объединенный
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дубна

E2-94-459

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GRAVITATIONAL FIELD OF THE SCALAR-CHARGED MASS IN THE LOBACHEVSKI SPACE FAR FROM SOURCE

Report at the International School-Seminar «Multidimensional Gravity and Cosmology», Yaroslavl, 1994

1994

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Гравитационное поле скалярного заряда на фоне пространства Лобачевского на больших расстояниях от источника

В варианте теории Черникова с двумя связностями, в котором фоновая связность задается по Лобачевскому, приближенно решена задача о гравитационном поле локализованного источника статических сферически-симметричных скалярного и гравитационного полей. Приближение соответствует малому скалярному заряду источника $GG_s^2 << G^2m^2$, здесь G — константа тяготения Ньютона, G_s — скалярный заряд, m — масса системы, и большим расстояниям от источника $r >> Gm/c^2$. Тем самым оно является приближением к точному решению Черникова статической задачи о поле локализованного источника гравитационного поля на фоне пространства Лобачевского. Результат справедлив для варианта биметрической теории гравитации Розена, в котором фоновая пространственная метрика описывает пространство Лобачевского.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1994

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Gravitational Field of the Scalar-Charged Mass in the Lobachevski Space Far from Source

A variant of the Chernikov gravitational theory with two connections, in which the background connection describes the Lobachevski space, is treated. A localized source of static spherically symmetric gravitational and massless scalar fields is present. An approximate solution is given, when the scalar charge is supposed to be small, $GG_s^2 << G^2m^2$, and the distance from the source is larger than the gravitational radius, $r >> Gm/c^2$ (G is the Newton gravitational constant, G_s — the scalar charge, m — the mass of the system). The result is valid for the Rosen bimetric general relativity when the background space metric describes the Lobachevski space.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1994

Recently, Chernikov [1] has suggested a generalization of the Einstein theory, the theory with two connections but one metric. The main aim of this theory was to obtain a covariant generalization of the Einstein gravitational energy–momentum pseudotensor. For this aim it occurs necessary and sufficient to introduce a second connection. A more early bimetric approach by Rosen [2] for the same aim proved, in this way, to be sufficient. Both theories have the general relativity as a limiting case, and this fact is to be taken into account if one appeals to an experiment.

The following variant of the Chernikov theory is considered. First, when the gravitation is absent ($G = 0$), the field connection $\Gamma^{\lambda}_{\mu\nu}$ is put equal to the background connection $\hat{\Gamma}^{\lambda}_{\mu\nu}$. Second, under the same condition, the metric in the spherical space coordinates is

$$ds^2 = (cdt)^2 - dr^2 - (k \sinh \frac{r}{k})^2 d\Omega^2, \quad d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2, \quad (1)$$

so its spatial part describes the Lobachevski space (k is the Lobachevski constant). Third, the background connection chosen is christoffelian and defined by the metric (1). A consequence of the latter is the symmetry of the background connection Ricci tensor $\hat{R}_{\mu\nu} = \hat{R}_{\nu\mu}$ and field equations become

$$R_{\mu\nu} - \hat{R}_{\mu\nu} = \frac{8\pi G}{c^4}(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T), \quad T \equiv g^{\rho\sigma}T_{\rho\sigma}. \qquad (2)$$

$T_{\mu\nu}$ is the Einstein material tensor. Note that these equations coincide with equations of the Rosen bimetric general relativity with the background metric (1). For the static spherically symmetric space these equations have been solved by Chernikov [1] when a concentrated massive source of the gravitational field is present.

This exact external (i.e. $T^{(m)}_{\mu\nu} = 0;\ r \neq 0$) solution is of the form

$$ds^2 = V^2(r)(cdt)^2 - F^2(r)dr^2 - H^2(r)d\Omega^2, \tag{3}$$

$$V^2(r) = F^{-2}(r) = P^{-2}\frac{\sinh\frac{r-\hat{r}}{k}}{\sinh\frac{r+\hat{r}}{k}}, \quad P = \exp(-\frac{\hat{r}}{k}), \tag{4}$$

$$H(r) = Pk\sinh\frac{r+\hat{r}}{k}, \quad \frac{k}{2}\sinh\frac{2\hat{r}}{k} = \frac{Gm}{c^2}, \tag{5}$$

and,when $k \to \infty$ it turns into the Schwarzschied solution written in the spherical space coordinates that are conventionally related to the rectangular harmonic coordinates [3].

Now we consider an analogous static problem when a concentrated source of the gravitational and massless scalar fields is present. For the scalar field U we assume the simplest generalization of the Schrödinger–Klein–Gordon–Fock equation, which outside the source gives

$$\nabla^\sigma\nabla_\sigma U = -F^{-2}[U'' + (\ln\frac{H^2V}{F})'U'] = 0, \tag{6}$$

the prime denotes d/dr. The static solution of it is

$$U' = -G_S\frac{F}{H^2V}. \tag{7}$$

The scalar field material tensor is

$$T^{(SC)}_{\mu\nu} = -\frac{1}{4\pi}(\nabla_\mu U\nabla_\nu U - \frac{1}{2}g_{\mu\nu}\nabla_\sigma U\nabla^\sigma U). \tag{8}$$

By using (7) and (8), field equations (2) give

$$\left(\frac{H^2V'}{F}\right)' = 0, \tag{9}$$

$$\left(\frac{HH'V}{F}\right)' - FV\cosh\frac{2r}{k} = 0, \tag{10}$$

$$H'' - \frac{H}{k^2} - H'\frac{(FV)'}{FV} = -\frac{GG_S^2}{c^4}\frac{F^2}{V^2H^3}. \tag{11}$$

We adopt that the scalar constant is small, $G_S^2 \ll Gm^2$, and the distance from the source is sufficiently large, $r \gg Gm/c^2$. In this case, in the limit $k \to \infty$, we should have the approximate general relativity solution of the problem in harmonic coordinates, namely, [4]

$$H(r) = r + \frac{Gm}{c^2} - \frac{GG_S^2}{2c^4r} + ..., \ r \to \infty, \tag{12}$$

which is the first approximation to the Lanczos-Fock quantity $r + Gm/c^2$, and

$$FV = 1 + O(\frac{1}{r^3}), \tag{13}$$

$$\frac{F}{V} = 1 + \frac{2Gm}{c^2r} + ... \tag{14}$$

Since for $k \to \infty$, the constant $\hat{r}$ tends to Gm/c^2, equations (5) and (12) prompt to suppose a solution of the form

$$H(r) = Pk\sinh\frac{r + \hat{r} + A(k)/r + ...}{k}, \ \lim_{k\to\infty} A(k) = -\frac{GG_S^2}{2c^4}. \tag{15}$$

If so, the equation (11) gives

$$FV = 1 - \frac{2A}{r^2}\ln\cosh\frac{r + \hat{r} + A/r + ...}{k} + ..., r \to \infty, \tag{16}$$

and

$$(FV)' = \frac{2A}{kr^2}\tanh\frac{r + \hat{r} + A/r + ...}{k} + ..., \ r \to \infty. \tag{17}$$

It follows from the equation (9), that

$$\frac{H^2VV'}{FV} = N/2, \quad N = \text{const},$$

whence, using (15) and (16), we get

$$(V^2)' = N\frac{FV}{H^2} = N\frac{1 + \frac{2A}{r^2}\ln\cosh\frac{r+\hat{r}+A/r+...}{k} + ...}{P^2k^2\sinh^2 k^{-1}(r+\hat{r}+A/r+...)}. \tag{18}$$

It is possible at this step to omit the second term of the numerator and integrate

$$V^2 = M - \frac{N}{P^2k}\coth\frac{r+\hat{r}+A/r+...}{k} + ... \tag{19}$$

Comparing with the Einstein limit ($k \to \infty$) we find $M = 1$, $N = 2Gm/c^2$. However, we can determine these values more exactly after comparing with the Chernikov solution,

$$M = \frac{1}{P^2}\cosh\frac{2\hat{r}}{k}, \quad N = k\sinh\frac{2\hat{r}}{k}. \tag{20}$$

Finally, equation (15), (16), (19) and (20) give the answer to the question. In this approximation it is not possible to obtain a detailed expression for the function $A(k)$, but here it is sufficient to have its limiting value as $k \to \infty$, $A = -\frac{GG_S^2}{2c^4}$.

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Received by Publishing Department
on November 30, 1994.