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INDEPENDENT PRODUCTION AND POISSON DISTRIBUTION

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Распределение Пуассона и независимое рождение

Известное утверждение о факторизации инклюзивных сечений в случае независимого рождения частиц (или кластеров, струй и т.д.) и вытекающий из него вывод о пуассоновом распределении по их множественности никак не следуют из теории вероятностей. При аккуратном применении теоремы о произведении независимых вероятностей получаются совсем другие уравнения и не получается никаких следствий относительно распределений по множественности.

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Independent Production and Poisson Distribution

The well-known statement of factorization of inclusive cross-sections in case of independent production of particles (or clusters, jets etc.) and the conclusion of Poisson distribution over their multiplicity arising from it do not follow from the probability theory in any way. Using accurately the theorem of the product of independent probabilities, quite different equations are obtained and no consequences relative to multiplicity distributions are obtained.

The investigation has been performed at the Laboratory of High Energies, JINR.

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In the physics of multiple processes it is customary to assume that the multiplicity distribution of particles (as well as clans, clusters, jets and other objects of multiplicity production) must be the Poisson one if they are produced independently (e.g., [1–3]).

It is "proved" with the help of inclusive cross-section factorization: if the particles are independent, then, e.g., rapidity many-particle distribution can be presented as a product of one-particle distributions (e.g., [4,5]):

$$\rho(y_1, y_2, \ldots y_i) = \rho(y_1)\rho(y_2)\ldots\rho(y_i), \qquad \text{where:} \tag{1}$$

$$\rho(y_1) \equiv \frac{1}{\sigma_{in}}\frac{d\sigma}{dy_1}; \qquad \rho(y_1, y_2, \ldots y_i) \equiv \frac{1}{\sigma_{in}}\frac{d^i\sigma}{dy_1 dy_2 \ldots dy_i}. \tag{2}$$

And as the integrals of these quantities over the whole rapidity space are equal respectively to

$$\int \rho(y_1)dy_1 = <n>;$$

$$\int \rho(y_1, y_2, \ldots y_i)dy_1 dy_2 \ldots dy_i = <n(n-1)\ldots(n-i+1)>, \tag{3}$$

then integration of eqs.(1) results in simultaneous equations which are equivalent to the Poisson distribution (e.g., [6,7]):

$$<n(n-1)\ldots(n-i+1)> = <n>^i. \tag{4}$$

The result of integration over the whole interval of y_k variation (3), (4) will not change if instead of particle rapidity, by y_k in (2) is meant its momentum, velocity, exit angle or any other variable connected with it, e.g., a random number generated by a computer and attributed to this particle. But the results (3), (4), i.e., the Poisson distribution, will not be any longer obtained if, e.g., a constant factor is introduced into the right hand side of (1) although it will not affect factorizability.

The point is that the factorizability of just probabilities and not of any quantities follows from independence. The probability density of a composite event is equal to the product of elementary ones if they are independent. Eqs.(1) and hence the Poisson distribution (4) do not arise from particle independence as quantities (2) are not probability densities even if because the integrals of these quantities are not equal to 1.

If these quantities (2) are normalized, i.e., divided by their integrals (3) and then substituted into (1), the Poisson distribution (4) will disappear. Moreover, all the same there are no grounds to substite them into (1) as normalization does not yet make them probabilities.

Interpretation of quantities (2) as densities of probability to find at least one (or exactly one) of particles at given value of y_k deprives these quantities of additivity property with respect to y_k, inalienable property of probability density (e.g., [8]). For example, the probability of at least one of the particles to fall into the whole phase volume is not equal to the sum of probabilities to fall into each of its halves. That is, not only the results of integration (3) and hence the Poisson distribution (4) disappear but even the possibility of integration.

Quantities (2) are measured experimentally as ($\Delta y_k \to 0$):

$$\rho(y_1)\Delta y_1 = \frac{N_{particles\ in\ \Delta y_1}}{N_{events}} = \sum_{n_1} n_1 P_{n_1} = <n_1>_{\Delta y_1}, \qquad (5)$$

P_{n_1} is the probability of event (interaction) where n_1 particles fall into Δy_1. That is, $\rho(y_1)$ is the density of mean multiplicity at the point y_1. Certainly, when $\Delta y_1 \to 0$, the mean multiplicity is equal to the probability of at least one (or exactly one) of the particles to

fall into Δy_1: $\sum n_1 P_{n_1} \to \sum P_{n_1} \to P_1$ because then $P_2,\ P_3 \ldots \to 0$. However, unlike these probabilities, the density of mean multiplicity is additive with respect to y_1, and when integrating it over the whole rapidity interval, the mean multiplicity (3) is obtained. By the way, the probability of at least one of the particles to fall into the whole interval is equal to 1, and the probability of exactly one of the particles to fall there equals 0 (for $n > 1$).

In the same way:

$$\rho(y_1, \ldots y_i)\Delta y_1 \ldots \Delta y_i = \sum_{n_1,\ldots n_i} n_1 \ldots n_i P_{n_1 \ldots n_i} = <n_1 \ldots n_i>_{\Delta y_1 \ldots \Delta y_i}, \quad (6)$$

$P_{n_1 \ldots n_i}$ is the probability of an event where n_1 particles fall into Δy_1, ... and n_i particles into Δy_i. So, $\rho(y_1, \ldots y_i)$ is many-particle multiplicity density; $\rho(y_1, y_2)$ is the multiplicity density of pairs of particles when one of the particles of a pair is within Δy_1 and other one is within Δy_2; $\rho(y_1, y_2, y_3)$ is the density of threes and so on. The many-particle multiplicity density is also additive with respect to each y_k, and when integrating it over the whole phase volume, the mean many-particle multiplicity (3) is obtained: $<n_1 \ldots n_i> = <n(n-1)\ldots(n-i+1)>$, where n is the number of particles in an event. Each of n particles can be the first one in a pair, in a three etc., each of $(n-1)$ remaining particles can be the second one and so on to $(n-i+1)$.

So, eqs.(1)–(4) are obtained from the statement of independence of mean multiplicity densities at different points of the phase volume and not of probability densities independence.

One can simulate a set of events with wittingly independent particles and spectra independence of multiplicity giving a multiplicity distribution and simulating each particle in random manner over the same one-particle spectrum. Despite this, trivial reasons for correlations of mean multiplicity at different points of the phase volume will remain. For example ($i = 2$), selecting a sub-set of events with large multiplicity within Δy_1, we thereby select the events with large total multiplicity, and hence we increase multiplicity within Δy_2. On the other hand, selecting the events with large multiplicity within Δy_1 at fixed total multiplicity, we decrease multiplicity within Δy_2. So, this multiplicity correlation is negative at a very narrow total multiplicity distribution, and it is positive at a very wide one. In case of the multiplicity distribution with $D^2 = <n>$, i.e. $<n(n-1)> = <n>^2$, these contrary tendencies are precisely compensated according to (4).

Let us obtain accurately the consequences from independence of particles at first for semy-inclusive events containing exactly n particles which independence is being investigated. The *probability density* that one particle, randomly chosen from an event with n particles (e.g., by means of a random number choosing the particle number), has rapidity y_1 is equal to

$$\rho'_n(y_1) \equiv \frac{1}{n\sigma_n}\frac{d\sigma_n}{dy_1}; \qquad \int \rho'_n(y_1)dy_1 = 1. \tag{7}$$

The *probability density* that i random particles, successively chosen from an event with n particles ($n \geq i$), have respectively $y_1,\ y_2,\ \ldots y_i$ (each following particle is chosen from the lesser number of remaining ones) is written as [9]:

$$\rho'_n(y_1, y_2, \ldots y_i) \equiv \frac{1}{n(n-1)\ldots(n-i+1)\ \sigma_n}\ \frac{d^i\sigma_n}{dy_1 dy_2 \ldots dy_i};$$

$$\int \rho'_n(y_1, y_2, \ldots y_i)dy_1 dy_2 \ldots dy_i = 1. \tag{8}$$

If all particles are produced independently, i.e. if in a sub-ensemble of events, where the first randomly chosen particle has rapidity y_1, the second one y_2 and so on to y_{i-1}, the y distribution of the rest of particles is the same as in the total ensemble, then the compound probability density is equal to the product of elementary ones:

$$\rho'_n(y_1, y_2, \ldots y_i) = \rho'_n(y_1)\rho'_n(y_2)\ldots\rho'_n(y_i). \tag{9}$$

These equalities correspond to the well-known procedure of studying correlations when real events are compared with mixed ones consisting of particles randomly chosen from different events [10] (but with the same n).

From (9), i.e. assuming the independence of produced particles, using approximations of one-particle semi-inclusive spectra (7) only (and multiplicity distributions), a good description of experimental data on diverse inclusive "correlations" of π^- mesons in pp interactions is obtained: two-particle C, C', C_{sh}, R correlations, forward–backward correlations with different rapidity cuts, right–left correlations, multiplicity distributions within diverse rapidity intervals and intervals separated by empty gaps [11].

Eqs.(9) can be averaged over n (P_n is the probability of an event with multiplicity n):

$$\sum_{n=i}^{\infty} P_n\rho'_n(y_1, y_2, \ldots y_i) = \sum_{n=i}^{\infty} P_n\rho'_n(y_1)\rho'_n(y_2)\ldots\rho'_n(y_i). \tag{10}$$

For probabilistic reasons one can also obtain relations for "more inclusive" events, however for that one has to use the same semi-inclusive cross sections. The *probability density* that one particle, randomly chosen from a *random* event (but with $n \geq i$) has y_1, is equal to (averaging over n):

$$\hat{\rho}(y_1) \equiv \sum_{n=i}^{\infty} P'_n \rho'_n(y_1); \qquad \int \hat{\rho}(y_1) dy_1 = 1. \qquad (11)$$

Here P'_n is the multiplicity distribution for a sub-set of events with $n \geq i$. The *probability density* that i random particles, successively chosen from a *random* event, have respectively $y_1, y_2, \ldots y_i$, is equal to:

$$\hat{\rho}(y_1, \ldots y_i) \equiv \sum_{n=i}^{\infty} P'_n \rho'_n(y_1, \ldots y_i); \quad \int \hat{\rho}(y_1, \ldots y_i) dy_1 \ldots dy_i = 1. \qquad (12)$$

Using the theorem of the product of independent probability densities, we obtain:

$$\sum_{n=i}^{\infty} P'_n \rho'_n(y_1, \ldots y_i) = \sum_{n=i}^{\infty} P'_n \rho'_n(y_1) \ldots \sum_{n=i}^{\infty} P'_n \rho'_n(y_i). \qquad (13)$$

If $\rho'_n(y)$ did not depend on n, then these equalities would coincide with (10).

Certainly, equalities (10) and (13) can be slightly altered. For example, one can try to start summation in (11)–(13) from $n = 1$. Averaging in (10) can be performed with a weight, e.g. proportional to statistics at each multiplicity: $n(n-1)\ldots(n-i+1)$, then the left hand side of (10) will be equal to the left hand side of (1): $\rho(y_1, y_2 \ldots y_i)$. However, in any case these equations cannot be reduced to (1) or only constructed of inclusive spectra in any other way. Formulating accurately the statement of particle independence, one has to use semi-inclusive probability densities. Therefore no consequences relative to multiplicity distributions are obtained from the independence of particles.

However, the Poisson multiplicity distribution for independent particle production can be obtained in the following case. For events with fixed total multiplicity, the multiplicity distribution within a limited phase volume is a binomial one. If this volume is very small, i.e. if for randomly chosen particle the probability to fall into this volume is very small, then the distribution turns into the Poisson one. The distribution over the number of decays of a radioactive source over an interval of time is an accurate analogy to this case if the time interval is much smaller than the source lifetime.

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REFERENCES

1. T.T. Chou, C.N. Yang, Phys.Rev.Lett. 55 (1985) 1359; Phys.Lett.B 167 (1986) 453; 193 (1987) 531.
2. A. Giovannini, L. Van Hove, Z.Phys.C 30 (1986) 391; Acta Phys.Pol.B 19 (1988) 495; 917; 931.
3. A. Capella, J. Tran Thanh Van, Z.Phys.C 23 (1984) 165; A. Capella et al., Phys.Rev.D 32 (1985) 2933; 37 (1988) 1763.
4. K.G. Wilson, Cornell preprint CLNS-131 (1970).
5. C.E. DeTar, Phys.Rev.D 3 (1971) 128.
6. A.H. Mueller, Phys.Rev.D 4 (1971) 150.
7. A. Bassetto, M. Toller, L. Sertorio, Nucl.Phys.B 34 (1971) 1.
8. H. Cramer, Mathematical methods of statistics. Princeton (1954).
9. W. Ko, Phys.Rev.Lett. 28 (1972) 935.
10. G.I. Kopylov, Phys.Lett.B 50 (1974) 472.
11. A.I. Golokhvastov, preprint JINR E2-94-27. Dubna (1994).

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