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DYNAMICAL PARITY VIOLATION IN THE TWO-DIMENSIONAL YUKAWA THEORY

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Динамическое нарушение четности
в двумерной модели Юкавы

Режим сильной связи и фазовая структура двумерной теории поля с взаимодействием Юкавы и самодействием бозонного поля исследуются с помощью метода канонических преобразований и формализма ренормгруппы. Построена фазовая диаграмма в плоскости (Y, G), где Y и G — константы взаимодействия Юкавы и самодействия, соответственно. Получены гамильтонианы, описывающие систему в каждой из фаз. Показано, что четность динамически нарушена и фермионы имеют ненулевую массу, если $G >> Y$. В случае чисто юкавского взаимодействия ($G \equiv 0$) имеются две различные фазы (симметричная и с нарушенной четностью) с очень близкими по величине плотностями свободной энергии. Однако при любых Y симметричной фазе соответствует меньшая энергия. Динамическое нарушение четности и генерация массы фермиона обусловлены только самодействием бозонного поля. Анализируется соотношение этих результатов, полученных в рамках перенормированной теории, и известных ранее результатов решеточных вычислений.

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Dynamical Parity Violation in the Two-Dimensional
Yukawa Theory

The strong coupling regime and phase structure of the two-dimensional field theory with the Yukawa coupling and boson self-interaction are investigated by means of the methods of canonical transformations and renormalization group. The phase diagram in the (Y, G)-plane is constructed, where Y and G are Yukawa and self-interaction coupling constants. The Hamiltonians describing the system in each phase are obtained. It is shown that the parity is dynamically broken and fermion is massive necessarily, if $G >> Y$. In the case of the pure Yukawa interaction ($G \equiv 0$), there are two different phases (symmetric and with broken symmetry) which have very close free energy densities. However, the symmetric phase is preferable for any Y since it has the lower energy. One can conclude that in the model under consideration the symmetry breaking and fermion mass are generated dynamically by the boson self-interaction. The relationship between these results obtained within the renormalized formalism and the results of lattice calculations is analyzed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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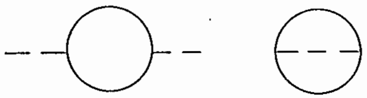

a b

Figure 1: Divergent diagrams.

identify a phase structure of the given QFT model with a set of nonequivalent CR representations realized in this model for different values of dynamical and external parameters.

This picture as well as the canonical formalism of QFT in general indicate the following **correct form** of the total Hamiltonian:

$$H = H_0 + H_I + H_{ct} + VE \; . \tag{1.1}$$

The standard free part H_0 describes a ground state of field system. The interaction Hamiltonian H_I does not contain terms linear or quadratic in fields and describes small corrections to H_0 if the coupling constants are small. The counter-term operator H_{ct} removes all UV divergencies. The form of H_{ct} is determined by H_0, H_I and renormalization scheme. The renormalization scheme should be fixed. The constant term E is the vacuum energy density.

The method we use is based on two ideas. First, the total Hamiltinian of field system should be written in the **correct form** in some particular representation which seems to be suitable for specific values of the dynamical and external parameters (e.g., in the weak coupling regime at zero temperature). Second, the **canonical transformations** of the field variables and the requirement that the Hamiltonian expressed in new variables has the **correct form** lead to equations defining unitary nonequivalent CR representations at any values of dynamical and external parameters. Each representation is characterized by the effective coupling constants and vacuum energy density. The system is considered to be in a definite phase if the effective coupling constants and vacuum energy density in the representation describing this phase are the smallest ones. The effective coupling constants are used to control an accuracy of approximation.

We will investigate the phase structure of the Yukawa model supplemented with the self-interaction of the pseudoscalar field φ

$$L(x) = \bar{\psi}(x) i\hat{\partial}\psi(x) + \frac{1}{2}\varphi(x)\left(\Box - m_B^2\right)\varphi(x) - \mathrm{y}\varphi(x)\bar{\psi}(x)i\gamma_5\psi(x) - \frac{g}{4}\varphi^4(x) \tag{1.2}$$

in space-time R^{1+1} ($x = (x_0, x_1)$). The fermion field ψ is massless. The parameters m_B^2, g and y are positive. The Dirac matrices are related to the Pauli matrices as

$$\gamma_0 = \sigma_3 \; , \quad \gamma_1 = i\sigma_2 \; , \quad \gamma_5 = \sigma_1 \; .$$

The Lagrangian is invariant under the parity (P) transformation

$$\varphi(x_0, x_1) \to -\varphi(x_0, -x_1), \quad \psi(x_0, x_1) \to \gamma_0\psi(x_0, -x_1).$$

Model (1.2) gives a simple but not trivial example for studying the dynamical P violation and generation of the fermion mass [11]. At the same time, there is a direct analogy with the models describing real systems in condensed state physics [12]. If the dimensionless coupling constants

$$G = \frac{g}{2\pi m_B^2} \quad \text{and} \quad Y = \frac{\mathrm{y}^2}{2\pi m_B^2} \tag{1.3}$$

are small enough, the Lagrangian (1.2) should describe in quantum theory the system symmetric under parity transformation. Is this statement really true and what happens in the strong coupling regime? We formulate the problem as follows:

what representation of CR is suitable for different values of G and Y and what physical picture corresponds to this representation?

The results of the present paper can be summarized as follows. We find the boson $M_B(G,Y)$ and fermion $M_F(G,Y)$ masses, effective coupling constants

$$G_{\text{eff}}(G,Y) = \frac{g}{2\pi M_B^2(G,Y)} \; , \quad Y_{\text{eff}}(G,Y) = \frac{\mathrm{y}^2}{2\pi M_B^2(G,Y)} \; , \tag{1.4}$$

order parameter and free energy density as functions of (G, Y) for different CR representations. The phase diagram in the (Y, G)-plane is constructed. The Hamiltonians describing system (1.2) in each phase are obtained. Two different symmetric phases and the phase with violated parity occur in the system. The parity breaking in the strong coupling regime $G \gg Y$ is conditioned by the boson self-interaction. This is in accordance with the vacuum structure of pure φ_2^4 theory [3, 8].

Another representation with the symmetry breaking coursed by the Yukawa coupling is not realized, since the vacuum energy in this representation is larger than the energy of symmetric phases for any Y, G. Therefore the Yukawa interaction does not lead to an instability of the symmetric phase. At the first glance this contradicts to the results of lattice calculations [2]. However, statement of the problem of the phase structure of a field system and investigation technique within the regularized (lattice) and renormalized (as in our case) formulations of quantum field theory are basically different. We analyze this point in the last section of the paper and show that the results of [2] and ours neither agree nor contradict to each other.

2 Hamiltonian and Renormalization

The quantized Hamiltonian corresponding to the Lagrangian (1.2) has the following form:

$$H = H_0 + H_{\rm I} + H_{\rm ct}\,, \tag{2.1}$$
$$H_0 = \int_V dx_1 \left\{ \frac{1}{2} : \left[\pi^2(x) + (\partial_1\varphi(x))^2 + m_B^2\varphi^2(x)\right] : + : \bar\psi(x) i\gamma_1\partial_1\psi(x) : \right\} .$$
$$H_{\rm I} = \int_V dx_1 \left\{ {\rm y}\varphi(x) : \bar\psi(x) i\gamma_5\psi(x) : + \frac{g}{4} : \varphi^4(x) : \right\} ,$$
$$H_{\rm ct} = \int_V dx_1 \left\{ \frac{1}{2}\delta m_B^2 : \varphi^2(x) : + \delta E \right\} .$$

Standard equal time canonical relations are postulated:

$$[\pi(x_0,x_1), \varphi(x_0,x_1')]_- = i\delta(x_1 - x_1')\,,$$
$$[i\psi^+(x_0,x_1), \psi(x_0,x_1')]_+ = i\delta(x_1 - x_1')\,. \tag{2.2}$$

The Hamiltonian (2.1) is constructed in such a way that CR (2.2) are represented in the Fock space of bosons with the renormalized mass m_B and massless fermions. The Hamiltonian is normally ordered with respect to the vacuum vector $|0\rangle$ of this Fock space.

The model under consideration is superrenormalizable. Boson mass and vacuum energy renormalization comes from the divergent diagrams given in Fig.1a and Fig.1b respectively. It is convenient to fix the renormalization scheme by the following prescription:

- **mass renormalization:** external momentum in the diagram in Fig.1a is on the mass shell ($p^2 = m_B^2$);
- **vacuum energy renormalization:** diagram in Fig.1b is subtracted completely.

Simple calculation gives the following result for the counter-terms:

$$\delta m_B^2 = {\rm y}^2\tilde\Pi^{\rm P}_{\rm reg}(m_B^2|0),\quad \delta E = \frac{{\rm y}^2}{8\pi}{\rm reg}\int_0^\infty \frac{du}{u+m_B^2}\tilde\Pi^{\rm P}_{\rm reg}(-u|0)\,,$$
$$\tilde\Pi^{\rm P}_{\rm reg}(p^2|0) = i{\rm reg}\int \frac{d^2q}{(2\pi)^2}{\rm Tr}\left\{ i\gamma_5\tilde S(q-p|0) i\gamma_5\tilde S(q|0)\right\}\,,$$

where $\tilde S$ is the fermion propagator

$$\tilde S(q|m_F) = 1/(m_F - \hat q - i\epsilon).$$

An appropriate regularization is implied in Eqs.(2.3). Now the S-matrix is defined, all terms of the perturbation series over $Y \ll 1$ and $G \ll 1$ are ultraviolet finite and can be calculated. The strong coupling regime $Y \gg 1$ and (or) $G \gg 1$ and self-consistency of this construction in the weak coupling regime will be investigated by means of the canonical transformation method.

3 Canonical transformation

Let us transform the canonical variables as

$$\{i\psi^+,\ \psi\} \longrightarrow \left\{ i\Psi^+ \exp\left(-i\frac{\alpha}{2}\gamma_5\right),\ \exp\left(i\frac{\alpha}{2}\gamma_5\right)\Psi \right\}\,,$$
$$\{\pi,\ \varphi\} \longrightarrow \{\Pi,\ \Phi + B\}\,. \tag{3.1}$$

Here Ψ is the fermion field with a new mass $M_F^2 = fm_B^2$, Φ is the boson field with the mass $M_B^2 = tm_B^2$, the angle α is a parameter of chiral transformations and B is a constant boson condensate. Such a transformation can be realized in terms of the creation and annihilation operators [10, 13]. Transformation (3.1) is the canonical one, i. e., the fields (Π, Φ) and ($i\Psi^+$, Ψ) obey the same canonical relations (2.2). New fields are defined on the Fock space with the vacuum state $|0\rangle\rangle$. This space is unitary nonequivalent to the initial one with the vacuum $|0\rangle$.

The Hamiltonian takes the following form in the new canonical variables

$$H = H_0' + H_{\rm I}' + H_{\rm ct}' + VE + \tilde H_1, \tag{3.2}$$
$$H_0' = \int_V dx_1 \left\{ \frac{1}{2} : \left[\Pi^2(x) + (\partial_1\Phi(x))^2 + M_B^2\Phi^2(x)\right] : \right.$$
$$\left. + : \bar\Psi(x)\left[i\gamma_1\partial_1 + M_F\right]\Psi(x) : \right\},$$
$$H_{\rm I}' = \int_V dx_1 \left\{ {\rm y}\Phi(x) : \bar\Psi(x)\left[\sin\alpha - i\gamma_5\cos\alpha\right]\Psi(x) : + \frac{g}{4} : \left[\Phi^4(x) + 4B\Phi^3(x)\right] : \right\},$$
$$H_{\rm ct}' = \int_V dx_1 \left\{ \frac{1}{2}\delta M_B^2 : \Phi^2(x) : + \delta E' \right\}.$$

Here the sign :: means normal ordering with respect to $|0\rangle\rangle$. The counter-terms $H_{\rm ct}'$ are determined by the new Hamiltonians H_0' and $H_{\rm I}'$ and correspond to the renormalization scheme which is equivalent to the initial one: the inner lines in the diagram Fig.1a correspond to the new fermion propagator and external momentum is on the mass shell $p^2 = M_B^2$, the vacuum diagram Fig.1b with the new propagators is subtracted completely. We get

$$\delta M_B^2 = {\rm y}^2\tilde\Pi^{\rm PS}_{\rm reg}(M_B^2|M_F)\,, \tag{3.3}$$
$$\delta E = \frac{{\rm y}^2}{8\pi}{\rm reg}\int_0^\infty \frac{du}{u+M_B^2}\tilde\Pi^{\rm PS}_{\rm reg}(-u|M_F)\,, \tag{3.4}$$

$$\tilde{\Pi}^{\rm PS}_{\rm reg}(p^2|M_F) = i{\rm reg}\int \frac{dq}{(2\pi)^2}{\rm Tr}\{(\sin\alpha - i\gamma_5\cos\alpha)\tilde{S}(q-p|M_F)$$
$$\times(\sin\alpha - i\gamma_5\cos\alpha)\tilde{S}(q|M_F)\}$$

The quantity E in Eq.(3.2) is the vacuum energy density and looks like

$$\begin{aligned}
E &= E_0 + E_{\rm I} + E_{\rm ct} \\
E_0 &= \frac{1}{2}m_B^2 B^2 + L(t) + \langle\langle 0|\bar{\Psi}(i\partial_1\gamma_1 + M_F)\Psi|0\rangle\rangle - \langle 0|\bar{\psi}i\partial_1\gamma_1\psi|0\rangle \,, \\
E_{\rm I} &= \frac{g}{4}\left[B^4 - 6B^2D(t) + 3D^2(t)\right] + {\rm y}B\sin\alpha\langle\langle 0|\bar{\Psi}\Psi|0\rangle\rangle \,, \qquad (3.5)\\
E_{\rm ct} &= \delta E - \delta E' - \frac{1}{2}\delta m_B^2 D(t) + \frac{1}{2}\delta m_B^2 B^2 \,,
\end{aligned}$$

$$D(t) = \int\frac{d^2k}{(2\pi)^2 i}\left[\frac{1}{m_B^2 - k^2 - i\varepsilon} - \frac{1}{M_B^2 - k^2 - i\varepsilon}\right] = \frac{1}{4\pi}\ln t \,. \qquad (3.6)$$

The function $L(t)$

$$L(t) = \frac{1}{2}\langle\langle 0|\Pi^2 + (\partial_1\Phi)^2 + m_B^2\Phi^2|0\rangle\rangle - \frac{1}{2}\langle 0|\pi^2 + (\partial_1\varphi)^2 + m_B^2\varphi^2|0\rangle$$
$$= \frac{m_B^2}{8\pi}(t - 1 - \ln t)$$

comes from the normal reordering of the free Hamiltonian. The last term $H_{\rm I}$ in Eq.(3.2) has the form

$$\begin{aligned}
H_{\rm I} = \int_V dx_1 \Big\{ &\frac{1}{2} :\Phi^2: \left[m_B^2 - M_B^2 - 3gD(t) + 3gB^2 + \delta m_B^2 - \delta M_B^2\right] \\
&+\Phi\left[m_B^2 B - 3gBD(t) + gB^3 + \delta m_B^2 B - {\rm y}\sin\alpha{\rm Tr}S(0|M_F)\right] \\
&+\left[{\rm y}B\sin\alpha - M_F\right] : \bar{\Psi}\Psi : - {\rm y}B\cos\alpha : \bar{\Psi}i\gamma_5\Psi : \Big\} \,.
\end{aligned}$$

To preserve the correct form (1.1) of the total Hamiltonian in the new representation we demand that $H_{\rm I} \equiv 0$. This requirement leads to equations for the parameters M_F, M_B, B and α of the canonical transformation:

$$\begin{aligned}
&{\rm y}B\sin\alpha - M_F = 0 \,, \\
&{\rm y}B\cos\alpha = 0 \,, \qquad (3.7)\\
&m_B^2 - M_B^2 - 3gD(t) + 3gB^2 + \delta m_B^2 - \delta M_B^2 = 0 \,, \\
&m_B^2 B - 3gBD(t) + gB^3 - {\rm y}\sin\alpha{\rm Tr}S(0|M_F) + \delta m_B^2 B = 0 \,.
\end{aligned}$$

Using Eqs.((2.3),(3.3)) and introducing the dimensionless quantities (1.3) and

$$f = \frac{M_F^2}{m_B^2}\,; \quad t = \frac{M_B^2}{m_B^2}\,, \quad b = \sqrt{\pi}B$$

one can rewrite Eqs.(3.7) in the form

$$\begin{aligned}
&\sqrt{2Y}b\sin\alpha - \sqrt{f} = 0 \,, \\
&\sqrt{2Y}b\cos\alpha = 0 \,, \qquad (3.8)\\
&1 - t - \frac{3}{2}G\ln t + Y\ln f + Y\left(1 - 4\frac{f}{t}\right)F\left(\frac{f}{t}\right) + 6Gb^2 = 0 \,. \\
&b\left[1 - \frac{3}{2}G\ln t + Y\ln f + 2Gb^2\right] = 0 \,.
\end{aligned}$$

where

$$F(z) = \int_0^1 \frac{dx}{x(1-x) - z} = \begin{cases} \frac{1}{\sqrt{1-4z}}\ln\left(\frac{1+\sqrt{1-4z}}{1-\sqrt{1-4z}}\right), & \text{if } z \le \frac{1}{4} \\ \frac{2}{\sqrt{4z-1}}{\rm arctg}\sqrt{4z-1}\,, & \text{if } z \ge \frac{1}{4}\,. \end{cases} \qquad (3.9)$$

Using these equations we can rewrite the energy density (3.5) as

$$E = \frac{m_B^2}{8\pi}\Big\{4b^2 + t - 1 - \ln t + 2f\ln f + G\left[4b^4 - 6b^2\ln t + \frac{3}{4}\ln^2 t\right] \qquad (3.10)$$
$$-\frac{1}{2}Y\ln^2 t + YJ(t/f)\Big\}\,,$$

$$J(s) = 2\int_0^1 \frac{dx(1-x^2)}{x((1-x)^2 + sx)}\left[\frac{x}{x-1}\ln x - \ln(1-x)\right] \,.$$

Equations (3.8) do not minimize the energy density (3.10) in the variables t, f, b. These equations do not relate to any variational principle. They follow from the demand of the correct form of the total Hamiltonian. This demand, combined with the canonical transformations, provides a regular prescription for dealing with the highest ultraviolet divergencies (like the diagrams in Fig.1). At the same time, the results of our and variational methods coincide in the case $Y = 0$ (the pure φ_2^4 theory) when the variational approach is well-defined [3, 8].

4 Phase structure

Different solutions of Eqs.(3.8) define the nonequivalent representations of the CR or different phases of the model (1.2). The proper Hamiltonians in these phases are given by Eqs.(3.2). It is convenient to formulate the following definitions. Let us suppose that Eqs.(3.8) have N different solutions, which can be denoted as

$$S_j(Y,G) = \{t_j(Y,G), f_j(Y,G), b_j(Y,G), \alpha_j(Y,G)\} \quad (j = 1, \ldots, N)\,.$$

The effective coupling constants (1.4) and energy density (3.10) corresponding to the j-th solution are denoted by

$$Y_{\rm eff}^{(j)}(Y,G) = \frac{Y}{t_j(Y,G)}\ ,\quad G_{\rm eff}^{(j)}(Y,G) = \frac{G}{t_j(Y,G)}\ ,$$
$$E_j(Y,G) = E\left(t_j(Y,G), f_j(Y,G), b_j(Y,G), \alpha_j(Y,G), Y, G\right)\ .$$

We shall say that in the region $\Gamma_k \subset R_+^2 = \{(Y,G) : Y \geq 0, G \geq 0\}$ the Yukawa system (1.2) exists in the phase described by the solution $S_k(Y,G)$ if for $(Y,G) \in \Gamma_k$

$$\min_j E_j(Y,G) = E_k(Y,G)\ , \tag{4.1}$$

$$\min_j Y_{\rm eff}^{(j)}(Y,G) = Y_{\rm eff}^{(k)}(Y,G)\ ,\quad \min_j G_{\rm eff}^{(j)}(Y,G) = G_{\rm eff}^{(k)}(Y,G)\ . \tag{4.2}$$

The regions Γ_k cover all the space R_+^2, i.e., $\cup\Gamma_k = R_+^2$. It is quite possible that some solutions are not realized as actual phases of the system, since they do not minimize the effective coupling constants and energy density for any Y and G.

Usually, criterion (4.1) based on comparison of the vacuum energy densities is used in the phase transition theory. Meanwhile, in QFT the demand of the weak effective coupling (4.2) seems to be more suitable. From a physical viewpoint the quantity E does not play any role, since it does not contribute to the S-matrix elements. Besides that, the energy density can not be calculated exactly or at least with similar accuracy in different phases, so that comparison of the energies loses its meaning. At the same time, it is natural to suppose that large coupling constants in H mean that representation determined by H_0 does not describe real states and can not be considered as a suitable representation for the total Hamiltonian H. Nevertheless, our calculations show that both criteria give similar results [8]-[10].

4.1 The pure Yukawa interaction

First of all, let us study the case $G = 0$, i.e., the pure Yukawa model. *We will show that for any coupling constant Y the Yukawa interaction does not lead dynamical generation of the fermion mass and parity violation.*

For $G = 0$ equations (3.8) are reduced to the form

$$\begin{aligned}
&\sqrt{2Y} b \sin\alpha = \sqrt{f}\ ,\quad \sqrt{Y} b\cos\alpha = 0\ ,\\
&1 - t + \frac{Y}{2}\ln f + Y\left(1 - 4\frac{f}{t}\right) F(f/t) = 0\ ,\\
&b\left[1 + Y\ln f\right] = 0\ .
\end{aligned} \tag{4.3}$$

Energy density (3.10) looks in this case like

$$E = \frac{m_B^2}{8\pi}\left\{4b^2 + t - 1 - \ln t - \frac{Y}{2}\ln^2 t + 2f\ln f + YJ(t/f)\right\}\ . \tag{4.4}$$

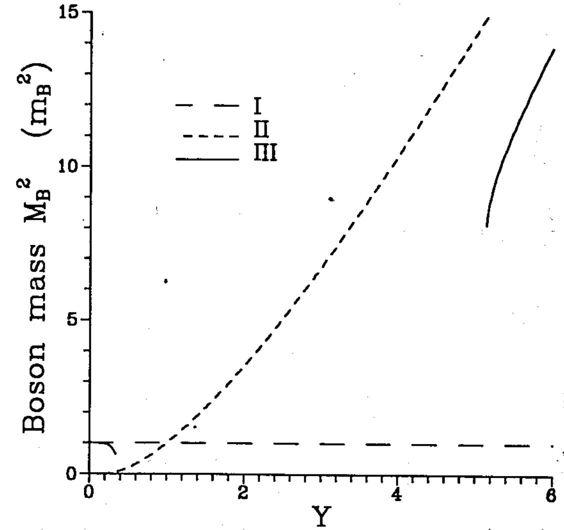

Figure 2: Boson mass for different phases of the pure Yukawa model.

Equations (4.3) has three different solutions.

I. $b_1 \equiv 0,\ t_1 \equiv 1,\ f_1 \equiv 0,\ \sin\alpha_1 = 0,\ Y_{eff}^{(1)} \equiv Y,\ E_1 \equiv 0.$
This is the initial representation (2.1).

II. $b_2 \equiv 0,\ t_2(Y),\ f_2 \equiv 0,\ \sin\alpha_2 = 0,\ Y_{eff}^{(2)}(Y),\ E_2(Y).$
Equation for the boson mass can be represented in the form:

$$\frac{t_2 - 1}{\ln t_2} = Y\ . \tag{4.5}$$

Using this equation one can represent energy density (4.4) as

$$E_2 = \frac{m_B^2}{8\pi}\left\{t_2 - 1 - \frac{1}{2}(t_2 + 1)\ln t_2\right\}\ . \tag{4.6}$$

The functions $t_2(Y)$ and $E_2(Y)$ are plotted in Figs.2,3 by the short-dashed lines. In the strong coupling regime $Y \gg 1$ we get from Eqs.(4.5) and (4.6)

$$\begin{aligned}
&t_2(Y) \to Y\ln Y\ ,\quad Y_{\rm eff}^{(2)}(Y) \to \frac{1}{\ln Y} \ll 1\ ,\\
&E_2(Y) \to -\frac{m_B^2}{8\pi}\frac{1}{2}Y\ln^2 Y\ .
\end{aligned} \tag{4.7}$$

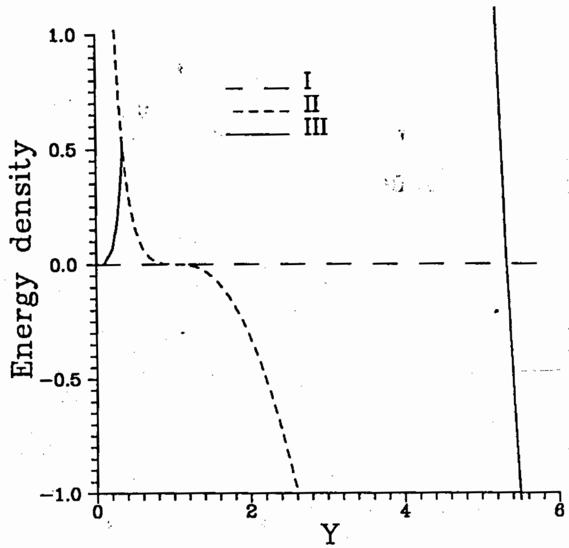

Figure 3: Energy density for different phases of the pure Yukawa model.

Neither Eq.(4.5) for boson mass t nor the energy density (4.6) depends on the angle α. We have a family of degenerate (in the masses and energy density) vacua enumerated by the angle α. Representations with $\sin\alpha \neq 0$ correspond to the symmetry broken by the interaction of the *pseudoscalar field* Φ with the *scalar* fermion current (see the interaction Hamiltonian H'_I (3.2)). Below we will consider for definiteness only the symmetric representation with $\sin\alpha = 0$.

III.$b_3 = \pm\frac{1}{\sqrt{2Y}}\exp\{-1/2Y\}$, $t_3(Y)$, $f_3 = \exp\{-1/Y\}$, $\sin\alpha_3 = \pm 1$, $Y^{(3)}_{\rm eff}(Y)$, $E_3(Y)$. The sign "$\pm$" corresponds to two degenerate vacua connected by parity transformation. In this case an equation for the boson mass has the form

$$t_3 - Y\left(1 - 4\frac{f_3}{t_3}\right) F\left(\frac{f_3(Y)}{t_3}\right) = 0\,. \qquad (4.8)$$

The energy density (4.4) takes the form:

$$E_3 = \frac{m_B^2}{8\pi}\left\{t_3 - 1 - \ln t_3 - \frac{Y}{2}\ln^2 t_3 + YJ(t_3/f_3)\right\} \qquad (4.9)$$

The function $t_3(Y)$ is plotted in Fig.2 by the solid line. One can see that the following relation takes place

$$t_3(Y) < \max\left[1,\ t_2(Y)\right], \quad \forall Y > 0. \qquad (4.10)$$

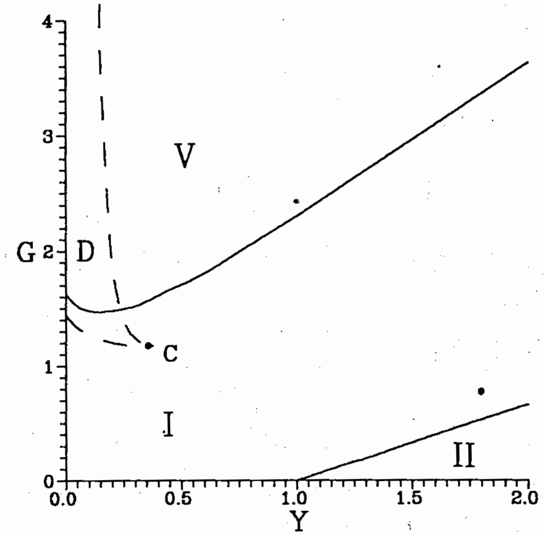

Figure 4: Phase diagram in the plane (Y, G). The dashed lines restrict the region D where Eqs.(4.20) have three solutions.

As a consequence of this relation, the effective coupling constant $Y^{(3)}_{\rm eff}$ is larger in the broken symmetry representation III than in the symmetric ones

$$Y^{(3)}_{\rm eff} > \min\left[Y,\ Y^{(2)}_{\rm eff}(Y)\right], \quad \forall Y > 0. \qquad (4.11)$$

The energy density $E_3(Y)$ (the solid line in Fig.3) is positive or larger than $E_2(Y)$ owing to inequality (4.10) and presence of the positive term $YJ(t/f)$ in Eq.(4.9)

$$E_3(Y) > \min\left[0, E_2(Y)\right], \quad \forall Y > 0\,. \qquad (4.12)$$

An asymptotic behavior of all functions in the weak ($Y \ll 1$) and strong ($Y \gg 1$) coupling regimes can be found from Eqs.((4.8),(4.9)).

For $Y \to 0$ we get:

$$t_3(Y) \to 1 - 2\exp\left\{-\frac{1}{Y}\right\}, \qquad (4.13)$$

$$Y^{(3)}_{\rm eff}(Y) = \frac{Y}{t_3(Y)} \to Y\left(1 + 2\exp\left\{-\frac{1}{Y}\right\}\right), \quad E_3(Y) \to \frac{m_B^2}{4\pi}\frac{1}{2Y}\exp\left\{-\frac{1}{Y}\right\}.$$

The asymptotic expression for the energy density originates from the term $YJ(t/f)$ in Eq.(4.9), i.e., it is conditioned by contribution of the diagram in Fig.1b. One can see that the energy density is non-analytic at $Y = 0$.

In the strong coupling regime $Y \gg 1$ one gets:

$$
\begin{aligned}
t_2(Y) - t_3(Y) &\to \frac{1}{Y \ln Y} > 0 \,, \\
Y_{\text{eff}}^{(2)}(Y) - Y_{\text{eff}}^{(3)}(Y) &\to -\frac{1}{\ln Y} < 0 \,, \\
E_2(Y) - E_3(Y) &\to -\frac{m_B^2}{8\pi} \ln Y < 0 \,.
\end{aligned}
\tag{4.14}
$$

Comparing the energy densities and effective coupling constants we get the following relations

$$
\min\left[Y, Y_{\text{eff}}^{(2)}(Y), Y_{\text{eff}}^{(3)}(Y)\right] = \begin{cases} Y & \text{if } Y \leq 1 \\ Y_{\text{eff}}^{(2)}(Y) & \text{if } Y \geq 1 \,, \end{cases} \tag{4.15}
$$

$$
\min\left[0, E_2(Y), E_3(Y)\right] = \begin{cases} 0 & \text{if } Y \leq 1 \\ E_2(Y) & \text{if } Y \geq 1 \,. \end{cases} \tag{4.16}
$$

Equations (4.15) and (4.16) show that according to both definitions (4.2) and (4.1) a kind of phase transition between the phases I and II occurs at $Y = 1$. The phase III with broken symmetry is not realized for any $Y > 0$.

Thus we conclude that parity is not violated dynamically in the two-dimensional Yukawa model. The fermion is massless for any values of the coupling constant Y. This conclusion differs from the results of the lattice calculations [2]. We discuss an origin of this difference in the last section of the paper.

The effects described in this subsection are determined by non-analyticity of the physical parameters of the system (like the masses and boson condensate) at $Y = 0$. Such a non-analytical behavior can be obtained neither in perturbation theory nor within the variational approach like the Gaussian effective potential which does not take into account a major contribution of the divergent diagrams given in Fig.1.

4.2 Yukawa model with boson self-interaction

The main effect of boson self-interaction is that the parity is dynamically violated and the fermion gets a nonzero mass in the strong coupling regime $G \gg Y$. This is illustrated by the phase diagram shown in Fig.4. The solid lines correspond to the phase boundaries. In the regime $G \gg Y$ the broken symmetry phase conditioned by the boson self-interaction exists, while for $Y \gg G$ the nontrivial symmetric phase caused by the Yukawa coupling is realized.

In the general case Eqs.(3.8) have five different solutions.

I. $b_1 \equiv 0,\ t_1 \equiv 1,\ f_1 \equiv 0,\ \sin\alpha_1 = 0,\ Y_{\text{eff}}^{(1)} \equiv Y,\ G_{\text{eff}}^{(1)} \equiv G,\ E_1 \equiv 0.$
This is the initial representation (2.1).

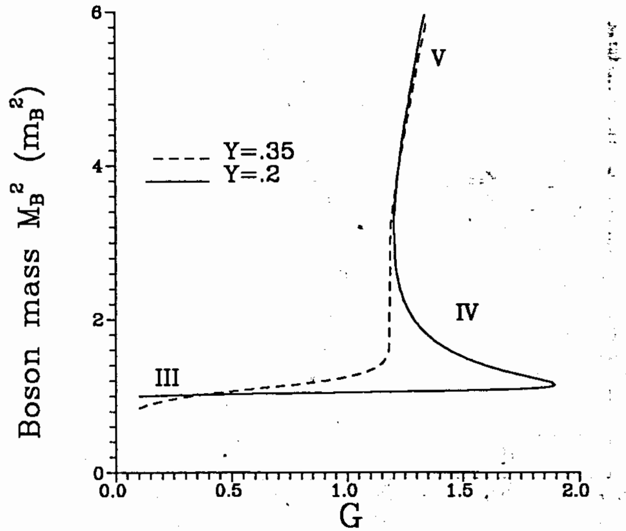

Figure 5: Boson mass for the phases III, IV and V with broken symmetry.

II. $b_2 \equiv 0,\ t_2(Y,G),\ f_2 \equiv 0,\ \sin\alpha_2 = 0,\ Y_{\text{eff}}^{(2)}(Y,G),\ G_{\text{eff}}^{(2)}(Y,G),\ E_2(Y,G)$
For $b = 0$ and $f = 0$ the equation for the boson mass can be written in the following form (see the third equation (3.8))

$$
\frac{t_2 - 1}{\ln t_2} = Y - \frac{3}{2} G \,. \tag{4.17}
$$

This equation has a unique solution for all Y and G obeying the condition

$$
Y - \frac{3}{2} G > 0
$$

and does not have solutions for other values of (Y, G). Using Eq.(4.17) in Eq.(3.10) one can reduce the energy density to the form

$$
E_2 = \frac{m_B^2}{8\pi} \left\{ t_2 - 1 - \frac{1}{2}(t_2 + 1) \ln t_2 \right\} , \tag{4.18}
$$

which coincides with Eq.(4.6). In the strong coupling regime $Y \gg G$ one finds

$$
\begin{aligned}
& t_2(Y,G) \to Y \ln Y \,, \\
& Y_{\text{eff}}^{(2)}(Y,G) \to \frac{1}{\ln Y} \ll 1 \,, \quad G_{\text{eff}}^{(2)}(Y,G) \to \frac{G}{Y \ln Y} \ll 1 \,, \\
& E_2(Y,G) \to -\frac{m_B^2}{8\pi} \frac{1}{2} Y \ln^2 Y \,.
\end{aligned}
\tag{4.19}
$$

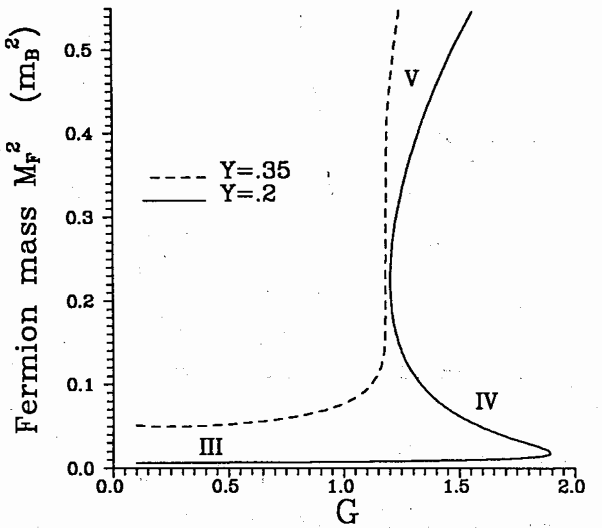

Figure 6: Fermion mass for the phases III, IV and V.

Using the formulas (4.18) and (1.4) we get the inequalities

$$E_2 \le 0, \quad G^{(2)}_{\rm eff} \le G, \quad Y^{(2)}_{\rm eff} \le Y \quad \text{for} \quad t_2 \ge 1.$$

Equation (4.17) indicates that $t_2 \ge 1$ if $Y - 3/2G \ge 1$. Thus, according to both criteria (4.2) and (4.1) the phase transition from the first symmetric phases I to the second symmetric phase II takes place on the curve $Y - 3/2G = 1$ shown in Fig.4 by the solid line starting at the point $(Y = 1, G = 0)$.

Solutions III, IV and V with nonzero boson condensate:

$b_j(Y,G) = \pm\sqrt{\frac{f_j(Y,G)}{Y}}$, $t_j(Y,G)$, $f_j(Y,G)$, $\sin\alpha_j = \pm 1$ $(j = 3,4,5)$.

The sign "$\pm$" corresponds to two degenerate vacua connected by the P-transformation. The P-symmetry breaking is provided by two reasons. These are the terms : Φ^3 : and Φ : $\bar\Psi\Psi$: in the interaction Hamiltonian H_I' (3.2). The energy density for the broken symmetry representations is defined in Eq.(3.10). For description of these solutions it is convenient to introduce the variable $s = f/t$, to subtract the last equation (3.8) from the third one and to rewrite Eqs.(3.8) in the form $(f = st)$

$$t\left(1 - 2\frac{G}{Y}s\right) = Y(1-4s)F(s) \,,$$

$$1 + (Y - \frac{3}{2}G)\ln t + Y\ln s + \frac{G}{Y}st = 0 \,. \qquad (4.20)$$

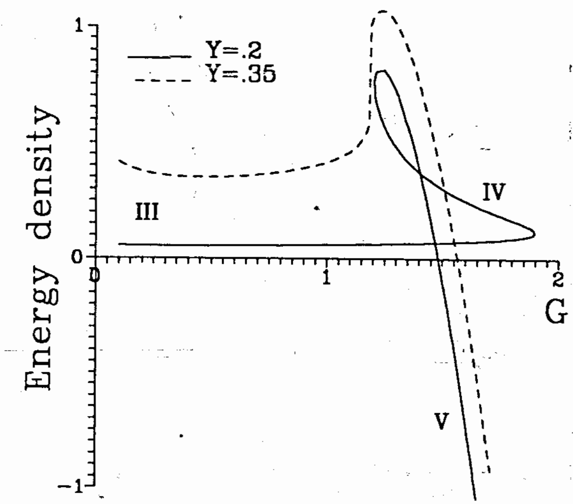

Figure 7: Energy density for the phases III, IV and V with broken symmetry.

The function $F(s)$ is defined by Eq.(3.9).

The analysis of the Eqs.(4.20) shows that there are two qualitatively different regions in the (Y,G) plane. There are three solutions inside the region D restricted by the G-axis and dashed lines in Fig.4, while outside this region only one solution exists. All solutions are equal to each other at the point C in Fig.4 which corresponds to $Y_C = .341...$ and $G_C = 1.12....$ Comparing the limit $G \to 0$ of Eqs.(4.20) and Eq.(4.8) for the pure Yukawa model we see that one of the three different solutions of Eqs.(4.20) is a continuation of the pure Yukawa solution III (see subsection 4.1) on the (Y,G) plane. This solution describes the Yukawa-type phase with broken symmetry. The existence of this phase is conditioned by the divergent diagrams (Fig.1) appearing due to the Yukawa coupling.

In the strong coupling regime $Y \gg G$, we get from Eqs.((4.20),(3.8)) the following asymptotic relations

$$t_2(Y,G) - t_3(Y,G) \to \frac{1}{Y\ln Y} > 0 \,,$$

$$Y^{(2)}_{\rm eff}(Y,G) - Y^{(3)}_{\rm eff}(Y,G) \to -\frac{1}{\ln Y} < 0 \,,$$

$$G^{(2)}_{\rm eff}(Y,G) - G^{(3)}_{\rm eff}(Y,G) \to -\frac{G}{Y\ln Y} < 0 \,,$$

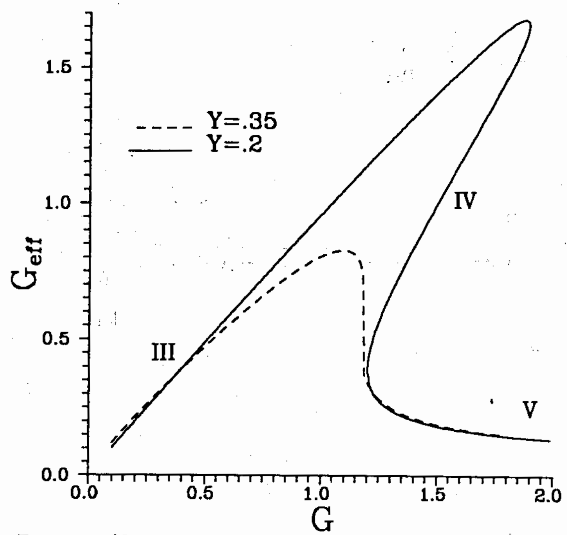

Figure 8: Boson self-coupling effective constant for the phases III, IV and V.

$$f_3(Y,G) \to \exp\left\{-\frac{1}{Y}\right\} \;,\quad b_3 \to \pm\frac{1}{\sqrt{2Y}} \;,$$

$$E_2(Y,G) - E_3(Y,G) \to -\frac{m_B^2}{8\pi}\ln Y < 0 \;,$$

which are exactly the same as Eqs.(4.14). The boson mass t_3 approaches t_2 from below. At the same time, the divergence between energy densities E_3 and E_2 grows due to the contribution of the term $YJ(f/t)$ in Eq.(3.8).

When $Y < Y_C$ and G grows two additional solutions of Eqs.(4.20) appear at the lower dashed line restricting the region D in Fig.4. This solutions are of the φ_2^4-type since they are a continuation of the pure φ_2^4 broken symmetry representations [8] on the (Y,G) plane. The φ_2^4-type phases originate from the divergences caused by the boson self-interaction (the bubble diagrams). On the upper dashed line in Fig.4 solutions III and IV terminate and above this line we have only the φ_2^4 type phase V with broken symmetry.

The following asymptotic solutions can be obtained from Eqs.(4.20) for $G \gg Y$

$$t_5(Y,G) \to 3G\ln G \;, \tag{4.21}$$

$$f_5(Y,G) \to \frac{3}{2}Y\ln G \;,\quad b_5 \to \pm\sqrt{\frac{3}{4}\ln G} \;,$$

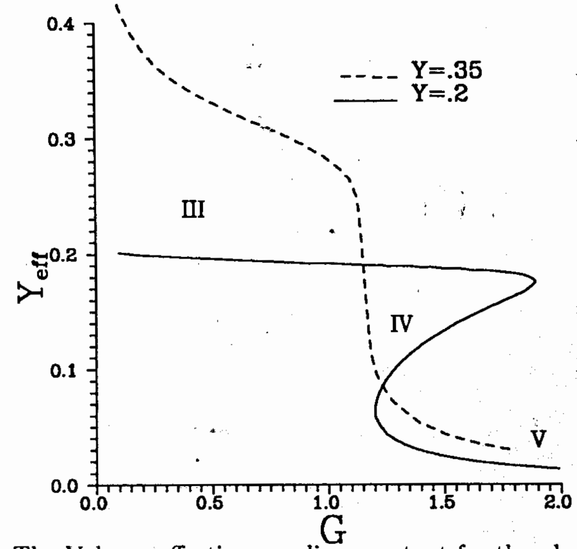

Figure 9: The Yukawa effective coupling constant for the phases III, IV and V.

$$E_5(Y,G) \to -\frac{m_B^2}{8\pi}\frac{3}{2}G\ln^2 G$$

The asymptotic behavior of the boson mass and energy density is the same as in the pure φ_2^4 theory [8].

The point C in Fig.4 is quite analogous to a critical point known in the classical thermodynamical systems like gas-liquid [14]. Different phases do not exist and the system is always homogeneous outside the region D. One can say that at the critical point (Y_C, G_C) the difference between phases disappears (solutions of Eqs.(4.20) are equal to each other at the critical point C). As soon as the critical point exists, a continuous transition between the phases III and V is possible, in which the separation into phases does not occur at any point. To do this, the change of coupling constants must take place along some curve in the (Y,G) plane nowhere cutting the lower dashed line in Fig.4. This curve may pass through the critical point C.

Boson and fermion masses as the functions of G for a fixed value of Y are shown in Figs.5,6 for two different paths in the (Y,G) plane. The solid line represents the case $Y < Y_C$: the path cuts the region D and we see the separation into the phases III, IV and V. The dashed line corresponds to $Y > Y_C$: the path does not cut the region D, the separation does not occur and a continuous transition from the Yukawa-type phase III to the φ_2^4-type phase V takes place. The difference between

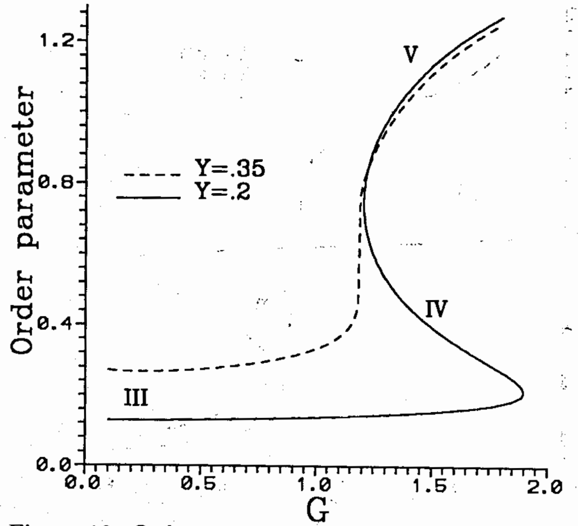

Figure 10: Order parameter for the phases III, IV and V.

these two phases is purely quantitative. Strictly speaking, one can speak of two phases only in the case when they exist at the same time touching each other, i.e., for points (Y, G) situated inside the region D.

In order to find the phase boundaries in the (Y, G) plane we have to compare the effective coupling constants and the energy densities of all the possible phases of the system. The energy densities and effective coupling constants for the phases with violated parity are shown in Figs.7-9. The solid lines correspond to $Y = .2 < Y_C$, the dashed lines represent the case $Y = .35 > Y_C$. Following the definition (4.2) we get the phase diagram given in Fig.4 by the solid lines. On the right hand side from the boundary starting at the point $(Y = 1, G = 0)$ the nontrivial symmetric phase II is realized, while the φ_2^4-type phase V with violated parity occurs above the line starting at the G-axis. The transition from the initial phase I to the phase V is of the first order since the order parameter (see Fig.10) has a jump at the boundary.

Asymptotic relations (4.19) and (4.21) shows that the description of the phases is quite accurate outside the critical regions, since the effective coupling constants are small and tend to zero when the coupling constant G or Y grows. At the same time our description of the phase boundaries and the region in Fig.4 where the phase I is realized is very approximate, the effective coupling constants are large enough as can be seen from Figs. 8, 9.

In any case, we can conclude that parity is violated and the fermion has a

dynamical mass in the strong coupling regime $G \gg Y$ owing to the self-interaction of the pseudoscalar field. The Yukawa coupling does not lead to dynamical generation of the fermion mass and parity violation but only courses the phase transition I→II without symmetry rearrangement.

5 Discussion

At the first glance above conclusion disagrees with the result of the lattice calculations [2] which claims that even very small Yukawa coupling generates a nonsero fermion mass. In this section we would like to clarify a relationship between results of the lattice approach and the present paper. The central point here is a basic difference between the renormalized and regularized (like the lattice QFT) formulations of the quantum field theory. In order to explain what we mean let us compare the main ideas of calculations in these two formulations of QFT for the simplest Yukawa model with the classical Lagrangian

$$L(x) = \bar{\psi}(x) i\hat{\partial}\psi(x) + \frac{1}{2}\varphi(x)\left(\Box - m^2\right)\varphi(x) + y\varphi(x)\bar{\psi}(x)\psi(x)$$

in the two-dimensional space-time. For this purpose it is more convenient to deal with the functional integral approach.

5.1 The Renormalized Formulation

First of all let us reformulate our method of description of the phase structure in application to the functional integral. We will do this quite schematically that is sufficient for above mentioned comparison. The vacuum amplitude for the model (5.1) can be written in the form

$$Z = \lim_{\Lambda\to\infty} \mathrm{reg}\, N \int \delta\psi\delta\bar{\psi}\delta\varphi \exp\left\{iA_R[\psi, \bar{\psi}, \varphi]\right\}, \tag{5.1}$$

where the action A_R corresponds to the renormalized Lagrangian

$$L_R(x) = \bar{\psi}(x) i\hat{\partial}\psi(x) + \frac{1}{2}\varphi(x)\left(\Box - m^2\right)\varphi(x) + y\varphi(x)\bar{\psi}(x)\psi(x) - \frac{1}{2}\delta m^2\varphi^2(x) - \delta E.$$

in which the mass and vacuum energy counter-terms corresponding to the divergent diagrams in Fig.1 are incorporated. The quantity m is the renormalized mass of the scalar field φ within the on-shell renormalization scheme. Definition (5.1) of the functional integral implies some appropriate ultraviolet regularization with a parameter Λ and definite rule for removing this regularization at the final stage of calculations. These two points are denoted in (5.1) by the sign $\lim_{\Lambda\to\infty}$ reg. Integrating out the fermion fields we can represent the vacuum amplitude Z as

$$Z = \lim_{\Lambda\to\infty} \mathrm{reg}\, N \int \delta\varphi \exp\left\{\mathrm{i}A_{\mathrm{R}}^{\mathrm{eff}}[\varphi]\right\},$$

$$A_{\mathrm{R}}^{\mathrm{eff}}[\varphi] = \int \mathrm{d}^2x \left[\frac{1}{2}\varphi(x)\left(\Box - m^2\right)\varphi(x) - \frac{1}{2}\delta m^2\varphi^2(x)\right] + \mathrm{Tr}\ln\left(i\hat{\partial} + \mathrm{y}\varphi\right). \quad (5.2)$$

Now we look for a constant field configuration $\varphi(x) = \phi_0$ =const which minimizes the action (5.2). We have to solve the equation

$$\frac{\mathrm{d}A_{\mathrm{R}}^{\mathrm{eff}}[\phi_0]}{\mathrm{d}\phi_0} = m^2\phi_0 + \delta m^2\phi_0 - \mathrm{y}\mathrm{Tr}\frac{1}{i\hat{\partial} + \mathrm{y}\phi_0} = 0. \quad (5.3)$$

This equation is divergence-free since the ultraviolet divergences in the last two terms eliminate each other. Formally equation (5.3) coincides with the last equation (3.7) for $g = 0$. Equation (5.3) has two solutions for all y:

$$\phi_0 = 0 \qquad \text{and} \qquad \phi_0^2 = \frac{m^2}{\mathrm{y}^2}\exp\left\{-\frac{2\pi m^2}{\mathrm{y}^2}\right\}. \quad (5.4)$$

Now we have to study which of the solutions (5.4) provides a minimum of the free energy density. In order to do this self-consistently we should change the integration variable in the functional integral (5.2) $\varphi \to \Phi + \phi_0$ and, at the same time, take into account the quantum corrections to the boson mass. In other words we have to find how the boson mass depends on the coupling constant y in the representations with $\phi_0 = 0$ and $\phi_0 \neq 0$. This dependence can be calculated approximately (taking into account leading corrections) by the use of demand of the correct form of the total Lagrangian. In the simple case under consideration this demand results in the equation

$$m^2 - M^2 + \delta m^2(m, \mathrm{y}) - \delta M^2(M^2, y, \phi_0) = 0, \quad (5.5)$$

where M is the renormalized boson mass and δM^2 is the counter-term corresponding to the diagram in Fig.1a in which the fermion propagators contain the mass term $\mathrm{y}\phi_0$ and the external momentum is subjected to the condition $p^2 = M^2$. Equation (5.5) is divergence-free and is equivalent to the third equation (3.7) for $g = 0$. The general form of the vacuum amplitude Z can be written as

$$Z = \exp\{-\mathrm{i}TV \cdot E(\mathrm{y})\} \lim_{\Lambda\to\infty} \mathrm{reg}\, N' \int \delta\Psi\delta\bar{\Psi}\delta\Phi \exp\left\{\mathrm{i}A_{\mathrm{R}}^{\mathrm{new}}[\Psi, \bar{\Psi}, \Phi]\right\},$$

$$L_{\mathrm{R}}^{\mathrm{new}}(x) = \bar{\Psi}(x)\left(i\hat{\partial} - M_{\mathrm{F}}\right)\Psi(x) + \frac{1}{2}\Phi(x)\left(\Box - M^2\right)\Phi(x) + \mathrm{y}\Phi(x)\bar{\Psi}(x)\Psi(x)$$

$$-\frac{1}{2}\delta M^2\Phi^2(x) - \mathrm{y}\Phi\mathrm{Tr}\frac{1}{i\hat{\partial} - M_{\mathrm{F}}} - \delta E'. \quad (5.6)$$

In this representation the fermion mass M_F is

$$M_{\mathrm{F}}^2 = \mathrm{y}^2\phi_0^2 = \begin{cases} 0 & \text{for } \phi_0 = 0, \\ m^2\exp\left\{-\frac{2\pi m^2}{\mathrm{y}^2}\right\} & \text{for } \phi_0 \neq 0. \end{cases} \quad (5.7)$$

The free energy density $E(\mathrm{y})$ in Eq.(5.6) is ultraviolet finite, can be computed and looks like the energy density in the subsection 4.1. Different solutions of the coupled system of equations (5.3) and (5.5) give physically different representations for Z and describe possible phases of the system. Comparing the energy densities $E(\mathrm{y})$ corresponding to the solutions of (5.3) and (5.5) we choose the phase which has minimal energy and, hence, is realized for given value of y. Even for $\phi_0 = 0$ equation (5.5) can have several solutions. In the case under consideration two such solutions exist for all y and they have the lower energy than the phase with $\phi_0 \neq 0$ (like solutions I and II in sect. 4.1).

Thus, solving equation (5.5) for $\phi_0 = 0$ and $\phi_0 \neq 0$ and comparing the free energy densities for different solutions we see that the phase with massive fermion has larger free energy than the symmetric phases and, hence, is not realized for all y. the phase with massive fermion is not realized in the system.

The following should be stressed here.

- We have two coupled equations (5.3) and (5.5) describing different phases of the system. This equations take into account leading quantum contributions both to the fermion and boson masses.

- The phase structure of the system is described in terms of the renormalized (physical) parameters. In partiqular, the fermion mass (5.7) is expressed through renormalized mass of the scalar field.

- The fermion is massles for any y.

Now let us consider the regularized formulation.

5.2 The Regularized Formulation

The vacuum amplitude for the model (5.1) in the regularized formalizm can be represented in the form

$$Z = \mathrm{reg}\, N \int \delta\psi\delta\bar{\psi}\delta\varphi \exp\left\{\mathrm{i}A[\psi, \bar{\psi}, \varphi]\right\}, \quad (5.8)$$

where the action A corresponds to the Lagrangian

$$L(x) = \bar{\psi}(x)i\hat{\partial}\psi(x) + \frac{1}{2}\varphi(x)\left(\Box - m_0^2\right)\varphi(x) + \mathrm{y}\varphi(x)\bar{\psi}(x)\psi(x)$$

with the bare boson mass m_0. Some regularization is implied in (5.8) but the rule for its removing is not defined. For example, the lattice approximation of the integral can be used, that is equivalent, roughly speaking, to cutoff of the integrals in the momentum space. An integration over the fermion fields leads to the expression

$$Z = \text{reg } N \int \delta\varphi \exp\left\{iA^{\text{eff}}[\varphi]\right\},$$
$$A^{\text{eff}}[\varphi] = \int d^2x \left[\frac{1}{2}\varphi(x)\left(\Box - m_0^2\right)\varphi(x)\right] + \text{Tr}\ln\left(i\hat{\partial} + y\varphi\right). \tag{5.9}$$

Looking for a constant field $\varphi(x) = \phi_0$ =const which minimizes the action (5.9) one has to solve the equation

$$\frac{dA^{\text{eff}}[\phi_0]}{d\phi_0} = m_0^2\phi_0 - y\text{Tr}\frac{1}{i\hat{\partial} + y\phi_0} = 0. \tag{5.10}$$

It is easy to check that Eq.(5.10) has two solutions for all y: $\phi_0 = 0$ and

$$\phi_0^2 = \frac{\Lambda^2}{y^2}\exp\left\{-\frac{2\pi m_0^2}{y^2}\right\}.$$

Here we used the regularization by cutoff of the momentum integrals at the scale Λ. As the next step one shifts the field $\varphi \to \Phi + \phi_0$ and gets the fermion mass in the form

$$M_{0F}^2 = \Lambda^2 \exp\left\{-\frac{2\pi m_0^2}{y^2}\right\}, \tag{5.11}$$

which is analogous to corresponding expression written in the paper [2]. In the new representation the vacuum functional Z takes the form

$$Z = \text{reg } N' \exp\left\{-iTV \cdot E_0(y)\right\} \int \delta\Psi\delta\bar{\Psi}\delta\Phi \exp\left\{iA^{\text{new}}[\Psi, \bar{\Psi}, \Phi]\right\}, \tag{5.12}$$
$$L^{\text{new}}(x) = \bar{\Psi}(x)\left(i\hat{\partial} - M_{0F}\right)\Psi(x) + \frac{1}{2}\Phi(x)\left(\Box - m_0^2\right)\Phi(x) + y\Phi(x)\bar{\Psi}(x)\Psi(x).$$

The vacuum energy density $E_0(y)$ in Eq.(5.12) is a bare quantity depending on the regularization parameter Λ. We see that

- only one equation (5.10) describes different phases of the system. This equation takes into account quantum contributions to the fermion mass but the boson mass m_0 is fixed;
- the phase structure of the system is described in terms of the bare mass m_0 of the scalar field and ultraviolet cutoff parameter Λ; in partiqular, the fermion mass (5.11) is expressed through the bare mass m_0 and parameter Λ;

- the fermion is massive for all y.

Comparing the content of the current and previous subsections (especially the concluding remarks) one could get a quite definite impression that comparison of the results obtained within the renormalized and regularized formulations of QFT is a subtle thing. In these two cases we have qualitatively different sets of possible phases. After all this is explaned by the basic difference in definitions (5.1) and (5.8) of the vacuum amplitude Z. Thus we can finish the paper by a conclusion that our results and the results of the lattice approach [2] neither agree nor contradict to each other.

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