объединенный
институт
ядерных
исследований
дубна

E2-94-404

A.V.Kotikov[1]

LONGITUDINAL STRUCTURE FUNCTION F_L AS FUNCTION OF F_2 AND $dF_2/dlnQ^2$ AT SMALL x

Submitted to «Письма в ЖЭТФ»

[1]e-mail: KOTIKOV@SUNSE.JINR.DUBNA.SU

1994

Котиков А.В. E2-94-404

Продольная структурная функция F_L как функция F_2 и $dF_2/dlnQ^2$ при малых значениях x

Представлено уравнение для выделения продольной структурной функции F_L глубоконеупругого рассеяния из данных для поперечной структурной функции F_2 и ее производной $dF_2/dlnQ^2$ в области малых значений x в ведущем порядке теории возмущений. Детальный анализ выполнен для данных группы $H1$. Значения F_L и отношения R найдены для $10^{-3} \le x \le 2 \cdot 10^{-2}$ и $Q^2 = 20$ ГэВ^2.

Работа выполнена в Лаборатории сверхвысоких энергий ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1994

Kotikov A.V. E2-94-404

Longitudinal Structure Function F_L as Function of F_2 and $dF_2/dlnQ^2$ at Small x

A formula is presented to extract the longitudinal deep inelastic structure function F_L from transverse structure function F_2 and its derivative $dF_2/dlnQ^2$ at small x in the leading order of perturbation theory. The detailed analysis is given for new data of $H1$ group from HERA. The values of F_L and DIS structure function ratio R are found at $10^{-3} \le x \le 2 \cdot 10^{-2}$ and $Q^2 = 20$ GeV^2.

The investigation has been performed at the Laboratory of Particle Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1994

For the experimental studies of hadron-hadron processes on new powerful **LHC** collider, it is necessary to know in detail the values of parton (quarks and gluon) distributions (PD) of nucleon, especially at small x. The basic information on quark structure of nucleon is extracted from the process of deep inelastic lepton-hadron scattering (DIS). Its differential cross-section has the following form:

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi\alpha_{em}^2}{xQ^4} \left[\left(1-y+y^2/2\right)F_2(x,Q^2) - \left(y^2/2\right)F_L(x,Q^2)\right],$$

where $F_2(x,Q^2)$ and $F_L(x,Q^2)$ are the transverse and longitudinal structure functions (SF), respectively. The longitudinal SF $F_L(x,Q^2)$ and the ratio

$$R(x,Q^2) = \frac{F_L(x,Q^2)}{F_2(x,Q^2) - F_L(x,Q^2)} \tag{1}$$

are a good QCD characteristics because they equal to zero in parton model. Moreover, the value of SF F_2, whose data is usually deduced from experiment, depends essentially on the corresponding values of F_L (or R). We note that the value of the SF F_L (or the ratio R) is very important also in the case for polarized SF which are taken from experimentally measured asymmetry of cross-sections of both: polarized leptons and nucleons.

The modern DIS experimental data (see [1] for review) are not reasonably accurate to determine $F_L(x,Q^2)$ (or $R(x,Q^2)$). Moreover, at small x the data for SF F_L are still absent.

In the present letter we are studying the behaviour of $F_L(x,Q^2)$ at small x using the new **H1** data and the method (see [2]) of replacement of Mellin convolution by ordinary products.

Let us introduce the standard parametrizations of singlet quark $s(x,Q^2)$ and gluon $g(x,Q^2)$ PD [2] (see, for example, [3])

$$\begin{aligned} s(x,Q^2) &= A_s x^{-\delta}(1-x)^{\nu_s}(1+\epsilon_s\sqrt{x}+\gamma_s x) \equiv x^{-\delta}\tilde{s}(x,Q^2) \\ g(x,Q^2) &= A_g x^{-\delta}(1-x)^{\nu_g}(1+\gamma_g x) \equiv x^{-\delta}\tilde{g}(x,Q^2), \end{aligned} \tag{2}$$

with Q^2 dependent parameters in the r.h.s.. We use the similar small-x behaviour for gluon and sea quarks PD that follows from the form of the kernel of Gribov- Lipatov-Altarelli-Parisi (GLAP) equation (see also recent fits of experimental data in [3]).

The "conventional" choice is $\delta = 0$. It leads to nonsingular behaviour of PD (see D_0' fit in [3]) when $x \to 0$. Another value $\delta \sim \frac{1}{2}$ has been obtained in papers [4] as the sum of leading powers of $\ln(1/x)$ in all the orders of perturbation theory (PT) (see also D_-' fit in ref.[3]). Recent **NMC** data [5] agree with small values of δ . This choice correspond to the present experimental data for **pp** and $\overline{\mathbf{p}}\mathbf{p}$ total cross-sections (see [6]) and the model of Landshoff and Nachtmann pomeron [7] with exchange of the pair of a nonperturbative gluons, yielding $\delta = 0.086$. However, the new **H1** data [8] from **HERA**, prefers $\delta \sim 0.5$. With help GLAP equation some attempts (see [9]) have been undertaken to obtain an agreement between the results of **NMC** at small Q^2 and **H1** group at large Q^2.

[2] We use PD multiplied by x and neglect the nonsinglet quark distribution at small x

1. Assuming the Regge-like behaviour for gluon and singlet quark PD (see eq.(2)), we get the following equation for the longitudinal SF F_L and for Q^2 derivative of the SF F_2[3]:

$$F_L(x,Q^2) = \delta_s x^{-\delta} \sum_{p=s,g} \Big(\overline{B}^p_{L,1+\delta}(\alpha)\, \tilde{p}(0,Q^2) + \overline{B}^p_{L,\delta}(\alpha)\, x\tilde{p}'(0,Q^2)\Big) + O(x^{2-\delta}),$$

$$\frac{dF_2(x,Q^2)}{dlnQ^2} = -\frac{\alpha(Q^2)}{2}\delta_s x^{-\delta} \sum_{p=s,g} \Big(\tilde{\gamma}^{1+\delta}_{sp}(\alpha)\, \tilde{p}(0,Q^2) + \tilde{\gamma}^{\delta}_{sp}(\alpha)\, x\tilde{p}'(0,Q^2)\Big) + O(x^{2-\delta}) \quad (3)$$

where $\overline{B}^p_{L,\eta}(\alpha)$ and $\tilde{\gamma}^{\eta}_{sp}(\alpha)$ are the longitudinal Wison coefficients and some combinations of the transverse Wilson coefficients and anomalous dimensions, respectively, of the η "moment" of Wilson operators (i.e., the corresponding variables expanded from integer values of argument to noninteger ones) and

$$\tilde{p}'(0,Q^2) \equiv \frac{d}{dx}\tilde{p}(x,Q^2) \text{ at } x=0$$

Here δ_s is the coefficient depending on the process and number of quarks f: $\delta_s = 5/18$ for **ep** collision and $f=4$.

Further we restrict our consideration to the leading order (LO) of perturbation theory (where $F_2(x,Q^2) \equiv \delta_s s(x,Q^2)$, $\overline{B}^p_{L,\eta}(\alpha)$ are the one-loop longitudinal Wilson coefficients $\alpha(Q^2)B^p_{L,\eta}$ and the $\tilde{\gamma}^{\eta}_{sp}(\alpha)$ are equal to the LO anomalous dimensions γ^{η}_{sp}) and case $\delta = 0.5$ corresponding to Lipatov pomeron that is supported by **H1** data. Both: including the case $\delta = 0$ corresponding to standard pomeron into our consideration and the expansion of this analysis to the next-to-leading order (NLO) of perturbation theory, will be done in future.

For the gluon parts from r.h.s of eq.(3) with the accuracy of $O(x^2)$ we have the following form:

$$B^g_{L,3/2}\tilde{g}(x/w_g,Q^2) \quad \text{with} \quad w_g = B^g_{L,3/2}/B^g_{L,3/2}$$
$$\gamma^{3/2}_{sg}\tilde{g}(x/\xi_{sg},Q^2) \quad \text{with} \quad \xi_{sg} = \gamma^{3/2}_{sg}/\gamma^{1/2}_{sg} \quad (4)$$

Hence, the eq.(3) may be represented in the form

$$F_L(x,Q^2) = \alpha(Q^2)\delta_s x^{-1/2} *$$
$$\Big(B^g_{L,3/2}\, \tilde{g}(x/w_g,Q^2) + B^s_{L,3/2}\, \tilde{s}(0,Q^2) + B^s_{L,1/2}\, x\tilde{s}'(0,Q^2)\Big) + O(x^{3/2}), \quad (5)$$
$$\frac{dF_2(x,Q^2)}{dlnQ^2} = -\frac{\alpha(Q^2)}{2}\delta_s x^{-1/2} *$$
$$\Big(\gamma^{3/2}_{sg}\, \tilde{g}(x/\xi_{sg},Q^2) + \gamma^{3/2}_{ss}\, \tilde{s}(0,Q^2) + \gamma^{1/2}_{ss}\, x\tilde{p}'(0,Q^2)\Big) + O(x^{3/2}) \quad (6)$$

Extracting the gluon distribution from eq.(6) and substituting it to eq.(5) we obtain the following equation

$$F_L(x,Q^2) = -2\, t_g^{1/2}\frac{dF_2(x/r_g,Q^2)}{dlnQ^2} +$$
$$\alpha(Q^2)\delta_s x^{-1/2}\Big(\tilde{B}^s_{L,3/2}\, \tilde{s}(0,Q^2) + \tilde{B}^s_{L,1/2}\, x\tilde{s}'(0,Q^2)\Big) + O(x^{3/2}), \quad (7)$$

[3]Hereafter contrary to the standard case we use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$.

where

$$t_g = \frac{B^g_{L,3/2}B^g_{L,1/2}}{\gamma^{3/2}_{sg}\gamma^{1/2}_{sg}}, \quad r_g = \frac{B^g_{L,1/2}\gamma^{3/2}_{sg}}{B^g_{L,3/2}\gamma^{1/2}_{sg}} \text{ and } \tilde{B}^s_{L,\eta} = B^s_{L,\eta} - B^g_{L,\eta}\frac{\gamma^{\eta}_{ss}}{\gamma^{\eta}_{sg}}$$

By analogy with eq.(4) for the quark part from r.h.s of eq.(7) with the accuracy of $O(x^2)$ we have the following form:

$$\tilde{B}^s_{L,3/2}\tilde{s}(x/r_s,Q^2) \text{ with } r_s = \tilde{B}^s_{L,3/2}/\tilde{B}^s_{L,3/2},$$

that leads to the following equation for the longitudinal SF

$$F_L(x,Q^2) = -2\, t_g^{1/2}\frac{dF_2(x/r_g,Q^2)}{dlnQ^2} + \alpha(Q^2)t_s^{1/2}F_2(x/r_s,Q^2) + O(x^{3/2}), \quad (8)$$

where $t_s = \tilde{B}^s_{L,1/2}\tilde{B}^s_{L,3/2}$

Thus, using the exact values of Wilson coefficients and anomalous dimensions, we get

$$F_L(x,Q^2) = \frac{24}{\sqrt{23*33}} *$$
$$\Big(\frac{dF_2(\frac{23}{33}x,Q^2)}{dlnQ^2} + \frac{8}{3}\alpha(Q^2)\sqrt{(\frac{13}{3} - 4ln2)(3-4ln2)}F_2(\frac{23}{33}\frac{3-4ln2}{\frac{13}{3}-4ln2}x,Q^2)\Big) + O(x^{3/2})$$
$$\approx 0.87\frac{dF_2(0.70x,Q^2)}{dlnQ^2} + 1.39\alpha(Q^2)F_2(0.10x,Q^2) + O(x^{3/2}) \quad (9)$$

Note that the arguments of the transverse SF and its derivative in r.h.s of eq.(9) are different. This is not convenient because the experimental data are known (see [8, 10]) for both variables in the similar range of x. To overcome this problem we note, that with the accuracy of $O(x^2)$ the quark part from r.h.s. of eq.(7) may be represented as a sum of two terms like eq.(4) with some coefficients and arguments shifts. Choosing the shifts as 1 and r_g^{-1} we have the following representation for the quark part:

$$c_1\, \tilde{s}(x,Q^2) + c_2\, \tilde{s}(x/\xi_{sg},Q^2),$$

where

$$c_1 = \frac{\tilde{B}^s_{L,3/2}\tilde{B}^g_{L,1/2} - \tilde{B}^s_{L,1/2}\tilde{B}^g_{L,3/2}}{\tilde{B}^g_{L,1/2} - \tilde{B}^g_{L,3/2}} \text{ and } c_2 = \tilde{B}^g_{L,3/2}\frac{\tilde{B}^s_{L,1/2} - \tilde{B}^s_{L,3/2}}{\tilde{B}^g_{L,1/2} - \tilde{B}^g_{L,3/2}} \quad (10)$$

with $\tilde{B}^g_{L,\eta} = B^g_{L,\eta}/\gamma^{\eta}_{sg}$

Thus, from eq.s (7)-(10) using the exact values of Wilson coefficients and anomalous dimensions, we get

$$F_L(x,Q^2) = \frac{24}{\sqrt{23*33}}\frac{dF_2(\frac{23}{33}x,Q^2)}{dlnQ^2} +$$
$$\frac{128}{15}\alpha(Q^2)\Big(\frac{6}{\sqrt{23*33}}(\frac{8}{5} - ln2)F_2(\frac{23}{33}x,Q^2) - F_2(x,Q^2)\Big) + O(x^{3/2})$$
$$\approx 0.87\frac{dF_2(0.70x,Q^2)}{dlnQ^2} + 10.29\alpha(Q^2)\Big(F_2(0.70x,Q^2) - 0.83F_2(x,Q^2)\Big) + O(x^{3/2}) \quad (11)$$

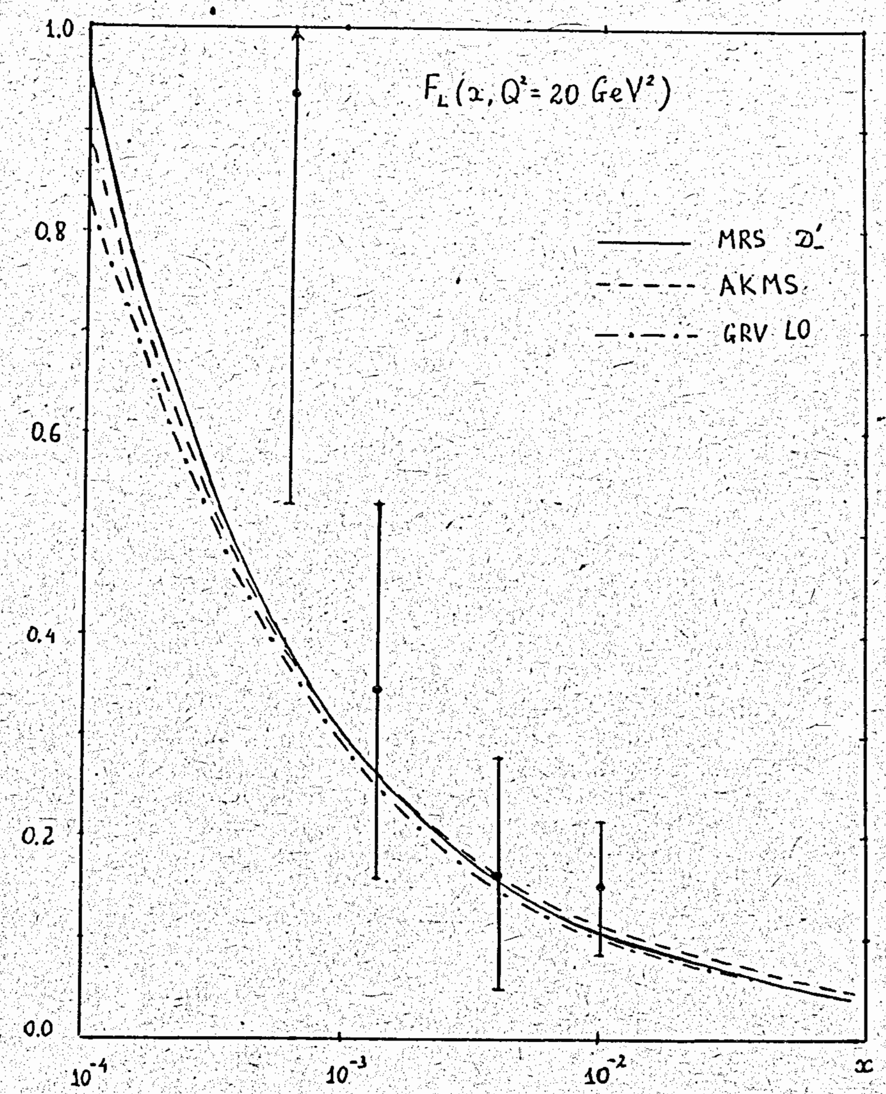

Figure 1. The longitudinal SF $F_L(x, Q^2)$ at $Q^2 = 20$ GeV^2. The black circles indicate the values extracted with the help of eq.(11). Only statistical errors are presented. The curves represent different parametrizations of $F_L(x, Q^2)$ [3, 11, 12]. The *GRV* curve is leading order parametrization, and the *MRS* parametrization is given in the *DIS* renormalization scheme. The *AKMS* curve is the solution of Lipatov equation and used at $Q^2 = 30$ GeV^2.

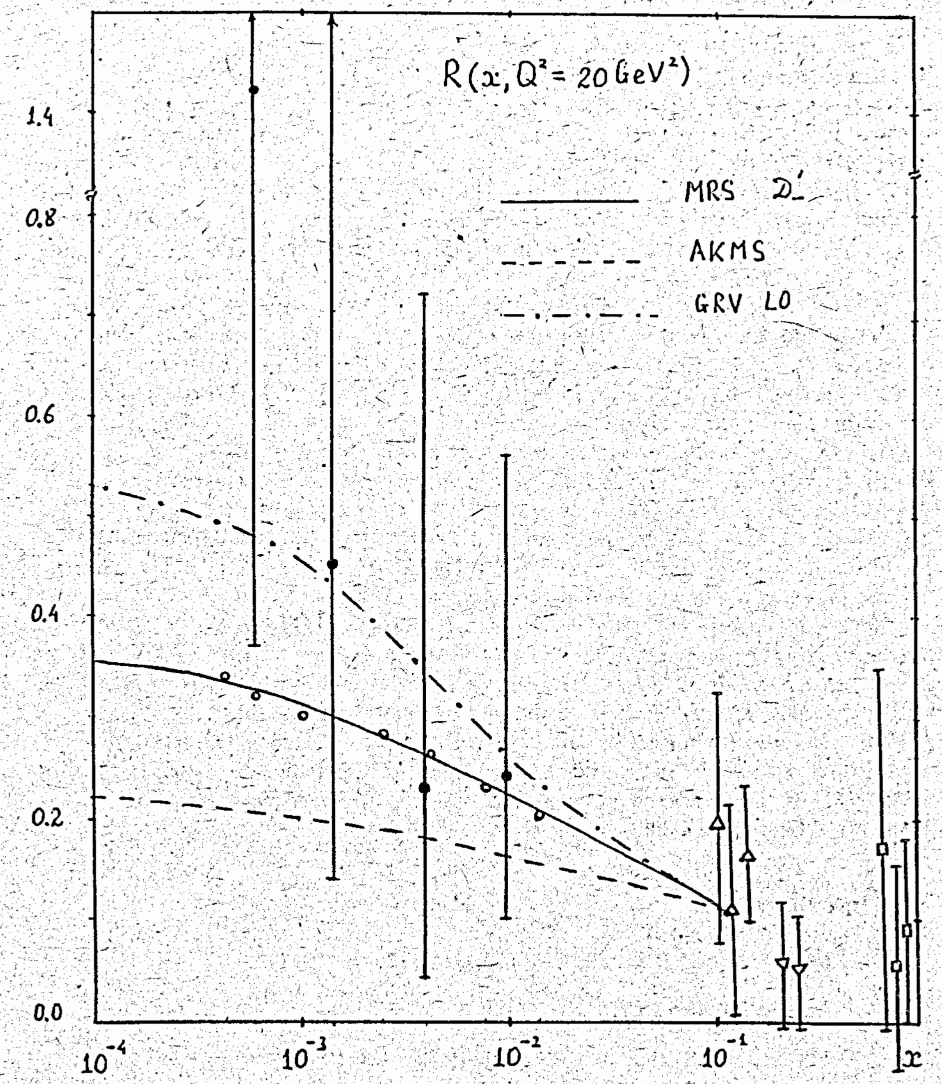

Figure 2. The same is as in Fig.1 but the $R(x, Q^2)$ values are extracted with help of eq.(1). The white circles indicate the $R(x, Q^2)$ values used by $H1$ group in [10]. The symbols $\triangle$, $\bigtriangledown$ and $\square$ are represented the *BCDMS* [14], *CDHSW* [15] and *SLAC* [16] data, respectively.

2. Let us study the predictions inspired by eq.(11). We use the values of SF F_2 and its Q^2 derivative found by **H1** collaboration (see [8] and [10], respectively). We present the extracted $F_L(x,Q^2)$ and $R(x,Q^2)$ values in Fig.1 and Fig.2, respectively, and compare them with theoretical predictions. As in [10], the hypothesis concerning the approximate linear lnQ^2 dependence of F_2 at small x as well as the value of QCD scale $\Lambda^{f=4}_{\overline{MS}} = 200$ MeV^2, have been used. As one can see in the above Figures, $R(x,Q^2)$ is closer to GRV predictions [11][4] only. The predictions of other groups (see [3, 12]) lead to the smaller $R(x,Q^2)$ values at $x \sim 10^{-4}$. Note that all group predict also smaller $F_L(x,Q^2)$ values at $x \leq 10^{-3}$. Moreover, as one can see in Fig.2, there is also disagreement between the $R(x,Q^2)$ values used by $H1$ group in [10] and the ones following from our eq.s (1) and (11). This is due to the very large values of $d(F_2(x,Q^2))/(dlnQ^2)$ at $x \sim 10^{-4}$ found in [10] which show themselves also in comparison between the gluon distribution at $x \leq 10^{-3}$ and the corresponding theoretical predictions (see [10, 13]). This disagreement may be overcome by including of NLO corrections to our equations and this work is in progress.

Note that the basic contribution to $F_L(x,Q^2)$ (and $R(x,Q^2)$) is given by $d(F_2(x,Q^2))/(dlnQ^2)$ part. However, the one from $F_2(x,Q^2)$ increases the values of $F_L(x,Q^2)$ from several percents at $x \approx 10^{-4}$ to 30% at $x \approx 2 \cdot 10^{-2}$.

Resume. We have presented formulae (9) and (11) to extract the longitudinal SF F_L at small x from SF F_2 and its Q^2 derivative. The addition of NLO contribution into eq.s (9) and (11) can be done in analogy with papers [2, 13].

The application of eq.(11) to the analysis of **H1** data from **HERA** has been performed. The values of $F_L(x,Q^2)$ and $R(x,Q^2)$ for small x: $10^{-4} \leq x \leq 2 \cdot 10^{-2}$ have been found. The expansion of this analysis for the case $\delta \sim 0$ which is in agreement with **NMC** data and the evaluation of the NLO contributions will be done in the future.

This work was supported, in part, by the Russian Fund for the Fundation Investigation (94-02-03665-a), by International association for the promotion of cooperation with scientists from the independent states of the FSU (INTAS grant 93 1180) and by a Soros Foundation Grant (RFL 000).

References

[1] R.G.Roberts and M.R.Whalley, *J. of Phys.* **G17**, D1 (1991).

[2] A.V.Kotikov, *Yad.Fiz.* **57**, N1,142 (1994).

[3] A.D.Martin, W.S.Stirling and R.G.Roberts, *Phys.Lett.* **B306**, 145 (1993).

[4] E.A.Kuraev, L.N.Lipatov and V.S.Fadin, *ZHETF* **53**, 2018 (1976) , **54**, 128 (1977); Ya.Ya.Balitzki and L.N.Lipatov, *Yad.Fiz.* **28**, 822 (1978); L.N.Lipatov, *ZHETF* **63**, 904 (1986).

[5] P. Amandruz et al, *Phys.Lett.* **295B**, 159 (1992).

[4] To find the values of $F_L(x,Q^2)$ predicted by GRV approach, we use the values of $g(x,Q^2)$ and $F_2(x,Q^2)$ from [11] and then the equation from [2] to extract F_L from g and F_2. The values of $R(x,Q^2)$ are obtained from the F_L and F_2 via (1).

[6] A.Donnachie and P.V.Landshoff, *Nucl.Phys.* **244B**, 322 (1984); **267B**, 690 (1986).

[7] P.V.Landshoff and O.Nachmann, *Z.Phys.* **C35**, 405 (1987).

[8] I. Abt et al, *Nucl.Phys.* **407B**, 515 (1993).

[9] L.L. Enkovszky, A.V. Kotikov and F. Paccanoni, *Phys.Lett.* **314B**, 421 (1993).

[10] I. Abt et al, *Phys.Lett.* **321B**, 161 (1994).

[11] M. Gluck, E. Reya and A.Vogt, *Z.Phys.* **C53**, 127 (1992); **306B**, 391 (1993);

[12] A.J.Askew, J.Kwecinski, A.D.Martin, and P.J.Sutton, *Phys.Rev.* **D49**, 4402 (1994).

[13] A.V.Kotikov, *JETP Lett.* **59**, 635 (1994).

[14] A.C.Benvenuti et al, *Phys.Lett.* **195B**, 91 (1987); **223B**, 485 (1989); **237B**, 592 (1990).

[15] P.Berge et al, *Z.Phys.* **C49**, 187 (1991).

[16] A.Bodek et al, *Phys.Rev.* **D20**, 1471 (1979); M.D.Mesteyer et al, *Phys.Rev.* **D27**, 285 (1983).

Received by Publishing Department
on October 19, 1994.