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PIONIUM IN THE ELEMENTARY PARTICLE DECAYS

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1 Introduction

The pionium ($\pi^+\pi^-$-dimesoatom) was firstly considered by Uretsky and Palfrey [1], although only after two decades an experimental study of such Coulomb dimesoatoms, initiated greatly by L. L. Nemenov [2], became real. Recently about 200 pionium atoms, produced in a Ta target by 70 Gev protons, were observed [3]. But an accurate pionium lifetime measurement is still absent, and it is expected that this will be done in a more elaborate version of this [4] or other [5] proposed experiments. From this measurement a model independent information about pion scattering lengths can be extracted [1, 6, 7].

Already in [1] it was realized that pionium can be formed in the elementary particle decays. Among them the $K^+ \to \pi^+ A_{2\pi}$ decay is most promising [8]. The branching ratios of this and other similar atomic decays are very small, making their experimental study an elaborate task. In the case of pionium, matters are further complicated by the fact that it is a very short lived object.

Let us note that the only atomic decays of mesons, experimentally studied at yet, are $\pi^0 \to \gamma A_{2e}$ [9] and $K_L \to \nu A_{\mu\pi}$ [10]. Their measured branching ratios agrees well to what is expected theoretically [2, 11, 12]. Even $O(\alpha)$ corrections for them were calculated [13, 14], although some part of $O(\alpha)$ corrections for $K_L \to \nu A_{\mu\pi}$, originated from the normalization constant, was omitted in [14].

It is expected that several mesonic factories with very high luminosity will begin to operate in a near future. The study of atomic decays, even with pionium in a final state, can become real at these factories. Below we will consider some atomic decays of the elementary particles which can be interesting in this respect.

Further information about dimesoatoms, not presented in this article, can be found in [15, 16, 17].

2 Pionium in the η, η' and K-meson decays

In the nonrelativistic approximation pionium state vector coincides (up to irrelevent phase factor) to the following superposition [12] :

$$|P_A; A_{2\pi}\rangle = \int \frac{d\vec{q}}{(2\pi)^3} \, |\frac{P_A}{2} + q; \pi^+\rangle |\frac{P_A}{2} - q; \pi^-\rangle \frac{\Psi(\vec{q})}{\sqrt{m}} \quad , \tag{1}$$

where $\Psi(\vec{q})$ is a momentum space wave function for $A_{2\pi}$ Coulomb atom and $\sqrt{m} \equiv \sqrt{m_{\pi^+}} \approx \sqrt{\frac{M_A}{2}}$ appears because of relativistic normalization

$$\langle \vec{p} \, | \vec{q} \rangle = 2E_p (2\pi)^3 \delta(\vec{p} - \vec{q}) \quad , \quad E_p = \sqrt{\vec{p}^{\,2} + m^2} \quad . \tag{2}$$

which we will use throughout this paper for any one-particle (composed or elementary) state vector.

(1) means that an amplitude for the atomic decay $P_1 \longrightarrow P_2 + A_{2\pi}$ is given by the following expression

$$\langle P_2 A_{2\pi} | P_1 \rangle = I(Q_1, \frac{P_A}{2}, \frac{P_A}{2}, Q_2) \frac{i\Psi(\vec{r} = 0)}{\sqrt{m}} \quad , \tag{3}$$

where $\Psi(\vec{r} = 0)$ is a Schrödinger wave function for a hydrogenlike atom at the origin, and $I(Q_1, p_+, p_-, Q_2)$ is an amplitude for the nonatomic decay $P_1(Q_1) \longrightarrow \pi^+(p_+) + \pi^-(p_-) + P_2(Q_2)$. If this amplitude is known, (3) allows us to calculate a ratio of atomic and nonatomic decay widths.

If P_1 and P_2 are pseudoscalar mesons, then in a P_1-meson rest frame $I(Q_1, p_+, p_-, Q_2)$ depends only on two independent energies, and it is convenient to use instead of them the conventional Dalitz variables:

$$x = \frac{\sqrt{3}(E_+ - E_-)}{M_1 - 2m - M_2} \qquad y = 3\frac{M_1 - E_+ - E_- - M_2}{M_1 - 2m - M_2} - 1 \quad . \tag{4}$$

If $M_1 - 2m - M_2 \ll M_1$, it is expected [18] that higher order terms in a power series expansion of $I(x,y)$ will be suppressed and in a good approximation

$$|I(x,y)|^2 \sim 1 + ay + by^2 + cx^2 \quad .$$

Note that terms linear in x are forbidden by CP-invariance. After integrating over two and three particle phase spaces:

$$\Gamma(P_1 \to P_2 A_{2\pi}) =$$

$$= \int |\langle P_2 A_{2\pi} | P_1 \rangle|^2 \frac{1}{2M_1}(2\pi)^4 \delta(Q_1 - Q_2 - P_A) \frac{d\overrightarrow{Q_2}}{(2\pi)^3 2E_2} \frac{d\overrightarrow{P}_A}{(2\pi)^3 2E_A}$$

$$\Gamma(P_1 \to P_2 \pi^+ \pi^-) =$$

$$= \int |I(Q_1, p_+, p_-, Q_2)|^2 \frac{1}{2M_1}(2\pi)^4 \delta(Q_1 - Q_2 - p_+ - p_-) \frac{d\overrightarrow{Q_2}\, d\overrightarrow{p_+}\, d\overrightarrow{p_-}}{(2\pi)^9 8E_2 E_+ E_-}$$

and using $|\Psi(\overrightarrow{x} = 0)| = \frac{\alpha^3 m^3}{8\pi}$, we obtain:

$$\frac{\Gamma(P_1 \to P_2 A_{2\pi})}{\Gamma(P_1 \to P_2 \pi^+ \pi^-)} = \frac{\pi}{R} \alpha^3 (\frac{m}{M_1})^2 |I(\tilde{x}, \tilde{y})|^2 \sqrt{r^2 - \frac{M_2^2}{M_1^2}} \quad . \tag{5}$$

$\tilde{x}$ and $\tilde{y}$ are Dalitz variables, which correspond to the atomic decay:

$$\tilde{x} = 0 \quad \tilde{y} = 3 \frac{\tilde{E}_2 - M_2}{M_1 - 2m - M_2} \quad \tilde{E}_2 = \frac{M_1^2 - 4m^2 + M_2^2}{2M_1} \quad . \tag{6}$$

R is a dimensionalless remnant of the three-particle phase space integral:

$$R = \int_{(x_+)_{min}}^{(x_+)_{max}} dx_+ \int_{(x_-)_{min}}^{(x_-)_{max}} dx_- \, |I(x_+, x_-)|^2 \quad , \tag{7}$$

where $x_+ = \frac{E_+}{M_1}, x_- = \frac{E_-}{M_1}$, and the integration limits in (7) are given by

$$(x_+)_{\rm min} = \frac{m}{M_1} \qquad (x_+)_{\rm max} = \frac{1}{2}(1 - \frac{M_2(2m + M_2)}{M_1^2})$$

$$(x_-)_{\substack{min \\ max}} = \frac{1}{2(1 - 2x_+ + \frac{m^2}{M_1^2})} \{(1 - x_+)(1 - 2x_+ + \frac{2m^2 - M_2^2}{M_1^2}) \mp$$

$$\mp \sqrt{(x_+^2 - \frac{m^2}{M_1^2})(1 - 2x_+ + \frac{M_2(2m - M_2)}{M_1^2})(1 - 2x_+ - \frac{M_2(2m + M_2)}{M_1^2})}\}. \tag{8}$$

At last

$$r = \frac{1}{2}(1 - 4\frac{m^2}{M_1^2} + \frac{M_2^2}{M_1^2}) \quad . \tag{9}$$

The Dalitz-plot distribution for $\eta \to \pi^+\pi^-\pi^0$ decay had been measured [19] and the result is

$$|I_\eta(x,y)|^2 \sim 1 - (1.08 \pm 0.014)y + (0.03 \pm 0.03)y^2 + (0.05 \pm 0.03)x^2. \tag{10}$$

Inserting this into (5) and (7), we get (it is assumed that $A_{2\pi}$ is produced in a $1S$ state. If we sum up over all nS states, the result will increase $\sum_{n=1}^{\infty} \frac{1}{n^3} \approx 1.2$ times):

$$\frac{\Gamma(\eta \to \pi^0 A_{2\pi})}{\Gamma(\eta \to \pi^+\pi^-\pi^0)} \approx 0.91 \cdot 10^{-7}. \tag{11}$$

Let us note, that a value $3.9 \cdot 10^{-7}$, cited in [16], corresponds to a theoretical prediction from effective chiral lagrangian [20] $I(x,y) \sim 1 - 0.55y$ and seems to be too optimistic, though the accuracy of quadratic terms determination in (10) allows, in principle, to increase (11) several times.

The results, analogous to (10), exist for $K^+ \to \pi^+\pi^+\pi^-$ decay [21]:

$$|I_{K^+}(x,y)|^2 \sim 1 + (0.2814 \pm 0.0082)y - (0.001 \pm 0.023)y^2 - \\ -(0.099 \pm 0.019)x^2 \quad ,$$

for $K_L \to \pi^+\pi^-\pi^0$ decay [22]:

$$|I_{K_L}(x,y)|^2 \sim 1 - (0.917 \pm 0.013)y + (0.149 \pm 0.013)y^2 + \\ +(0.055 \pm 0.010)x^2 \quad ,$$

and for $\eta' \to \eta\pi^+\pi^-$ decay [23]:

$$|I_{\eta'}(x,y)|^2 \sim |1 - (0.08 \pm 0.03)y)|^2 \quad .$$

Using them, we get

$$\frac{\Gamma(K^+ \to \pi^+ A_{2\pi})}{\Gamma(K^+ \to \pi^+\pi^-\pi^0)} \approx 10^{-5}$$

$$\frac{\Gamma(K_L \to \pi^0 A_{2\pi})}{\Gamma(K_L \to \pi^+\pi^-\pi^0)} \approx 8.6 \cdot 10^{-7} \qquad \frac{\Gamma(\eta' \to \eta A_{2\pi})}{\Gamma(\eta' \to \eta\pi^+\pi^-)} \approx 1.4 \cdot 10^{-6} \tag{12}$$

For $K^+ \to \pi^+ A_{2\pi}$ decay, it is necessary to take into account the identity of π^+-mesons, which increases the result two times. An extra ~ 5 times difference between K^+ and K_L decays is due to $I_{K^+}(\tilde{x},\tilde{y})/I_{K_L}(\tilde{x},\tilde{y}) \approx 2.15$. Note that $K^+ \to \pi^+ A_{2\pi}$ decay was considered earlier in [8] with slightly different result.

Taking nonatomic decays' branching ratios from [24], (11) and (12) can be transformed to

$$Br(\eta \to \pi^0 A_{2\pi}) \approx 2 \cdot 10^{-8} \qquad Br(\eta' \to \eta A_{2\pi}) \approx 6.2 \cdot 10^{-7}$$
$$Br(K^+ \to \pi^+ A_{2\pi}) \approx 5.5 \cdot 10^{-7} \qquad Br(K_L \to \pi^0 A_{2\pi}) \approx 1.1 \cdot 10^{-7}. \quad (13)$$

For ϕ-factory (13) means about 10^4 K-meson atomic decays with pionium per year. So, we think, the study of such atomic decays at ϕ-factory is not only realistic, but a desirable task.

3 Pionium in the ψ and Υ-meson decays

For $c-\tau$ and B-factories $\psi(2S) \to \psi(1S)A_{2\pi}$ and $\Upsilon(2S) \to \Upsilon(1S)A_{2\pi}$ decays can be interesting since the corresponding nonatomic decays have large branching ratios.

In the nonrelativistic approximation the most general form for the $V_1 \to V_2\pi^+\pi^-$ decay amplitude, which follows from PCAC, is [25]

$$\langle V_2\pi^+\pi^-|V_1\rangle = \vec{\epsilon_1}\cdot\vec{\epsilon_2}[-A\, q_+\cdot q_- + B\, E_+E_-] +$$
$$+C\,(\vec{\epsilon_1}\cdot\vec{q_1}\,\vec{\epsilon_2}\cdot\vec{q_2} + \vec{\epsilon_1}\cdot\vec{q_2}\,\vec{\epsilon_2}\cdot\vec{q_1}), \quad (14)$$

where A,B,C are approximately constant and ϵ_1, ϵ_2 are V_1, V_2 vector meson polarization vectors.

There are some theoretical indications [26] and experiment confirms [27] that $C = 0$. Then to calculate the (5) ratio, we only need B/A ratio and it can be extracted from the $\pi^+\pi^-$ invariant mass distribution in the $V_1 \to V_2\pi^+\pi^-$ decay. The results are [27]:

$$\frac{B}{A} = -0.21 \pm 0.01 \quad (15)$$

for the $\psi(2S) \to \psi(1S)A_{2\pi}$ decay, and

$$\frac{B}{A} = -0.154 \pm 0.019 \quad (16)$$

for the $\Upsilon(2S) \to \Upsilon(1S)A_{2\pi}$ one.

After summing over vector meson polarizations, it follows from (14) that (if C=0)

$$\langle V_2\pi^+\pi^-|V_1\rangle \sim \frac{1}{2}(1+\frac{M_2^2}{M_1^2}-2\frac{m^2}{M_1^2})-1+x_+ + x_- - \frac{B}{A}x_+x_- \quad , \qquad (17)$$

x_+ and x_- were defined earlier.

Using this instead of $I(x,y)$ in (5) and (7), we get from (15) and (16)

$$\frac{\Gamma(\psi(2S)\to\psi(1S)A_{2\pi})}{\Gamma(\psi(2S)\to\psi(1S)\pi^+\pi^-)} \approx 4.6\cdot 10^{-8}$$
$$\frac{\Gamma(\Upsilon(2S)\to\Upsilon(1S)A_{2\pi})}{\Gamma(\Upsilon(2S)\to\Upsilon(1S)\pi^+\pi^-)} \approx 5.2\cdot 10^{-8} \quad , \qquad (18)$$

which correspond to the following branching ratios

$$Br(\psi(2S)\to\psi(1S)A_{2\pi}) \approx 1.4\cdot 10^{-8}$$
$$Br(\Upsilon(2S)\to\Upsilon(1S)A_{2\pi}) \approx 10^{-8} \qquad (19)$$

Unfortunately this is too small for B-factory. We can expect only several events per year. So it seems unrealistic to study Υ-meson atomic decays at B-factory.

4 $O(\alpha)$ order corrections to the pionium lifetime

The main decay mode for pionium is $A_{2\pi}\to\pi^0\pi^0$ and its amplitude according to Mandelstam [28] can be expressed in the form

$$\langle\pi^0\pi^0|A_{2\pi}\rangle = \int\frac{dp}{(2\pi)^4}J(p_1,p_2,\frac{P_A}{2}+p,\frac{P_A}{2}-p)\chi(p) \quad , \qquad (20)$$

where $\chi(p)$ is $A_{2\pi}$-dimesoatom bound state Bethe-Salpeter wave function and $J(p_1,p_2,p_+,p_-)$ stands for $\pi^+\pi^-$-irreducible kernel for the reaction $\pi^+(p_+)+\pi^-(p_-)\to\pi^0(p_1)+\pi^0(p_2)$.

Up to $O(\alpha)$ terms, J is constant defined through pion scattering lengths a_0 and a_2 [29]:

$$J=\frac{32}{3}\pi m(a_0-a_2) \quad . \qquad (21)$$

So (20) can be rewritten as

$$\langle\pi^0\pi^0|A_{2\pi}\rangle = J\cdot\chi(x=0) \quad , \qquad (22)$$

where

$$\chi(x=0)=\int\frac{dp}{(2\pi)^4}\chi(p) \tag{23}$$

is a configuration space Bethe-Salpeter wave function at the origin.

The Bethe-Salpeter equation for $\chi(p)$ in a $A_{2\pi}$ rest frame, up to $O(\alpha)$ terms, takes the form

$$\left[m^2+\vec{p}^{\,2}-(\frac{M_A}{2}+p_0)^2\right]\left[m^2+\vec{p}^{\,2}-(\frac{M_A}{2}-p_0)^2\right]\chi(p)= \\ =\frac{i\lambda}{\pi^2}\int dq\,\frac{\chi(q)}{(p-q)^2-i\epsilon} \tag{24}$$

where $\lambda=\frac{\alpha M_A^2}{4\pi}$.

(24) corresponds to the Wick-Cutkosky model [30]. Let us note that this fact was firstly noticed and used to calculate $O(\alpha)$ order corrections to the $K_L\to\nu A_{\mu\pi}$ decay width in [14].

According to [30] (see also [31] for a review), a ground state ($1S$ in the nonrelativistic limit) solution of (24) corresponds to

$$\chi(p)=\int_{-1}^{1}\frac{g(z)\,dz}{[A+Bz]^3}\qquad, \tag{25}$$

where

$$A=m^2-\frac{1}{4}M_A^2-p^2\equiv\Delta^2-p^2\qquad B=p_0M_A\quad,$$

and g(z) spectral function satisfies the following integral equation

$$g(z)=\frac{\lambda}{2}\int_{-1}^{1}\frac{1}{\Delta^2+\frac{1}{4}M_A^2y^2}[\frac{1-z}{1-y}\Theta(z-y)+\frac{1+z}{1+y}\Theta(y-z)]g(y)\,dy. \tag{26}$$

If $M_A=2m-E$, $E\ll m$, then $\Delta^2\approx mE$ and

$$\frac{1}{\Delta^2+\frac{1}{4}M_A^2y^2}=\frac{\pi}{m\sqrt{mE}}\delta(y)\qquad, \tag{27}$$

because

$$\frac{\epsilon}{\epsilon^2+y^2}\longrightarrow\pi\delta(y)\quad.$$

when $\epsilon\to 0$.

Substituting (27) into (26), we get

$$g(z)=\frac{\lambda\pi}{2m\sqrt{mE}}(1-|z|)\,g(0)\qquad. \tag{28}$$

So

$$\frac{\lambda\pi}{2m\sqrt{mE}} = 1 \quad ,$$

which really gives a hydrogenlike atom nonrelativistic ground state energy level $E = \frac{m\alpha^2}{4}$.

But the solution (28), found in [30], is not a complete $O(\alpha)$ order solution. Indeed, taking $g(z) = g_0(z) + \alpha g_1(z)$ and

$$\frac{1}{\Delta^2 + \frac{1}{4}M_A^2 y^2} = \frac{2\pi}{m^2\alpha}\delta(y) + \sigma(y) \quad , \tag{29}$$

where $\sigma(y)$ has $O(\alpha)$ order smallness compared to the first $\sim \delta(y)$ term, we get from (26) (N_0 and N_1 are constants)

$$g_0(z) = N_0(1 - |z|) \quad g_1(z) = N_1(1 - |z|) + \frac{\lambda}{2\alpha}\int_{-1}^{1} \sigma(y)R(z, y)g_0(y)\,dy, \tag{30}$$

where

$$R(z, y) = \frac{1 - z}{1 - y}\Theta(z - y) + \frac{1 + z}{1 + y}\Theta(y - z) \quad .$$

Calculating in the $\alpha \to 0$ limit the integral in (30), we get

$$g_1(z) = N_1(1 - |z|) + \frac{N_0}{\pi}\{(1 - |z|)\ln(\alpha) + (1 + |z|)[\ln(2|z|) - \ln(1 + |z|)]\} \ .$$

Therefore, the complete $O(\alpha)$ order solution of (26) looks like

$$g(z) = N\{(1 - |z|) + \frac{\alpha}{\pi}(1 + |z|)[\ln(2|z|) - \ln(1 + |z|)]\} \tag{31}$$

Substituting this in (25), we get the $O(\alpha)$ order pionium Bethe-Salpeter wave function

$$\chi(p, P_A) = \\ = \frac{N}{(\Delta^2 - p^2)\left[m^2 - (\frac{P_A}{2} + p)^2\right]\left[m^2 - (\frac{P_A}{2} - p)^2\right]}\left\{1 + \frac{\alpha}{\pi}\chi_1(p, P_A)\right\}, \tag{32}$$

where

$$\chi_1(p, P_A) = \frac{m^2 - (\frac{P_A}{2} - p)^2}{2(\Delta^2 - p^2)}\ln(m^2 - (\frac{P_A}{2} - p)^2) + \\ + \frac{m^2 - (\frac{P_A}{2} + p)^2}{2(\Delta^2 - p^2)}\ln(m^2 - (\frac{P_A}{2} + p)^2) - \ln(\Delta^2 - p^2) + O(\alpha\ln(\alpha)). \tag{33}$$

A normalization constant N is defined from the normalization condition [32], which in the $O(\alpha)$ order takes the form ($A_{2\pi}$ rest frame is assumed)

$$-2m^2\alpha^2 \approx \alpha\frac{dM^2}{d\alpha} =$$
$$= \frac{iN^2}{(2\pi)^4}\int\frac{dp}{(\Delta^2-p^2)^2\left[\left(\frac{M_A}{2}+p_0\right)^2-m^2-\vec{p}^2\right]\left[\left(\frac{M_A}{2}-p_0\right)^2-m^2-\vec{p}^2\right]} \quad (34)$$

But the integral in the r.h.s. of (34) equals to

$$L = -\frac{i\pi}{2}\frac{\partial^2}{\partial(\Delta^2)^2}\int d\vec{p}\int\frac{dp_0}{2\pi i}\frac{1}{\Delta^2+\vec{p}^2-p_0^2}\left[\frac{1}{(\frac{M_A}{2}+p_0)^2-m^2-\vec{p}^2}+\right.$$
$$\left.+\frac{1}{(\frac{M_A}{2}-p_0)^2-m^2-\vec{p}^2}\right]$$

Remembering that in fact m^2 in the above expression should be replaced by $m^2-i\epsilon$, we can perform an integration over dp_0 and obtain

$$L = \frac{2i\pi^2}{M_A^2}\int_0^\infty dx\,\frac{x^2}{(\Delta^2+x^2)^2}\left\{\frac{1}{\sqrt{m^2+x^2}}\left(\frac{4(m^2+x^2)}{\Delta^2+x^2}-1\right)-\frac{3}{\sqrt{\Delta^2+x^2}}\right\}\approx$$
$$\approx\frac{i\pi^3}{m^4\alpha^3}(1-2\frac{\alpha}{\pi}+O(\alpha^2)).$$

So (34) takes the form

$$-2m^2\alpha^2 = \frac{i}{(2\pi)^4}N^2\frac{i\pi^3}{m^4\alpha^3}(1-2\frac{\alpha}{\pi}) \quad ,$$

and therefore

$$N = 32\sqrt{\pi m}(\frac{1}{2}m\alpha)^{5/2}(1+\frac{\alpha}{\pi}) \quad . \quad (35)$$

Analogously

$$\int\frac{dp}{(2\pi)^4}\chi(p) = \frac{iN}{8\pi^2M_A^2}\int_0^\infty dx\,\frac{x^2}{\Delta^2+x^2}\left\{\frac{2}{\sqrt{m^2+x^2}}-\frac{2}{\sqrt{\Delta^2+x^2}}+\right.$$
$$\left.+\frac{M_A^2}{\sqrt{(m^2+x^2)(\Delta^2+x^2)}}\right\}\approx\frac{iN}{16\pi m^2\alpha}(1+\frac{\alpha}{\pi}) \quad .$$

Therefore

$$\chi(x=0) = \frac{iN}{32m\pi(\frac{1}{2}m\alpha)}(1+\frac{\alpha}{\pi})\approx\frac{i}{\sqrt{m}}\Psi(\vec{x}=0)\,(1+2\frac{\alpha}{\pi}) \quad (36)$$

Substituting this in (22), we finally get

$$\Gamma(A_{2\pi} \to \pi^0\pi^0) = \Gamma_0(A_{2\pi} \to \pi^0\pi^0)(1 + 4\frac{\alpha}{\pi}) \qquad , \tag{37}$$

where [6]

$$\Gamma_0(A_{2\pi} \to \pi^0\pi^0) = \frac{16\pi}{9}(a_0 - a_2)^2 \sqrt{\frac{2(m_{\pi^+} - m_{\pi^0})}{m_{\pi^+}}} |\Psi(\vec{r} = 0)|^2 \quad .$$

$\sim \chi_1$ part of the Bethe-Salpeter wave function contributes neither in its value at the origin nor in its normalization, because in the $O(\alpha)$ order

$$\int \frac{\alpha\, dp_0}{\left[\left(\frac{M_A}{2} + p_0\right)^2 - m^2 - \vec{p}^2\right]\left[\left(\frac{M_A}{2} - p_0\right)^2 - m^2 - \vec{p}^2\right]} \cdots = \int \frac{i\pi\alpha\delta(p_0)\, dp_0}{M_A(\Delta^2 + \vec{p}^2)} \cdots$$

and $\delta(p_0)\chi_1(p; P_A) = 0$.

As a last remark, let us note that a nonrelativistic approximation (3) for the $M_1 \to M_2 + A_{2\pi}$ decay amplitude follows from

$$\chi(x = 0) \approx \frac{i}{\sqrt{m}}\Psi(\vec{r} = 0)$$

and [28]

$$(P_2 A_{2\pi}|P_1) = \int \frac{dp}{(2\pi)^4} I(Q_1, \frac{P_A}{2} + p, \frac{P_A}{2} - p, Q_2)\chi(p) \approx$$
$$\approx I(Q_1, \frac{P_A}{2}, \frac{P_A}{2}, Q_2)\chi(x = 0) \quad .$$

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