experiment. Therefore, we interpret the ratio $s_x^{1/2}/(dN(0)/d\eta)$ as a quantity proportional to the energy of a string peace which doesn't split already but during the hadronization it converts into the observed hadron. The process of string splitting is self-similar in the sense that the leading peace of a string forgets the string history and its hadronization doesn't depend on the number and behaviour of other peaces. From this self-similarity the scaling properties of cross sections follow. The factor ΔM in the definition of z is proportional to the kinetic energy of the two objects in the final state of the subprocess (3) and it can be considered therefore as something which reflects the tension of the string. So, we can interpret the variable z as a quantity proportional to the length of the elementary string, or to the formation length, on which the inclusive hadron is formed from its QCD ancestor.

Finally, in this language, we reformulate the dynamical hypothesis introduced in our contribution for $pp/p\bar{p}$ interactions as follows: The production cross sections of a hadron depend at high energies dynamically and in a self-similar manner only on its formation length.

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References

[1] V.S. Stavinsky: JINR Rapid Communications 18-86, 5 (1986)

[2] A.M. Baldin et al.: JINR, E1-82-472, Dubna, 1982

[3] V.S. Stavinsky : Proc. of the IX Intern. Seminar on Problems of High Energy Physics (Dubna, 1988), Vol. 1, p.190

[4] A.A. Baldin: Phys. At. Nucl. 56 (3), March 1993, p. 385

[5] C.W. Akerlof et al.: Phys. Rev. D3 (1974) 645

[6] Z. Koba, H.B. Nielsen, and P. Olesen: Nucl. Phys. B40 (1972) 317

[7] F. Abe et al.: Phys. Rev. D41 (1990) 2330

[8] D.R. Ward: CERN-EP/87-178, Septemper 29th, 1987

[9] F. Abe et al.: Phys. Rev. Lett. 61 (1988) 1819; C. Albajar et al.: CERN-EP/89-85, July 12th, 1989

[10] B. Alper et al.: Nucl. Phys. B87 (1975) 19; B. Alper et al.: Nucl. Phys. B100 (1975) 237

[11] D. Antreasyan et al.: Phys. Rev. D19 (1979) 764

[12] M.G. Albrow et al.: Nucl. Phys. B56 (1973) 333

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Плотности частиц и инклюзивные распределения в $pp/p\bar{p}$ взаимодействиях при высоких энергиях

Рассматривается феноменологический подход для анализа масштабно-инвариантных свойств взаимодействия адронов/ядер при высоких энергиях. Анализируются экспериментальные данные по инклюзивным распределениям заряженных частиц в $pp/p\bar{p}$ столкновениях. Введена новая скейлинговая переменная z и установлен скейлинговый закон для описания этих процессов. Построена скейлинговая функция $H(z)$, описывающая локальные свойства процесса адронизации.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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Particle Densities and Inclusive Distributions in $pp/p\bar{p}$ Collisions at High Energies

The phenomenological approach for the analysis of the scale-invariant properties of hadron/nuclear interactions at high energies is considered. The experimental data on inclusive charge particle distributions in $pp/p\bar{p}$ collisions are analyzed. The new scaling variable z is proposed and the scaling law for description of these processes is found. The scaling function $H(z)$ reflecting local properties of hadronization process was constructed.

The investigation has been performed at the Laboratory of High Energies, JINR.

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