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RELATIVISTIC QUARK MODEL AND NEW EXPERIMENTAL DATA ON π– AND ρ-MESON RESONANCES

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Релятивистская кварковая модель и новые экспериментальные данные по $\pi-$ и $\rho-$мезонным резонансам

Показано, что новые экспериментальные данные по легким резонансам свидетельствуют в пользу теоретического предсказания в возможности существования нового возбужденного состояния π-мезона с массой в интервале $M_{\pi} = 700 \div 800$ МэВ. Это предсказание было получено в рамках формализма ковариантных двухчастичных уравнений, использованного для построения релятивистской кварковой модели. Получены теоретические предсказания относительно орбитальных возбуждений $\pi-$ и ρ-мезонов.

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Ivanshin Yu.I., Skachkov N.B. E2-94-468
Relativistic Quark Model and New Experimental Data on $\pi-$ and $\rho-$Meson Resonances

It is shown that new experimental data do support the theoretical prediction on possible existence of a new $\pi-$ meson excited state with mass in the interval $M_{\pi} = 700 \div 800$ MeV. This prediction was obtained in the framework of a covariant formalism for two-particle equations used for constructing a relativistic quark model. New theoretical predictions on the position of the levels of possible orbital excitations of $\pi-$ and $\rho-$mesons are presented.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics and Laboratory of Particle Physics, JINR.

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1 Introduction

The problem of describing the mass spectrum of mesons composed of light quarks is tightly connected with the problem of relativistic systems composed of two and more particles. The apparatus of three-dimensional relativistic equations for a two-particle system that has been developed in the framework of quantum field theory [1] and admits the formulation in a three-dimensional relativistic configuration representation [2] has a clear geometric analogy with the apparatus of the three-dimensional Schrödinger equation. On its basis, a relativistic version of the potential quark model [1] has been proposed [3, 4].

In ref. [7], the relativistic potential quark model has been used for a simultaneous description of the mass spectra of radial excitations of π- and ρ-mesons. In the same paper, the existence of the π-meson radial excitation with the mass in the range $M_\pi = 700 \div 800 MeV$ has been predicted.

During the last two years, the situation with experimental data on the spectra of π- and ρ-mesons has changed essentially. Thus, the last year, an experimental evidence for a possible existence of the resonance structure with π-meson quantum numbers and the mass $M_\pi = 749 \pm 30 MeV$ has appeared [8]. This year, an experimental work [9] has been published where the evidence for the resonance structure with the mass $M = 1290^{+20}_{-30}$ MeV and quantum numbers of ρ-meson has been presented.

In view of the appearance of these new data, the question arises of how do these data agree with theoretical models describing meson spectra. Here, we shall compare new experimental results with the predictions made on the basis of the relativistic quark model [7].

[1] For recent reviews of the nonrelativistic potential quark model applications see [5, 6].

2 The Relativistic Potential Quark Model

The formulation of three-dimensional field-theoretical two-particle equations [1] in the relativistic configuration representation [2] allows the application of the JWKB-method for deriving an approximate (in powers of $\hbar$ rather than in v^2/c^2) solution of these equations. A quantization condition for the case of a bound state of two quarks of equal masses m looks (in the case of zero orbital momentum $l = 0$) as follows [3]:

$$\frac{1}{\pi}\int_{r_-}^{r_+} dr\, \chi(r) = \lambda(n + 3/2). \qquad (1)$$

Here n is the principal quantum number that numerates radial excitations of π- and ρ-mesons, $\lambda = \hbar/mc$ is the Compton wave length of a quark and $r_\pm$ are the turning points.

The integrand $\chi(r)$ is

$$\chi(r) = Arch\, X(r) = ln[X(r) + \sqrt{X(r)+1}], \qquad (2)$$

where $X(r) = [Mc^2 - V(r)]/2mc^2$ (M is a mass of a bound state of two quarks) has the physical meaning of rapidity of a particle that moves in the field of the potential $V(r)$ defined in the relativistic configurational representation [2, 3].

The mass spectra of π- and ρ-meson radial excitations with the zero orbital moment ($l = 0$) has been calculated in [7] by formula (1) with the potential $V(r)$ chosen as a sum of two parts: $V(r) = V_{centr}(r) + (\vec{\sigma}_1\vec{\sigma}_2)V_{ss}(r)$, where $V_{centr}(r)$ is the central potential and $V_{ss}(r)$ describes the form of a spin–spin interaction (the contribution of tensor forces responsible for the transition between the states with different l was neglected). For π- and ρ-mesons those potentials had the following form: $V^{\pi}(r) = V^{\pi}_{centr}(r) - 3V_{ss}(r)$, $V^{\rho}(r) = V^{\rho}_{centr}(r) + V_{ss}(r)$. The central potential in the relativistic configurational representation was chosen as a sum of Coulomb and confining potentials:

$$V_{centr}(r) = -\frac{\alpha}{r} + \lambda r^{\beta}. \qquad (3)$$

The confining part was considered in two variants: either as a linear potential ($\beta = 1$) or as an oscilator one ($\beta = 2$).

It is to be mentioned that the Coulomb potential given in the relativistic configurational representation has a "Fourier transform" in the momentum space with asymptotics [4]:

$$V_{Coul}(Q^2)_{Q^2\to\infty} = -\frac{8\pi\alpha}{Q^2 ln(Q^2/m^2)}. \qquad (4)$$

This form reproduces the "asymptotically free" behaviour of the one-gluon exchange amplitude including the running coupling constant $\alpha_s(Q^2)$ of QCD.

3 The Model Predictions and Experiment

Formula (1) has been used in ref. [7] to fit experimental data on the excited states of π- and ρ-mesons known at the moment of publication. It is to be remind that at that time two resonances with the quantum numbers of π-meson and masses $M_{\pi(2)} = 1240 MeV$ and $M_{\pi(3)} = 1770 MeV$ were

found [10] and three excited states of ρ-meson with the masses $M_{\rho(2)} = 1250\ MeV$, $M_{\rho(3)} = 1600\ MeV$ and $M_{\rho(4)} = 2150\ MeV$ were known.

Calculation by (1) has shown that with the numeration of the levels shown above one cannot well describe the data with any meaningful values of parameters. Thus, it was natural to study the dependence of fit quality on the choice of the numeration of levels. It turned out that if the level $\rho(1250)$ was omitted, i.e. the following sequence for ρ-meson took place: $n = 1$: $\rho(770)$; $n = 2$: $\rho(1600)$; $n = 3$: $\rho(2150)$; and for π-meson: $n = 1$: $\pi(139)$; $n = 2$: $\pi(1240)$; $n = 3$: $\pi(1770)$, a satisfactory quality of the fit would be achieved.

Another possibility of the numeration studied in [7] suggests that the resonance $\rho(1250)$ exists but the state $\pi(1240)$ is not the first radial excitation (see Table 1).

Table 1

$l = 0$	$M^{(n)}_{\pi}(MeV)$	$M^{(n)}_{\rho}(MeV)$
$n = 1$	139.6	770
$n = 2$	?	1250
$n = 3$	1240	1600
$n = 4$	1770	2150

In this case, the position of the suggested pion's first radial excitation depends on the following assumptions used in fitting:

1.If the mass of the third excited level of π-meson is fixed at the value $M_{\pi(4)} = 1770\ MeV$, the fit gives 710 MeV for the first excitation of π-meson.

2.If the position of $M_{\pi(4)}$ is not fixed, the fit gives 760 MeV for $M_{\pi(2)}$ and 1700 MeV for $M_{\pi(4)}$. Nowadays, the resonance state $\pi(1770)$ exists [11] and has been independently confirmed [12]. That is why the first value (710 MeV) seems to be more preferable.

For further consideration it is important to emphasize once more that the prediction of the position of the first radial excitation of π -meson as $\pi(710)$ is a consequence of the physical supposition that the first radial excitation of ρ-meson is $\rho(1250)$.

As it has been mentioned in the Introduction, new experimental data on π- and ρ-meson excitation states have appeared since 1983. Thus, the data of ref. [8] are interpreted as the indication of existence of the resonance state with the π-meson quantum numbers found in the pion diffractive production on nuclei

$$\pi^- + A \longrightarrow \pi^+ + \pi^- + \pi^- + A.$$

The mass value of the found structure is $749 \pm 30\ MeV$, that agrees well with the theoretical prediction $M_{\pi(2)} = 710\ MeV$. Nevertheless, due to the fact that the resonance $\rho(1250)$ was omitted by the Particle Data Group during the last 10 years, this result, in our opinion, could not be interpreted alone as a support of the prediction made in [7] on the basis of the relativistic quark model.

The publication of the LASS spectrometer collaboration from SLAC [9] which appeared this year does present the clear evidence for the resonance $\rho(1300)$ with the mass $1290 \pm 30\ MeV$ and the width $120^{+60}_{-30}\ MeV$.

Despite the fact that the experimental data used in [7] have slightly changed from $\pi(1240 \pm 30)$ to $\pi(1300 \pm 100)$ and from $\rho(1600 \pm 60))$ to $\rho(1700 \pm 20)$, we conclude that the evidence for a new state $\rho(1290)$ can be treated as a strong argument in favor of the interpretation of the resonance $\pi(749)$ as a state predicted in ref. [7] on the basis of the relativistic formulation of the potential quark model.

4 Orbital Excitations of π- and ρ-mesons

Formula (1) should be changed in the case of a non-zero orbital momentum. The formula for $l \neq 0$ has been derived in [3]. Here we use the formula

$$\frac{1}{\pi} \int_{r_-}^{r_+} dr\ \chi(r) = \lambda(n + l/2 + 3/4), \quad n = 1, 2, 3... \qquad (5)$$

which is a relativistic analog of the "modified" quantization condition, widely used in quantum mechanics [13, 14]. Its main merit, despite some approximations done in its deriving (quite analogous to the approximations used in a nonrelativistic theory [2]), is the elimination of the dependence on the angular momentum l from the integrand, and its localization in a simple form in r.h.s. of (5).

Here, we shall use this simple form of the quantization condition (leaving application of a more sophisticated formula for further publications) because it makes the physical picture of orbital excitation levels very transparent. Really, comparing formulae (5) and (1), we find that in the r.h.s. of (5) the orbital momentum l appears in a combination $n + l/2$. To this end, the position of the levels with $l = 2$ can be estimated from Table 1 (with $l = 0$) by shifting the value of the principal quantum number n by -1. So, we get Table 2 that can be treated as a rather approximate indication of the position of orbital excitations. It is interesting to

[2]For details see [13, 14].

note that for part of the levels of Table 2 there already exist candidates for these states[3] in the tables of the Particle Data Group [11].

Table 2

$l=2$	$M_\pi^{theor}(MeV)$	$M_\rho^{theor}(MeV)$
$n=1$	710	1250
$n=2$	1240	1600 *
$n=3$	1770 *	2150 *

Thus, for the state of π-meson with $l=2$ and $n=3$ one can find, in these tables, an analog $\pi_2(1670)$ with the total moment $J=2$ ($J^{PC}=2^{-+}$), and to the states of the ρ-meson with $l=2$ and $n=2,3$ there correspond states $\rho_3(1690)$ and $\rho_3(2250)$ with $J=3$ ($J^{PC}=3^{--}$).

It is evident that if one would use in the fit a new value of the mass of the first radial excitation of ρ-meson, i.e $M_{\rho^{(2)}} = 1300\,MeV$ [11] instead of the one assumed as an input in the old fit [7] $M_{\rho^{(2)}} = 1250\,MeV$, it would lead to increasing values of theoretical predictions for radial excitations of the state with $l=2$ and thus to a better agreement with the experimental values.

5 Conclusions

So, from our point of view, the new data on $\pi(749)$ [8] and the evidence for $\rho(1290)$ [9][4] can be considered as a serious indication that the picture of the sequence of levels of π-meson radial excitations, proposed in [7] and having predicted the existence of $\pi^{(2)}(700 \div 800)$ can be correct. New

[3]They are denoted in Table 2 by *.

[4]See also section 5 of ref. [15], where a certain structure around $M_{4\pi} \sim 1300\,MeV$ (named as "data excess") is mentioned.

theoretical predictions for the position of orbital excitations of π- and ρ-mesons open wide possibilities for checking the theoretical scheme for π- and ρ-meson spectra proposed in [7]. In subsequent publications, this question will be considered in more detail and on the basis of new calculations.

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