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ENHANCED OSCILLATION OF NEUTRINOS OF DIFFERENT MASSES IN MATTER

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Усиление осцилляции нейтрино разных масс в веществе

В работе оценивается вероятность осцилляции нейтрино, имеющих разные массы, при их прохождении через вещество различных толщин, в том числе и для Солнца.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Beshtoev Kh.M. E2-94-46

Enhanced Oscillation of Neutrinos of Different Masses in Matter

The oscillation probability is estimated for neutrinos of different masses in their passing through matter of different thickness, including the Sun.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1994

In the previous paper [1], where possible types of neutrino oscillation were analysed, it was found that:

1) if neutrinos of different types have equal masses, real oscillations are possible for different types of neutrinos by analogy with K^0, $\overline{K}^0$ oscillation;

2) if neutrino masses are different for different neutrino types, only virtual neutrino oscillations are possible while real oscillations require participation of neutrinos in interactions for their transition to the respective mass shells by analogy with transition of a γ-quantum to the ρ^0-meson in the vector dominance model.

In continuation of ref. [1] we shall estimate the probability for neutrinos to change from one type ν_l to another $\nu_{l'}$ ($m_{\nu_l} \neq m_{\nu_{l'}}$) in passing through matter. Neutrino transition to the mass shell will occur via weak neutrino-matter interaction (by analogy with the $\gamma - \rho^0$ transition or K_1^0, K_2^0 oscillation). We shall assume that the difference in mass of ν_l, $\nu_{l'}$ neutrinos is small enough to consider the probability of $\nu_{l'}$ transition to the mass shell proportional to the total elastic cross section $\sigma^{el}(k)$ for weak interaction (for simplicity we shall deal with oscillation of two neutrinos). Then the length of elastic interaction of the neutrino in the matter of density ρ, charge Z, atomic number A and momentum k will be defined as

$$\lambda_0 \cong \frac{1}{\sigma^{el}(k)\,\rho(Z/A)}. \quad (1)$$

If the neutrino mass difference is fairly large, it can be taken into account by the methods of the vector dominance model [2]. As pointed out above, we shall assume that this difference is very small and employ formula (1).

The real part of forward scattering amplitude Re $f_i(k, 0)$ is responsible for elastic neutrino scattering in matter. It is related to the refraction coefficient by

$$k(n_i - 1) \cong \frac{2\pi N_e f_i(k, 0)}{k}, \quad i = \nu_e, \nu_\mu, \nu_\tau. \quad (2)$$

Keeping in mind that [3]

$$f_i(k, 0) \cong \sqrt{2}\, G_F\, k\, (M_W^2 / M_i^2) \quad (3)$$

$$\text{if } i = \nu_e, \; M_i^2 = M_W^2;$$

$$\text{if } i = \nu_\mu, \nu_\tau, \; M_i^2 = M_Z^2 0,$$

we obtain

$$k(n_i - 1) \cong \sqrt{2}\, G_F\, k\, (M_W^2 / M_i^2)\, N_e.$$

The phase of the elastic scattering amplitude changes by 2π over the length

$$\Lambda_0 \cong \frac{2\pi}{\sqrt{2}\, G_F \rho\, (Z/A)} = 2\pi \lambda_0. \quad (4)$$

(Absorption or the imaginary part of the forward scattering amplitude can be ignored for low-energy neutrinos.)

Knowing that the length of elastic neutrino-matter interaction is Λ_0, we must estimate the oscillation probability for the neutrino passing through the matter of thickness L. The probability of the elastic ν_l interaction in the matter of thickness L is

$$P(L) = 1 - \exp(-2\pi\, L/\Lambda_0). \quad (5)$$

Then, using formulae (4), (5), we can find the neutrino oscillation probability $P_{\nu_l \nu_{l'}}(L)$ at different thickness L. Averaging the expression for neutrino oscillation probability [4] over R

$$\Pi_{\nu_l \nu_{l'}}(R) = \frac{\sin^2 2\theta_{\nu_l \nu_{l'}}}{2}\,(1 - \cos(2\pi R/L_0)),$$

$$L_0 = 4\pi k/\Delta m^2, \quad (6)$$

we obtain

$$\overline{\Pi}_{\nu_l \nu_{l'}} = \frac{\sin^2 2\theta_{\nu_l \nu_{l'}}}{2}.$$

Then the oscillation probability $P_{\nu_l\nu_{l'}}(L)$ or the mixing angle β at $\Lambda_0 \geq L_0$ will be defined by the expressions (for simplicity, then, it is supposed $\Lambda_0 = \Lambda_e \cong \Lambda_\mu = \Lambda_\tau$):

a) for L comparable with Λ_0,

$$P_{\nu_l\nu_{l'}}(L) = \frac{\sin^2 2\beta}{2} \cong \overline{\Pi}_{\nu_l\nu_{l'}}, \quad \beta \cong \theta_{\nu_l\nu_{l'}}; \tag{7}$$

b) for very large L,

$$\frac{L}{\Lambda_0} > \frac{1}{\sin^2 2\theta_{\nu_l\nu_{l'}}} >> 1,$$

$$P_{\nu_l\nu_{l'}}(L) = \frac{\sin^2 2\beta}{2} \cong \frac{1}{2}, \quad \beta \cong \frac{\pi}{4}; \tag{8}$$

c) for intermediate L,

$$\frac{\sin^2 2\theta_{\nu_l\nu_{l'}}}{2} \leq P_{\nu_l\nu_{l'}}(L) \leq \frac{1}{2} \quad \text{or} \quad \theta \leq \beta \leq \frac{\pi}{4}. \tag{9}$$

If $L_0 \geq \Lambda_0$, the expressions like (7)—(9) will also hold true, but Λ_0 should be replaced by L_0 and the thickness of matter will be determined in units of L_0. Also, since the oscillation length L_0 increases with the neutrino momentum (see (6)), the number of oscillation lengths $n = L/L_0$ fitting in the given thickness L decreases with increasing neutrino momentum as, accordingly, the neutrino oscillation probability $P_{\nu_l\nu_{l'}}(L)$ does.

Let us consider in detail the neutrino oscillation probability for intermediate interaction numbers n. The distribution probability of n-fold elastic neutrino interaction for thickness L whith the mean value $\overline{n} = L/\Lambda_0$ at not very large $\overline{n}$ is determined by the Poisson distribution

$$f(n, \overline{n}) = \frac{(n)^n}{n!} \exp(-\overline{n}). \tag{10}$$

At large $\overline{n}$ it changes to the Gaussian distribution

$$f(n, \overline{n}, \overline{n}) = \frac{1}{\sqrt{2\pi \overline{n}}} \exp(-(n - \overline{n})^2/2\overline{n}). \tag{11}$$

The probability of neutrino conversion from ν_l to ν_l and $\nu_{l'}$ in n-fold elastic interaction is determined by recursion relations

$$\theta = \theta_{\nu_l\nu_{l'}},$$

$$P^{(n)}_{\nu_l\nu_l} = 1 - (1 - \exp(-b))\frac{\sin^2 2\theta}{2},$$

$$P^{(n)}_{\nu_l\nu_{l'}} = (1 - \exp(-b))\frac{\sin^2 2\theta}{2},$$

$$P^{(n)}_{\nu_l\nu_l} = P^{(n-1)}_{\nu_l\nu_l}\left(1 - (1 - \exp(-b))\frac{\sin^2 2\theta}{2}\right) + $$
$$+ P^{(n-1)}_{\nu_l\nu_{l'}}(1 - \exp(-b))\frac{\sin^2 2\theta}{2},$$

$$P^{(n)}_{\nu_l\nu_{l'}} = P^{(n-1)}_{\nu_l\nu_{l'}}\left(1 - (1 - \exp(-b))\frac{\sin^2 2\theta}{2}\right) + $$
$$+ P^{(n-1)}_{\nu_l\nu_l}(1 - \exp(-b))\frac{\sin^2 2\theta}{2}. \tag{12}$$

If we take $b = 2\pi$, the term e^{-b} in (11) can be ignored. For mean estimation we can make use of the fact that

$$\overline{N} = \int f(n, \overline{n}, \overline{n})dn = \overline{n}. \tag{13}$$

Then the mean probabilities of neutrino oscillation will be

$$P(\nu_l \to \nu_l) \cong P^{(\overline{n})}_{\nu_l\nu_l},$$

$$P(\nu_l \to \nu_{l'}) \cong P^{(\overline{n})}_{\nu_l\nu_{l'}}. \tag{14}$$

Now let us make estimations for solar neutrinos. The mean number of elastic interactions of electron neutrinos produced in the Sun is

$$\Lambda_{\text{Sun}} \cong 1.7 \cdot 10^7 \text{ m}, \quad \overline{n}_{\text{Sun}} \cong 40.$$

Keeping in (12) the terms of the first order in $\sin^2 2\theta$, we get

$$P(\nu_e \to \nu_e) \cong 1 - \frac{\overline{n}}{2}\sin^2 2\theta,$$

$$P(\nu_e \to \nu_\mu) \cong \frac{\bar{n}}{2}\sin^2 2\theta. \tag{15}$$

If we assume that the solar neutrino deficit results from neutrino oscillation and use the neutrino flux data of the GALLEX experiments [5], the standard model predictions [6] ($K = P_{exp}/P_{theor} \cong 0.7$) and formula (14), we get

$$\sin^2 2\theta = 1.5 \cdot 10^{-2}.$$

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