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COLOR, COLORED QUARKS, QUANTUM CHROMODYNAMICS

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Few days prior to the beginning of our Seminar, the discovery of a sixth, top, quark was reported by Fermilab. Thirty years passed between the hypothesis of quark structure of hadrons and experimental discovery of six quarks. Thirty years, which were filled with new ideas, theoretical models and painstaking experimental searches. Colored quarks, colored gluons are discovered now and we have a good candidate for the quantum theory of colored quark-gluon interactions. Quantum Chromodynamics.

In this introductory talk, on the occasion of the first appearance of quarks on the world scene. I would like to overview some ideas that were, in my opinion, responsible for the creation of QCD.

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Nakano-Nishijima-Gell-Mann rule

Once the strange particles were experimentally discovered in 1950's, a great amount of new particles has appeared. Classifying already known hadrons in accordance with the strangeness S and third component of the isospin I_A . Nakano, Nishijima [1] and Gell-Mann [2] have pointed out an interesting tendency. It turned out that mesons baryons and antibaryons could produce groups of eight particles and that the change of hypercharge in each group on ± 1 led to the change of the third component of the isospin on $\pm 1/2$. As a result, Nakano, Nishijima and Gell Mann independently have proposed the formula which connects fundamental characteristics of particles: the electric charge Q, third component of the isospin I_3 and the hypercharge Y

$$Q = I_3 + \frac{1}{2}Y.$$
 (1)

The hypercharge Y is expressed through the baryon number B and strangeness S in the following way:

$$Y = B + S.$$

Formula (1) (hereafter the NNG formula) presupposed the existence of hadrons with specified properties. Soon, these hadrons were discovered and the NNG formula was formed into a law.

However, the fundamental question of why hadrons are arranged as octets remained open. The solution of this problem has led to the view on constituent structure of hadrons.

Searches in this direction were originated in papers by M.A. Markov [3], S. Sakata [4] and L.B. Okun. [5] where the constituent particle model was used for the classification of hadrons.

Sakata model

The Sakata model of composite particles [4] that goes back to the Fermi Yang model, was proposed in 1956. The fundamental particles in this model are protons, neutrons, lambda-hyperons and their antiparticles with the corresponding quantum numbers:

	5	Ŷ	1	I_3
р	0	1	1/2	1/2
n	0	1	1/2	-1/2
1	-1	0	0	0
p	0	-1	1/2	$\cdot 1/2$
n	0	-1	1/2	1/2
Δ	I	0	0	0

The rest of the hadrons were considered as some bound states of these particles. Later on, generalizing the ideas of isotopinvariance, lkeda. Ogawa, Ohnuki [6] and Yamaguchi [7] proposed to consider the Sakata triplet as a realization of the fundamental representation of the SU(3) group. Hence, the generalized SU(3) invariant Sakata model was first used for the classification of hadrons. Mesons in this theory were regarded as hadrons composed of particles of the Sakata triplet and antitriplet. However, attempts to classify baryons still encounter some basic difficulties.

In an alternative approach. Gell-Mann [8] and Neeman [9] independently have shown that the NNG formula is a direct consequence of the SU(3) symmetry and have proposed to consider baryons and mesons as particles of the octet representation of the SU(3) group. Gell-Mann has called his approach "The Eightfold Way". Certainly, the experimental discovery of the Ω^{+} -hyperon predicted in 1963, was a great success of the SU(3) invariant theory.

Innumerable attempts to improve the generalized SU(3) invariant Sakata model, which would be able to classify all hadrons (both baryons and mesons) in the same way, were not successful.

It was just the quark model of Gell-Mann [10] and Zweig [11] proposed in 1961 that replaced Sakata's theory.

Quark model of Gell-Mann and Zweig

Within the quark model of Gell-Mann and Zweig all hadrons are regarded as composed of various combinations of three quarks u, d, s and antiquarks u, d, s. Quarks are arranged in triplets of the fundamental representation of the $SU(3)_F$ flavor group. Imposing the requirement that the operator of charge Q and hypercharge Y should

be the generators of the $SU(3)_F$ group, one arrives at the following quantum numbers for Gell-Mann-Zweig quarks:

	Q	S	I	Ι,	В
11	2/3	0	1/2	1/2	1/3
d	-1/3	0	1/2	1/2	1/3
8	-1/3	- 1	0	()	173
u	-2/3	0	1/2	1/2	1/3
d	1/3	0	1/2	1/2	1/3
	1/3	1	0	0	1/3

As we have already pointed out, all hadrons within the Gell Mann-Zweig quark model are regarded as composed of the fundamental particles, mesons are nothing but pairs of quarks and antiquarks, while baryons consist of three quarks. All particle multiplets realized the irreducible representation of the $SU(3)_{T}$ group (though, there are some mixing between mesons).

The idea of unification of the internal and external symmetries was very essential for classification of hadrons. In particular, the group SU(3) has been combined with the group of the spin symmetry $SU(2)_S$ into the unified SU(6) group [12].

Within the SU(6) symmetric theory the octet of pseudoscalar mesons and the nonet of vector mesons

$$K^{+}, K^{0}, \pi^{+}, \pi^{0}, \pi^{-}, \eta, K^{0}, K^{-}$$
$$K^{*+}, K^{*0}, \phi^{0}, \rho^{+}, \rho^{-}, \rho^{0}, \omega, K^{*+}, K^{*0}$$

are arranged as the following multiplets:

$$6 * 6 = 1 + 35$$

$$35 = (8; 0) + (1 + 8; 1).$$

The baryon octet and the delta-decuplet, consequently,

$$n, p, \Sigma^{-}, \Sigma^{+}, \Sigma^{0}, \Lambda^{0}, \Xi^{-}, \Xi^{+},$$
$$\Delta^{-}, \Delta^{0}, \Delta^{+}, \Delta^{++}, Y^{*-}, Y^{*0}, Y^{*+}, \Xi^{*-}, \Xi^{*+}, \Omega^{*}$$

are arranged as the 56-plet

$$6 * 6 * 6 = 56 + 70 + 70 + 20$$

 $56 = (8; 1/2) + (10; 3/2).$

Let us present the summary of main results of the $SU(3)_F$ symmetric quark model.

Particles belonging to the same multiplets of the $SU(3)_F$ group have the same spin, comparable masses and satisfy the NNG formula.

Since, in fact, the masses of particles in multiplets are slightly different, the $SU(3)_F$ symmetry is not exact.

Having assumed that the strange quark is heavier [13], the violation of the unitary symmetry will obey the Gell-Mann-Okubo formulas [14].

This model allows one to establish the relation between magnetic moments and the amplitudes of reactions for particles belonging to the same multiplets. These relations are in good qualitative agreement with experimental data.

Thus, in 1961 an important stage of classification of the elementary particles on the basis of the quark model had been completed. Clearly, questions about the physical content of the $SU(3)_F$ model appeared. In particular, the question about actual existence of quarks as physical objects was very important. It was necessary to perform a thorough analysis of problems that emerged within the $SU(3)_F$ quark model. The following issues were especially challenging:

a) Why the relation between spin and statistics is seemingly violated within the quark model?

It was pointed out that when baryons were arranged into the 56-plet of $SU(6)_{FS}$ group, the three-quark states symmetric under the interchange of particles appeared. Hence, the contradiction with the Pauli principle arose and the interpretation of quarks as physical fermions became impossible.

b) Why baryons are constructed from three quarks and mesons from a quark and an antiquark and why multiquark bound states (the so called "exotic states") do not exist?

c) Why do additive laws work within the quark model for calculation of various physical quantities ?

d) Finally, why are there no free quark states in the Nature and, respectively, whose forces confine quarks inside the hadrons?

Searches for answers to these questions led to the creation of a modern dynamical theory of hadrons, Quantum Chromodynamics (QCD).

The basis for this theory is formed by fundamental discoveries: *color* and *colored quarks*, as sources of *colored gluons*, which in turn provide a strong interaction between quarks.

Color, Colored Quarks

To solve the problem of quark spin and statistics, in works by N.N. Bogoliubov, B.V. Struminsky and A.N. Tavkhelidze [15],[16], M. Han and Y. Nambu [17] and also Y. Miyamoto [18] a new quantum number for quarks was introduced. This quantum number could take three values and, thus, each type of quarks could be found in three independent states

$$q^A \equiv q^{a,\alpha}, \quad \alpha = 1, 2, 3,$$

where a is the flavor index and α is the new quantum number index.

At the time, only three types of quarks were known:

$$q^{1,\alpha} = u^{\alpha}, \quad q^{2,\alpha} = d^{\alpha} \quad q^{3,\alpha} = s^{\alpha}.$$

so the model was called the three-triplet quark model.

Under the requirement that quarks are ordinary Dirac fermions and that the additional quantum number should preserve the basic results of the $SU(3)_F$ symmetry, the wave functions of hadrons in the ground state were represented in the form

$$\begin{split} \psi_{abc}(x_1x_2x_3) &= \varepsilon^{\alpha,\beta,\gamma} \phi_{a,b,\varepsilon;\alpha,\beta,\gamma}(x_1x_2x_3) \quad - \quad for \quad baryons, \\ \phi^b_a(x_1x_2) &= \delta^\alpha_\beta \phi^B_A(x_1x_2) \quad - \quad for \quad mesons. \end{split}$$

As a result, the antisymmetricity with respect to the interchange of quarks was satisfied. Moreover, hadrons became neutral in terms of the new quantum number. On the other hand, it has been suggested [17] that each quark with the three values of the new quantum number could realize the fundamental representation of the $SU(3)_c$ group. As a consequence, the hadronic states were considered as singlet representations of the $SU(3)_c$ group [17]. By analogy with the optical mixing of three colors (red, blue and yellow) which yields the white one, the new quantum number was later named "color". Accordingly, quarks were named colored quarks. Within the model of colored quarks, fuldamental particles may have both fractional and integer values of the electric charge, hypercharge and baryon number. Possibility of the existence of quarks with integer charges is not excluded theoretically if one assumes that color symmetry is violated in electromagnetic interactions of quarks, being still exact in the world of observed hadrons [16], [17]. Present-day experimental data provide evidence in favor of fractional quark charges.

We would like to note that the first attempt to overcome the contradiction between spin and statistics for quarks was made by Greenberg who conjectured that quarks are parafermions of rank three [19]. There are several works on the relation between the Greenberg description of hadrons and the colored quark model [20].

The Model of Quasifree Quarks

The colored quark model in the framework of the SU(6), SU(6), group, essentially strengthened the point of view about the colored quarks as physical objects. The absence of quarks in free states, or "confinement", remained to be a principal problem of elementary particle physics.

Along with experimental studies the quest for the dynamical theory of hadrons as bound states of colored quarks was held.

The creation of Quantum Chromodynamics has been preceded by a number of phenomenological approaches. I would like to select the model of quasifree quarks and Current Algebra.

The dynamical model of quasifree quarks was developed in Dubna at the beginning of 1965 [15], [21]. This model was based on the assumption about the heavy quarks which are bound inside the hadrons by strong forces. These forces cause a large defect of quark masses and hold them in confinement, though inside the hadron they could be regarded as quasifree. The model was described by the relativistic invariant quasipotential equation for composite particles [23] with the scalar quasipotential. In the limit of very heavy quarks and large mass defects, the equation admits solutions that contain unitary spin and color structures in the part of the wave function describing the motion of a hadron as a whole with momentum p

$$\psi_A^B(p,q) \simeq \varphi(q)\phi_A^B(p) - for \quad mesons,$$

$$\varphi_{A,B,C}(p,q_1,q_2) \simeq \varphi(q_1,q_2)\phi_{A,B,C}(p) - for - baryons.$$

The wave functions describing the motion of hadrons as a whole satisfy the free equations

$$(p^{2} - m_{M}^{2})\phi_{A}^{H}(p) = 0,$$

$$(p^{2} - m_{H}^{2})\phi_{A,H,U}(p) = 0,$$

where m_M and m_B are masses of mesons and baryons, respectively. To avoid the mixing of solutions with positive and negative frequencies, one should require the fulfillment of additional conditions

$$(\hat{p} - m)^{A'}_{A} \phi^{B'}_{A'} = 0, \quad (\hat{p} + m)^{B'}_{B'} \phi^{B'}_{1} = 0$$

for mesons and

$$(\dot{p}-m)^{A'}_{A}\phi_{A'|B,C}=(\dot{p}-m)^{B'}_{A}\phi_{A|B',C}=(\dot{p}-m)^{C'}_{A}\phi_{A|B,C'}=0.$$

for baryons.

This choice for hadron wave functions within the quasifree quark model allows one to obtain the relativistically invariant generalization for the wave functions of the nonrelativistic SU(6) model. It is essential to emphasize that the wave function depends on hadron masses but not on heavy quark masses! So in the quasifree quark model the dynamical origin of the constituent quark masses has been recognized for the first time. In accordance with the Abdus Salam's saying, quarks are in the "Archimedean bath". Specifically, I wish to point out that the "nucleon magnetic moment enhancement phenomenon" has firstly been explained within the framework of the quasifree quark model. Namely, the expression for the proton magnetic moment

$$\mu_p \simeq \frac{5}{2} \frac{\epsilon}{2M_p}.$$

has been derived [15] which, contrary to the naive expectations, contains the nucleon mass instead of the heavy quark mass.

The following development of the quasifree quark model came up with the creation of the "Dubna bag" model [21] and the "MIT bag" model [22] for hadrons.

The Model with Spontaneously Broken Symmetry and Current Algebra

The fundamental phenomenon. Spontaneous Breaking of Symmetry (SBS), discovered and elaborated by N.N. Bogoliubov in Statistical Physics [24], plays an important role in the modern theory of elementary particles. For the first time the method of SB5 was used for the description of the dynamical fermion mass generation in the γ_{5^+} invariant models of Quantum Field Theory. [25], [26],[27]. In the paper by Nambu and Jona-Lasinio [25] it has been shown that there is another, very important effect beyond the dynamical fermion mass generation. Namely, pseudoscalar mesons appear as the Nambu Goldstone bosons. It is worthwhile to point out here that this phenomenon is in close relation with the Bogoliubov's distinguished theorem "About the $\pm iq^2$ type singularities" in Statistical Physics [28].

The method of SBS plays a crucial role for the Current Algebra application in hadron phenomenology.

Starting with the general conception that the strong interactions should possess chiral symmetry $SU(3)_L * SU(3)_R$, Gell-Mann in 1962 [29] has postulated relations for the hadronic vector $V^a_{\mu}(x)$ and axial-vector $A^a_{\mu}(x)$ currents in the form of commutation relations for the algebra of the $SU(3)_L * SU(3)_R$ group

$$\begin{split} & [V_a^0(x,t), V_b^0(y,t)] = i f_{abc} V_c^0(x,t) \delta^3(x-y), \\ & [V_a^0(x,t), A_b^0(y,t)] = i f_{abc} A_c^0(x,t) \delta^3(x-y), \\ & [A_a^0(x,t), A_b^0(y,t)] = i f_{abc} V_c^0(x,t) \delta^3(x-y). \end{split}$$

These currents being constructed in the quark model, automatically satisfy the commutation relations of the algebra of the $SU(3)_L * SU(3)_R$ group.

If one assumes that the Hamiltonian of strong interactions is invariant under the $SU(3)_L * SU(3)_R$ transformations, hadrons should be arranged in corresponding irreducible multiplets of this group. Moreover, the octet of scalar mesons would correspond to the octet of pseudoscalar mesons and the octet of baryons would have partners with the opposite parity. The absence of these particles-partners in the Nature led to the idea of spontaneous breaking of the $SU(3)_L * SU(3)_R$ symmetry. On the other hand, as is well known, the particle spectrum realizes just that part of symmetry which is realized both for the Hamiltonian and for the ground state. It was natural to suppose that the ground state realizes only the $SU(3)_V$ symmetry, but spontaneously violates the $SU(3)_A$ group. In accordance with the Namhu-Goldstone theorem, eight pseudoscalar mesons appear in the hadron spectrum and the observed SU(3) symmetry is the result of dynamical violation of the initial $SU(3)_L * SU(3)_R$ group.

The existence of the direct relation between the meson spectrum and the breaking of the axial symmetry led to the hypothesis about the partial conservation of the axial current (PCAC)

$$\partial^{\mu}A^{a}_{\mu} \simeq f_{a}m^{2}_{a}\phi^{a}, \quad a=1,2,...8.$$

where ϕ^a are the meson fields.

Having used the PCAC hypothesis, the low-energy theorems were obtained within the framework of Current Algebra and the values of masses for the so-called current quarks were calculated. These masses play the role of parameters of soft violation of the chiral symmetry in the initia' Hamiltonian and lead to the masses for pseudoscalar mesons.

The SBS method plays the key role for all unified theories of weak, electromag netic and strong interactions.

Quantum Chromodynamics

Besides the hadron classification coming from the quark model, the experiments in high energy physics were most important for the quark model confirmation.

Experiments at SLAC (USA) in 1967 on the deep-inelastic lepton-nucleon scattering and experiments on the inclusive hadron-hadron reactions conducted in Protvino (Russia) in 1969, suggested [30] and theoretically studied by A.A. Logunov with collaborators [31], have discovered the scaling properties present in the quark-parton model: J.D. Bjorken [32], R.P. Feynman [33], C.N. Yeng [34].

The explanation of the observed scaling properties in electromagnetic, week and strong processes has been done in [35] on the basis of the universal principle of automodelity (self-similarity) and dimensional analysis. Combining the requirements of automodelity with the principles of the quasifree quark model, the quark counting formulae have been derived [36], [37]. These formulae allowed one to obtain the high energy asymptotic behavior for cross-sections for any binary reaction with participation of hadrons, and for hadron form factors at large momentum transfers

$$\frac{d\sigma}{dt}(ab \to cd) \to s^{-(n_a+n_b+n_c+n_d-2)}f(\frac{s}{t}),$$
$$F_i(t) \to t^{-(n_d-1)}.$$

where $n_i = (i = a, b, c, d)$ is the number of quark flavors inside the corresponding hadron.

On the other hand, experiments on the $\pi^0 \rightarrow 2\gamma$ decay and $e^+e^- \rightarrow -hadrons$ annihilation have finally confirmed the existence of three color degrees of freedom.

The experimental confirmation of the theoretical predictions of the colored quark model at the beginning of 70's did not leave any doubt in physical actuality of colored quarks and the rich content of the color symmetry.

However, the question about the nature of the interquark forces confining them i.ito hadrons was still open.

Already in the work by M. Han and Y. Nambu [17], it has been supposed that the interaction between quarks could be realized by the octet of vector mesons belonging to the adjoint representation of the SU(3), group. It has also been noted that it is possible to construct a colorless ground state for the particles which could be identified with the known hadrons. Seconingly, the formulation of QCD was quite an hand. However, that time the quantum theory of classical Yang-Mills fields did not exist. The problem of quantization for vector gauge fields was solved in the works by R.P. Feynman [38], B. De Witt [39] and L.D. Faddeev, V.N. Popoy [40].

Under the requirement that strong interactions are described by the Yang-Mills theory with the SU(3) -symmetry group and with quark fields arranged as fundamental triplets of this group, the Lagrangian for QCD takes the form [41]

$$L_{QCD} = -\frac{1}{2} tr G_{\mu\nu} G^{\mu\nu} + \sum_{k=1}^{n_f} \hat{q}_k (i\gamma^{\mu} D_{\mu} - m_k) q_k.$$

where

$$G_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} - ig[A_{\mu}, A_{\nu}],$$
$$D_{\mu}q_{k} = (\partial_{\mu} - igA_{\mu})q_{k}, \quad A_{\mu} = \sum_{a=1}^{8} A_{\mu}^{a} \frac{\lambda^{a}}{2}$$

Quantum Chromodynamics has aroused great interest at once. This theory possesses all known symmetries of strong interactions, is asymptotically free [42] and the complicated structure of its ground state leads to the spontaneous breakdown of chiral symmetry. QCD combines all ideas of different phenomenological approaches in the hadron theory and at present is the generally accepted theory of strong interactions.

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